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Editor-in-Chief's Message

International Journal of Industrial Electronics, Control and Optimization (IECO) Volume 9, No. 4, 2026

It is my pleasure to present Volume 9, Issue 4 (2026) of the International Journal of Industrial Electronics, Control and Optimization (IECO).

This issue brings together a diverse collection of research contributions addressing several important and emerging challenges in industrial electronics, control, optimization, and modern power systems. The published works demonstrate the continuing evolution of these fields, with particular emphasis on intelligent methods, advanced control strategies, renewable-energy integration, power-system analysis, and computational optimization.

A prominent theme in this issue is the growing integration of data-driven and computational intelligence techniques with established engineering methodologies. For example, advanced forecasting and optimization methods are employed to improve energy management in renewable-based microgrids, while intelligent algorithms are applied to enhance the performance, reliability, and adaptability of complex engineering systems. These contributions highlight how the combination of mathematical modeling, optimization, machine learning, and engineering expertise can provide effective solutions to increasingly complex industrial problems.

The papers included in this issue also reflect the broad scope of IECO, ranging from power electronic converters and renewable energy systems to intelligent diagnostics, control methodologies, antenna design, and power-system stability. Collectively, they demonstrate the importance of interdisciplinary approaches in addressing the technical challenges associated with the rapidly changing landscape of modern electrical and industrial systems.

IECO remains committed to promoting high-quality research and providing an international platform for the dissemination of innovative theoretical developments and practical engineering applications. We are dedicated to maintaining rigorous and transparent peer-review standards while encouraging meaningful scientific collaboration among researchers from different institutions and countries.

I would like to express my sincere appreciation to all authors for their valuable contributions, to the reviewers for their careful and constructive evaluations, and to the members of the Editorial Board and the editorial team for their continued dedication and support.

We hope that the articles presented in this issue will stimulate further research, discussion, and collaboration and will contribute to the advancement of industrial electronics, control, optimization, and related areas.

Prof. S. Masoud Barakati

Editor-in-Chief

International Journal of Industrial Electronics, Control and Optimization

About Journal

The University of Sistan and Baluchestan entered into strategic partnership with Iranian Association of Electrical and Electronic Engineers (IAEEE) to publish the **International Journal of Industrial Electronics Control and Optimization (IECO)**. The IECO is a refereed international journal which presents to the international scientific community important results of work in these fields, whether in the form of modeling simulation, analysis, fundamental research, development, application, design or real-time implementation. The scope of IECO is broad, encompassing all aspects of Industrial Electronics, Control and Optimization.

Note: International Journal of Industrial Electronics, Control and Optimization (IECO) has qualified to **ACADEMIC RESEARCH JOURNAL (ELMI-PAJOHESHI)** status certified by the ministry of Science, Research and Technology of Iran (No. 231566/3/18 dated 1396/10/09), and is published by the University of Sistan and Baluchestan through a formal partnership (No. 952/2/1500 dated 1395/11/04) with Iranian Association of Electrical and electronic Engineers (IAEEE) in order to develop scientific and research cooperation.

Aims and Scope

International Journal of Industrial Electronics, Control and Optimization (IECO) is a Peer reviewed journal of advanced and state-of-the-art in the science and engineering of Industrial Electronics, Control and Optimization. Its Scope encompasses the applications of Industrial Electronics, power systems, control, optimization and computational intelligence for the enhancement of industrial and manufacturing system and processes. The scope of the journal include the following:

I. Industrial Electronics

- Low and high-power converters
- Renewable energy

- Drive control techniques
- Techniques for advanced power semiconductor devices
- Power quality and utility applications
- Communications
- Flexible AC Transmission Systems (FACTS)
- Control in power electronics
- Electromagnetic and thermal performance of electronic power converters
- Motion control, robotics, sensors and actuators
- Fault detection and diagnosis
- Power systems
- Factory automation, communication, and computer networks

II. Control

- Adaptive control
- Control of process systems
- Control theory
- Data processing
- Design of control systems
- Hybrid systems
- Identification and observation
- Intelligent systems
- Model-predictive control
- Optimal control
- Robust control
- Fractional order systems

III. Optimization

- Ant Colony
- Chaos Theory
- Evolutionary Computing
- Fuzzy Computing
- Hybrid Methods
- Immunological Computing
- Neuro Computing
- Particle Swarm
- Probabilistic Computing
- Rough Sets
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A Study on Optimal Energy Management of a Renewable-Based Microgrid: A Model Predictive Control Approach

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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 08-December-2025 Received in revised form: 03-February-2026 Accepted: 25-February-2026 Published online: 22-Dec-2026 Keywords: Energy Management, Energy Storage Systems, Microgrid, Model Predictive Control, Quadratic Interpolation Optimization.	Effective energy management in renewable-based microgrids demands advanced optimization and forecasting strategies capable of addressing the intrinsic uncertainties of solar generation and load demand. This paper proposes a comprehensive day-ahead scheduling framework based on Model Predictive Control (MPC) to enhance the operational efficiency of a grid-connected microgrid comprising photovoltaic units, microturbines, fuel cells, and battery energy storage. The MPC structure incorporates 24-hour-ahead forecasts of load and solar irradiance generated through multiple deep learning architectures, enabling dynamic adaptation to rapidly changing environmental and consumption conditions. To solve the underlying optimization problem, a novel Quadratic Interpolation Optimization (QIO) algorithm is employed, offering improved convergence behavior and robustness in comparison with conventional metaheuristic methods. The integrated forecasting–optimization methodology ensures optimal dispatch of distributed resources, effective utilization of storage systems, and economically efficient power exchange with the utility grid. Extensive simulations validate the superiority of the proposed MPC-QIO framework in reducing operational costs, enhancing renewable energy penetration, and maintaining system reliability even under forecasting uncertainties. The results confirm that the synergy between deep learning-based prediction and advanced optimization significantly strengthens the controllability and resilience of modern microgrids, highlighting the method's strong potential for real-world deployment.

I. Introduction

The proliferation of Distributed Energy Resources (DERs) has positioned microgrids as a cornerstone of modern, decentralized power systems, offering a pathway to enhanced resilience, cost efficiency, and localized energy control [1-3]. These systems are instrumental in harnessing Renewable Energy Sources (RES) like solar and wind power, thereby accelerating the global transition to a sustainable energy paradigm [4-6]. By decentralizing electricity generation and alleviating strain on traditional transmission networks, microgrids present a viable solution to pressing issues such as energy poverty, transmission congestion, and volatile electricity pricing [7]. Nevertheless, the integration of RES introduces significant operational complexities. The innate intermittency and stochastic nature of solar irradiance and wind speed pose substantial threats to the power balance and operational reliability of microgrids. To counteract these

challenges, Energy Storage Systems (ESS) are increasingly employed to mitigate power fluctuations, enhance energy time-shifting, and guarantee a reliable power supply. Concurrently, sophisticated Energy Management (EM) strategies now play a pivotal role in achieving optimal, safe, and economical microgrid operation under uncertainty [8, 9].

To address this challenge, researchers have recently developed efficient methodologies for solving the EM problem in microgrids. For example, reference [10] proposes a hybrid dynamic demand response program using the Pelican Optimization Algorithm for microgrid EM, effectively reducing peak demand and operating costs while maintaining system reliability. Additionally, reference [11] proposes a multi-layer optimization framework for integrated energy systems management. The first layer performs single-objective optimization to minimize operational costs, while the second layer employs multi-objective optimization to

simultaneously reduce environmental emissions and groundwater extraction. The third layer enhances system independence from the main grid through technical improvements in power loss reduction and voltage profile optimization. Expanding on this, Reference [12] introduces a hybrid optimization framework that merges metaheuristic algorithms with conventional power flow analysis for microgrid economic dispatch. The study implements a comprehensive parameter optimization strategy to evaluate five metaheuristic algorithms, identifying the most effective one for minimizing leveled energy costs. Reference [13] proposes an intelligent Golden Jackal Optimization algorithm to address distributed-generation EM in battery storage systems and hybrid energy sources. The method aims to minimize operating costs while considering constraints such as power balance, generation capacity, consumer loads, and energy storage dynamics. Additionally, reference [14] develops a quantum-enhanced particle swarm optimization approach for optimal microgrid EM, focusing on minimizing operational costs and emissions. The study formulates a deterministic scheduling model and applies QPSO to improve search performance in multi-objective dispatch.

Existing metaheuristic optimization approaches can provide acceptable solutions for day-ahead microgrid scheduling under deterministic conditions. However, modelling uncertainties in solar irradiance and load profiles using predefined scenarios or probabilistic assumptions does not offer a fully reliable representation of real-world behavior, as these approaches often fail to capture the stochastic and time-varying nature of distributed energy resources. To overcome these limitations, MPC has emerged as an effective solution, as it can learn the behavioral patterns of historical data and use this knowledge to predict future system behavior. Consequently, MPC provides a more robust and accurate framework for EM in microgrids, improving both operational efficiency and reliability under uncertain conditions [15].

For the aforementioned reasons, researchers have gradually turned to using MPC for microgrid EM. For instance, the authors in [16] apply MPC in an AC/DC microgrid as a solution for EM. The approach formulates a receding-horizon optimization problem that considers short-term predictions of load and renewable generation to schedule storage and load operations in real time. MPC explicitly handles system constraints and updates control actions dynamically to adapt to changing conditions. Furthermore, reference [17] implements a two-level hierarchical MPC for EM in a microgrid, where the high level performs long-term economic optimization and the low level carries out short-term economic optimization to manage storage and load operations efficiently. Additionally, reference [18] proposes an MPC-based EMS for an isolated electro-thermal microgrid, aiming to minimize operating costs, reduce battery degradation, and ensure reliable energy supply. The approach incorporates an estimated State of Health model for the ESS to extend its

lifespan, while MPC optimizes the scheduling of PV, diesel generator, battery, and thermal loads. Reference [19] presents an MPC-based EMS for cooperative optimization of interconnected grid-connected microgrids. The approach formulates scheduling and power exchange among distributed energy resources as a mixed-integer quadratically constrained programming problem, using forecasted inputs to optimize operations across multiple microgrids efficiently. In addition, the authors in [20] develop a hybrid constrained Particle Swarm Optimization–MPC algorithm for storage EM in a microgrid. The method formulates the operational scheduling problem as a constrained optimization, and uses a modified PSO, with constraint handling via Deb's rule, embedded in MPC to produce feasible, cost-effective control decisions. In a related study, the authors in [21] formulate and solve the optimization problem using quadratic programming, integrating a control penalty mechanism for battery and PEV state-of-charge (SoC) management and employing Monte Carlo simulation with scenario reduction to handle renewable generation uncertainties within an MPC framework. The authors in [22] formulate a MILP-based MPC optimization for islanded microgrids, integrating dynamic load shedding with time-of-use pricing and solving it via the CPLEX solver.

Although the aforementioned MPC and hybrid-based EMS approaches effectively mitigate some of the limitations in previous studies, they generally do not investigate different deep learning techniques for the prediction module and often rely on a single forecasting method. In many cases, the prediction model is treated as a black box, without providing explicit details, or when its structure is specified, no comparison with alternative predictive models is performed. Moreover, in the EM optimization stage, many studies either employ outdated linear programming formulations or fail to introduce novel optimization strategies, and seldom benchmark their results against standard or state-of-the-art solvers.

Additionally, a significant challenge and limitation of optimization algorithms, particularly conventional methods, lies in their susceptibility to converging to local minima. This phenomenon hinders the ability to identify the globally optimal solution, as the algorithm becomes trapped within a suboptimal configuration, preventing further improvement.

To address the limitations of previous EMS studies, this work integrates a comparative evaluation of multiple deep learning architectures to generate 24-hour-ahead forecasts for both load and solar irradiance generation within the MPC framework. For EM optimization, a novel Quadratic Interpolation Optimization (QIO) algorithm is employed, providing enhanced convergence, robustness, and solution quality compared with conventional metaheuristics. Comparative simulations against PSO and GA indicate that QIO consistently achieves lower operational costs and more stable scheduling outcomes, highlighting the effectiveness of combining deep learning-based forecasting with an advanced

optimization engine for reliable and economical microgrid operation.

The contributions of this study are detailed as follows:

1. The research systematically compares multiple deep learning architectures for 24-hour ahead forecasting within an MPC framework.
2. A novel Optimization algorithm is developed for day-ahead microgrid scheduling, offering enhanced convergence speed and solution quality relative to conventional metaheuristics.
3. The proposed integrated framework combining deep learning forecasts with MPC-QIO is benchmarked against PSO and GA, evidencing substantial reductions in operational costs and improved reliability of microgrid operations.
4. Battery health is explicitly managed through the enforcement of upper and lower SoC limits, mitigating the risks of overcharging and deep discharge and thereby extending battery lifespan.

The remaining sections of this paper are structured as follows: Section 2 presents the detailed modelling of the microgrid components and their operational constraints. Section 3 introduces the MPC framework along with the training of the deep learning-based forecasting models. Section 4 describes the proposed EM System integrated with the QIO algorithm. Section 5 reports the simulation results, performance comparisons, and discussion of the EMS under different scenarios. Finally, Section 6 concludes the study and highlights potential directions for future research.

II. Modelling of microgrid

Microgrids are localized power grids that offer enhanced reliability, resilience, and potential cost savings compared to traditional grid systems. This paper investigates a grid-connected microgrid designed to provide a stable and economical power supply to a local community. The microgrid under investigation operates in grid-connected mode and incorporates a photovoltaic (PV) system, battery energy storage (BESS), micro turbine (MT), and fuel cell (FC) as primary energy resources. This configuration enables the system to satisfy local electricity demand through an optimal combination of renewable generation and dispatchable units, while maintaining the capability to exchange power with the main grid to enhance operational flexibility and economic performance. The schematic diagram of the proposed microgrid architecture is presented in Fig 1.

A. Objective function

This paper formulates a day-ahead EM model for the microgrid with the primary objective of minimizing total operational costs under normal operating conditions. The optimization problem is mathematically expressed through the following objective function:

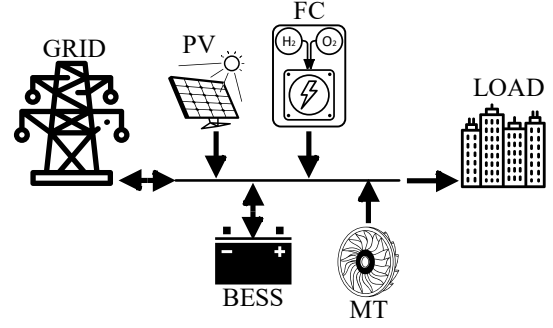


Fig. 1. diagram of the grid-connected microgrid configuration.

$$\min (\sum_{t=1}^{N_t} Cost_t^{BESS} + \sum_{t=1}^{N_t} Cost_t^{FC} + \sum_{t=1}^{N_t} cost_t^{MT} + \sum_{t=1}^{N_t} Cost_t^{PV} + \sum_{t=1}^{N_t} Cost_t^{GRID}) \quad (1)$$

The objective function minimizes the total operational cost of the microgrid over the 24-hour scheduling horizon. It includes the costs associated with energy supply from the BESS($Cost_t^{BESS}$), FC($Cost_t^{FC}$), MT($cost_t^{MT}$) and PV($Cost_t^{PV}$), along with the costs of power exchange with the main grid($Cost_t^{GRID}$). The complete formulation of each cost component is provided below:

$$Cost_t^{BESS} = \lambda^{BESS} \times P_t^{BESS} \quad (2)$$

$$Cost_t^{FC} = \lambda^{FC} \times P_t^{FC} \quad (3)$$

$$Cost_t^{MT} = \lambda^{MT} \times P_t^{MT} \quad (4)$$

$$Cost_{t,s}^{PV} = \lambda^{PV} \times P_{t,s}^{PV} \quad (5)$$

$$Cost_{t,s}^{GRID} = Market\ Price_t \times P_t^{GRID} \quad (6)$$

The operation and maintenance costs (λ) for each energy source are adopted from [23], while market price data are obtained from [24].

B. Problem Constraints

To ensure reliable microgrid operation, the power balance constraint must be strictly maintained at each time step. This fundamental requirement, expressed in Equation (7), guarantees that the total power generation from all sources exactly matches the total load demand, accounting for both local consumption and power exchange with the main grid [24].

$$P_t^{LOAD} = P_t^{GRID} + P_t^{BESS} + P_t^{FC} + P_t^{MT} + P_t^{PV} \quad (7)$$

The operational constraints for each distributed generation unit are defined by their minimum and maximum output limits, as specified in Equations (8) through (12). These technical constraints ensure that all components operate within their safe operational boundaries while maintaining system reliability and efficiency [24].

$$P_{Min}^{GRID} \leq P_t^{GRID} \leq P_{Max}^{GRID} \quad (8)$$

$$P_{Min}^{BESS} \leq P_t^{BESS} \leq P_{Max}^{BESS} \quad (9)$$

$$P_{Min}^{FC} \leq P_t^{FC} \leq P_{Max}^{FC} \quad (10)$$

$$P_{Min}^{MT} \leq P_t^{MT} \leq P_{Max}^{MT} \quad (11)$$

$$P_{Min}^{PV} \leq P_t^{PV} \leq P_{Max}^{PV} \quad (12)$$

Furthermore, to maintain battery health and ensure operational reliability, the SoC is constrained within specified minimum and maximum thresholds. These limits prevent overcharging and deep discharging, thereby extending the battery's lifespan and maintaining its performance throughout the scheduling horizon [2].

$$SoC_{Min}^{BESS} \leq SoC_t^{BESS} \leq SoC_{Max}^{BESS} \quad (13)$$

The proposed optimization framework ensures strict adherence to the equality and inequality constraints outlined in Equations (8-12) throughout the solution process. To further reinforce compliance with the SoC limitations specified in Equation (13) and maintain grid interconnection standards, a penalty-based mechanism is implemented. This approach assigns additional cost components for constraint violations, effectively steering the optimization toward feasible and physically realizable operating conditions while preserving system integrity.

C. Modelling of PV

The photovoltaic system's power generation is determined by incident solar irradiance, ambient temperature conditions, and the panel's rated characteristics. The relationship between these parameters and the resulting electrical output is mathematically represented as follows [25]:

$$P_{PV}(S_i(t)) = PV^{array} \cdot Df^{PV} \cdot \left(\frac{S_i(t)}{S_{i,STC}} \right) \quad (14)$$

$$[1 + \alpha_p(T_c - T_{c,STC})]$$

where $S_i(t)$ represents the solar irradiance at time t , $S_{i,STC}$ denotes the irradiance under standard test conditions, T_c and $T_{c,STC}$ are the cell temperature and its reference value under standard conditions, respectively. PV^{array} indicates the rated capacity of the PV system, Df^{PV} is the derating factor accounting for losses, and α_p is the temperature coefficient of power.

The power outputs of the BESS, FC, and MT are directly derived from the proposed algorithm's solution, and are constrained within pre-defined operational ranges. The algorithm effectively coordinates the power generation of these distributed energy resources to simultaneously satisfy the load demand, minimize grid energy procurement costs, and, when feasible, inject surplus power into the grid during periods of peak pricing. Crucially, the proposed methodology ensures that these achievements are consistently realized while rigorously satisfying all defined constraints.

III. Model predictive control

This research implements a Model Predictive Control (MPC) framework to effectively manage uncertainties inherent in

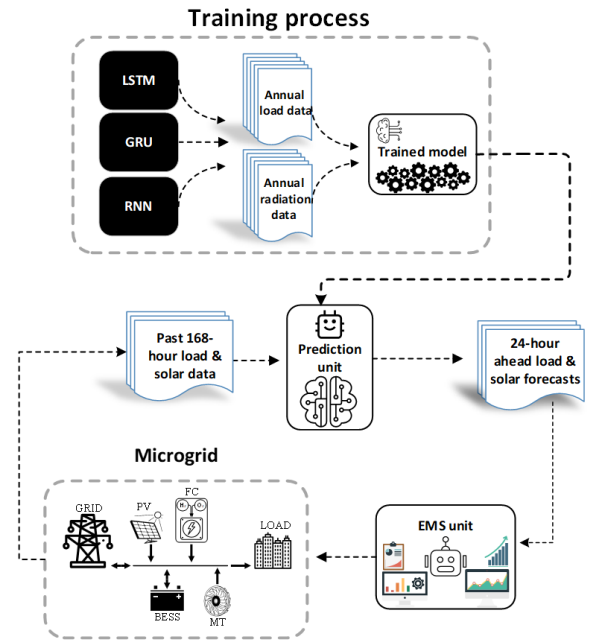


Fig. 2. Schematic of the predictive control methodology.

load demand and solar irradiance within microgrid operations. The proposed MPC methodology adopts a rolling horizon strategy that integrates real-time forecast updates with dynamic optimization of control actions. A comprehensive historical dataset, encompassing one full year of recorded load consumption and solar irradiance measurements, serves as the foundation for developing the prediction models. This extensive data collection captures critical seasonal fluctuations, recurring daily patterns, and anomalous operational scenarios, thereby establishing a robust basis for constructing reliable forecasting models.

The predictive component of the MPC architecture employs a sliding window mechanism that processes historical observations over a 168-hour window (equivalent to seven days), formulated as:

$$X_t = (x_{t-167}, x_{t-166}, \dots, x_t) \quad (15)$$

to generate 24-hour-ahead forecasts for both electrical load and solar irradiance. The forecasting process is mathematically represented as:

$$f_{\theta}(X_t) = \hat{Y}_{t+1:t+24} \quad (16)$$

where f_{θ} denotes the trained forecasting model with parameters θ , mapping the historical input sequence to future predictions. This architectural design facilitates the identification of weekly cyclical patterns while ensuring computational tractability. The forecasting engine transforms historical observation sequences into future projections through sophisticated deep learning structures, with these predictions subsequently informing the optimization module responsible for day-ahead scheduling decisions. The receding horizon implementation guarantees continuous controller adaptation to evolving operational conditions through the

integration of the most recent measurements at each optimization interval.

A. Comparative Analysis of Deep Learning Forecasting Architectures

A systematic evaluation of three advanced deep learning architectures, Long Short-Term Memory (LSTM), Recurrent Neural Network (RNN), and Gated Recurrent Unit (GRU), was conducted to determine the optimal forecasting methodology for integration within the MPC framework. To ensure equitable comparison, all models were standardized with identical input configurations, processing 168 historical time steps to generate 24-step-ahead forecasts, thereby maintaining consistency in architectural constraints and learning capabilities across the experimental framework. The overall flowchart of the proposed methodology, which integrates the forecasting process and optimization framework, is illustrated in Fig 2.

IV. Energy management system

The Energy Management System (EMS) constitutes the decision-making core of the proposed framework, where the 24-hour-ahead load and solar irradiance forecasts, generated by the deep learning models in the preceding stage, are transformed into optimal operational schedules for all microgrid assets. These forecasted profiles provide the temporal structure within which the EMS determines the charging-discharging strategy of the BESS, the dispatch levels of distributed generation units, and the interaction with the utility grid. Given the inherently nonlinear and constrained nature of microgrid scheduling, the performance of the EMS critically depends on the selection of a robust and computationally efficient optimization algorithm capable of navigating a highly complex search space. To address these challenges, this study adopts the Quadratic Interpolation Optimization (QIO) algorithm as the core optimization engine. QIO leverages the mathematical rigor of the Generalized Quadratic Interpolation mechanism to achieve an effective balance between global exploration and local exploitation, thus preventing premature convergence and improving solution quality compared with conventional metaheuristics. Owing to its demonstrated superiority in solving engineering optimization problems, the QIO algorithm is introduced here as the recommended optimization method. A detailed description of the algorithmic structure and its integration within the EMS is presented in the subsequent section.

A. Quadratic Interpolation Optimization

The QIO algorithm is a mathematically-driven metaheuristic that builds its search strategy upon the concept of Generalized Quadratic Interpolation (GQI). The fundamental idea is that, instead of randomly exploring the

search space or relying solely on evolutionary operators, QIO continuously constructs local quadratic models using triplets of candidate solutions. For any three sampled points in the decision space, the algorithm fits a quadratic curve that approximates the underlying cost function in that region. The estimated minimizer of this quadratic model is then used as a directional indicator to generate new candidate solutions.

Unlike classical quadratic interpolation, which only works properly when the three points satisfy specific geometric orderings, GQI expands the method to all possible point configurations. The algorithm systematically analyzes every case in which three points may be located relative to each other and reconstructs the quadratic model in a way that always produces a valid minimizer. This guarantees that the search direction is meaningful, even in irregular or high-dimensional landscapes where traditional interpolation fails or produces a maximizer instead of a minimizer.

QIO incorporates these interpolated minimizers into two complementary search modes:

Exploration Phase (Global Search):

Random individuals are selected from the population to generate interpolation-based candidate points. The algorithm uses the minimizer obtained from the GQI curve to jump into unexplored regions of the search space. When QIO performs exploration, the position of the current individual is updated as:

$$v_i(t+1) = x_{i,rand1,rand2}^*(t) + \omega_1 \cdot (x_{rand3}(t) - x_{i,rand1,rand2}^*(t)) + round(0.5 \cdot (0.05 + r_1)) \cdot \log\left(\frac{r_2}{r_3}\right) \quad (17)$$

$$x_{i,rand1,rand2}^*(t) = GQI(x_i(t), x_{rand1}(t), x_{rand2}(t), fit(x_i(t)), fit(x_{rand1}(t)), fit(x_{rand2}(t))) \quad (18)$$

where x_{rand1} , x_{rand2} , and x_{rand3} represent the positions of randomly chosen individuals from the current population, r_1 , r_2 , and r_3 are random numbers between 0 and 1, $fit()$ denotes the fitness function value and $GQI()$ is the GQI function. Weight ω_1 , which can be modified using an adaptive coefficient b , is expressed as follows:

$$\omega_1 = 3n_1b \quad (19)$$

$$b = 0.7a + 0.15a \left(\cos\left(\frac{5\pi t}{T}\right) + 1 \right) \quad (20)$$

$$a = \cos\left(\frac{t\pi}{2T}\right) \quad (21)$$

where n_1 follows the standard normal distribution, t is the number of current iteration, and T is the maximum number of iterations. This prevents the algorithm from getting trapped too early in local minimum.

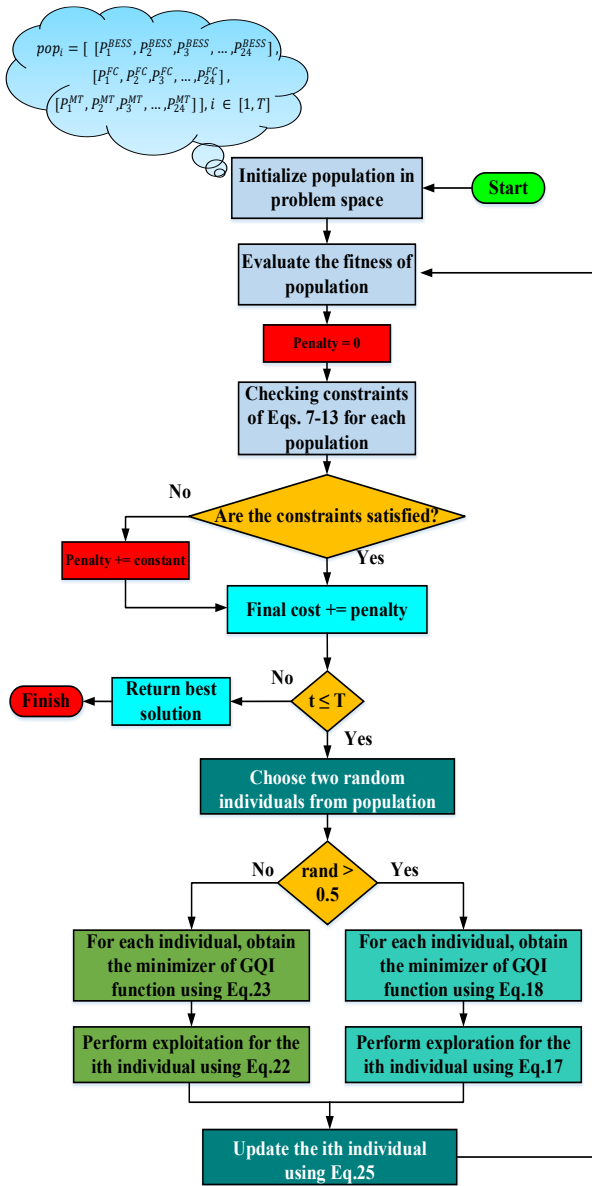


Fig. 3. flowchart detailing the implementation of the proposed method.

Exploitation Phase (Local Refinement):

The best individual found so far is combined with other population members to construct a new quadratic interpolation model. The resulting minimizer leads the search toward areas close to the current optimum, enhancing convergence speed and precision. When QIO performs exploitation, the position of the current individual is updated as:

$$v_i(t+1) = x_{best,rand1,rand2}^*(t) + \omega_2 \cdot (x_{best}(t) - \text{round}(1 + rand) \cdot \frac{ub - lb}{ub^{rD} - lb^{rD}} \cdot x_i^{rD}(t)) \quad (22)$$

$$x_{best,rand1,rand2}^*(t) = GQI(x_{best}(t), x_{rand1}(t), x_{rand2}(t), fit(x_{best}(t)), fit(x_{rand1}(t)), fit(x_{rand2}(t))) \quad (23)$$

$$\omega_2 = 3 \left(1 - \frac{t-1}{T}\right) n_2 \quad (24)$$

where n_2 follows a standard normal distribution and rD is a random integer within the range $[1, d]$, lb^{rD} and ub^{rD} represent the lower and upper boundaries at the rD th dimension. ω_2 serves as the exploitation weight, functioning as an adaptive coefficient.

According to Eq. (23), the best individual found so far (x_{best}) and two randomly selected individuals (x_{rand1} and x_{rand2}) are utilized within the GQI method to generate a minimizer ($x_{best,rand1,rand2}^*$), which typically outperforms the original three individuals. Based on Eq. (22), the i th individual is updated by exploring the neighborhood of $x_{best,rand1,rand2}^*$ relative to x_{best} , incorporating a random perturbation from x_i^{rD} . This search facilitates efficient exploitation and prevents stagnation within the algorithm.

After the exploration and the exploitation strategies are achieved, the position of the i th individual is updated according to the following equations:

$$x_i(t+1) = \begin{cases} x_i(t), & fit(x_i(t)) \leq fit(v_i(t+1)) \\ v_i(t+1), & fit(x_i(t)) > fit(v_i(t+1)) \end{cases} \quad (25)$$

If the fitness value at the candidate position of the i th individual is superior to its current fitness, the individual's position is updated to the candidate position. Otherwise, the individual's position remains unchanged.

Through alternating these two mechanisms, QIO adaptively adjusts between exploration and exploitation without requiring gradient information, penalty parameters, or complex control coefficients. The method benefits from the accuracy of local quadratic models while maintaining the stochastic diversity of metaheuristic search. As a result, QIO exhibits strong performance across nonlinear, constrained, and multi-modal optimization problems, including microgrid energy scheduling.

Given the considerable mathematical complexity of the full GQI derivations, spanning multiple interpolation cases and detailed analytical constructions, the present paper focuses on describing the conceptual workflow of QIO rather than reproducing its extensive formulation. For readers interested in the complete mathematical development, the full derivations are comprehensively presented in reference [26]. Fig 3 illustrates the complete flowchart detailing the implementation of the proposed method, including the QIO algorithm, as well as how the problem is modeled and constraints are considered.

V. Simulation and Results

To evaluate the performance of the proposed methodology, comprehensive simulations were conducted on a grid-connected microgrid configuration, illustrated in Fig. 1. All numerical experiments were developed using Python and

executed on a desktop computer equipped with an Intel Core i5 processor (1.60 GHz) and 16 GB of RAM. The technical specifications and rated parameters of the PV subsystem are reported in Table 1, while Table 2 summarizes the operational constraints and bidding prices associated with the distributed generation units participating in the scheduling process.

The historical data utilized in this study was obtained from the Renewable Ninja website, focusing on Denmark for the year 2022. To ensure a fair and unbiased comparison among the three forecasting architectures, LSTM, RNN, and GRU, all models were trained under identical experimental settings and hyperparameter configurations. Each network received the same input structure, was optimized using the same learning strategy, and was trained for an equal number of epochs to eliminate any performance variations arising from unequal training conditions. The complete set of training hyperparameters employed for all three models is summarized in Table 3.

TABLE I Numerical parameter values of the PV system

Variable	value
PV_{array}	15 kW
Df^{PV}	80 %
$S_{i,STC}$	1 kW/m ²
α_p	-0.5 %/°C
T_c	60 °C
$T_{c,STC}$	25 °C

TABLE II Data of DG units and main grid

Type of Source	Min power (kW)	Max power (kW)	Bid (€/ct/kWh)
PV	0	15	2.584
MT	6	30	0.457
FC	3	30	0.294
BESS	-30	30	0.380
Main Grid	-30	30	-----

As presented in Table 4, LSTM consistently provides the most accurate and stable forecasts for both solar irradiance and load demand, followed closely by GRU, while RNN shows comparatively lower precision, particularly for the more variable solar irradiance data. This comparison clearly demonstrates the relative effectiveness of the LSTM in capturing temporal patterns within the datasets. Therefore, the forecasts produced by the LSTM model are fed into the EMS, which utilizes the QIO algorithm to perform day-ahead scheduling. This integration ensures that the optimization algorithm has accurate and timely predictions, allowing for more efficient planning and reliable operation of the microgrid.

Figs 4&5 present the outcomes of the QIO algorithm for scenario 1 and 2 respectively. These figs show that the FC and MT units, driven by their competitive bid prices and high economic efficiency, frequently operate at or near their maximum output capacities. the results demonstrate that the energy resources are dispatched efficiently to meet the

demand while optimizing operational costs. Priority is given to units with higher economic efficiency, reducing reliance on the main grid during peak pricing periods. Moreover, any surplus generation is exported to the grid, providing additional revenue and enhancing the overall economic performance of the microgrid.

A critical aspect of battery EM is maintaining the SoC within defined boundaries to ensure safe and efficient operation. Fig 6 depicts the SoC evolution over the 48-hour scheduling horizon. As illustrated, the battery strategically charges during periods of low electricity prices (00:00-08:00) to ensure adequate SoC for subsequent discharge during peak price hours (09:00-16:00), thereby minimizing reliance on the grid. Furthermore, the battery demonstrates effective responsiveness to price fluctuations, as evidenced by its reaction to the price surge around 21:00. Starting from an initial SoC of 20%, the proposed algorithm consistently regulates the battery state within the permissible range across both days, supporting reliable performance and longevity.

TABLE III Training hyperparameters

Parameter	Value
Optimizer	Adam
Learning rate	0.002
Batch size	128
Epochs	50
Loss function	Mean Squared Error

TABLE IV Comparison of Forecasting Performance of Algorithms

Algorithm m	Dataset	R ²	RMSE	MAE
LSTM	Solar Irradiance	0.99	15.06	11.09
GRU		0.98	17.22	12.64
RNN		0.97	25.74	17.35
LSTM	Load Demand	0.988	1.94	1.56
GRU		0.984	2.23	1.82
RNN		0.97	2.56	1.97

Table 5 presents a comparison of the objective function values achieved by the tested algorithms over two consecutive days. The results indicate that the QIO algorithm consistently obtains superior solutions, with lower mean and worst-case values, as well as reduced variability compared to other methods. This demonstrates the effectiveness and stability of QIO in optimizing the system's performance across different daily scenarios.

To enable a clearer comparison, convergence curves for both scenarios are presented in Figs 7 and 8. These figs illustrate the progression of the algorithms toward optimal solutions across iterations in each scenario. By examining the convergence behavior, the relative speed, stability, and consistency of the methods can be more accurately assessed.

As illustrated in Fig 9, the costs achieved by each of the three algorithms are presented to compare their efficiency in reducing microgrid operational costs.

TABLE V Objective Function Comparison for the Tested Algorithms

Scenario	Algorithm	Best	Mean	Worst	SD
Day 1	QIO	57.19	65.33	77.36	5.17
	PSO	81.69	205.76	424.22	98.07
	GA	125.58	151.48	166.95	13.57
Day 2	QIO	81.04	88.47	97.17	5.04
	PSO	160.29	285.07	477.91	99.20
	GA	172.81	194.22	229.89	15.30

QIO results for Scenario 1.

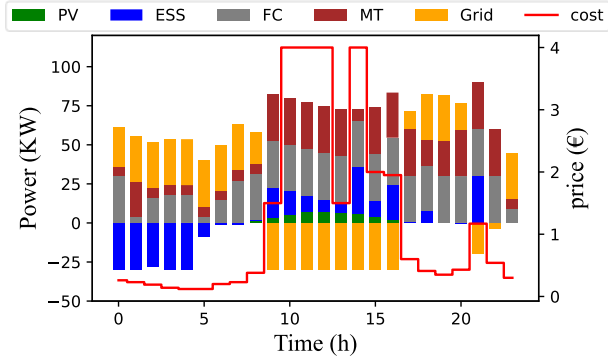


Fig. 4. QIO results for Scenario 2.

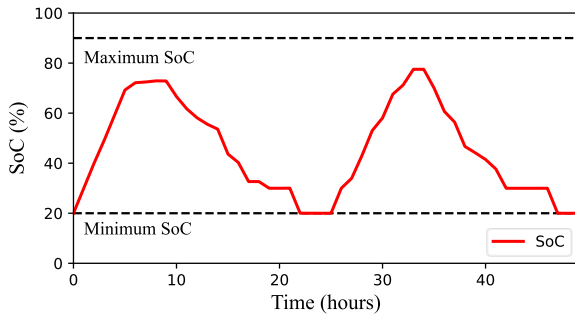


Fig. 5. SoC profile of test days.

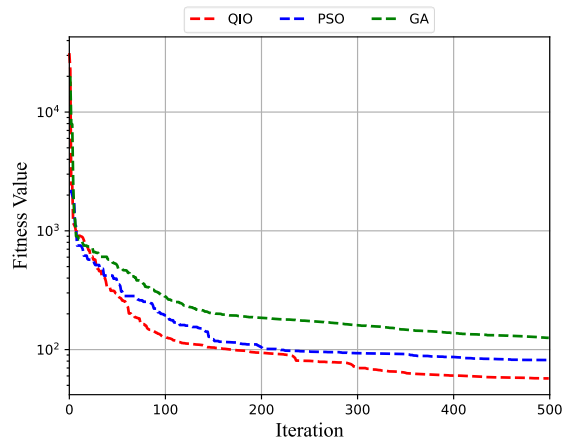


Fig. 6. Convergence curves of the algorithms for Scenario 1.

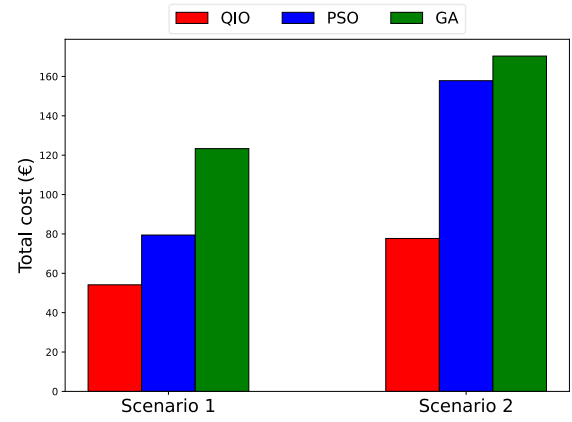


Fig. 7. Convergence curves of the algorithms for Scenario 2.

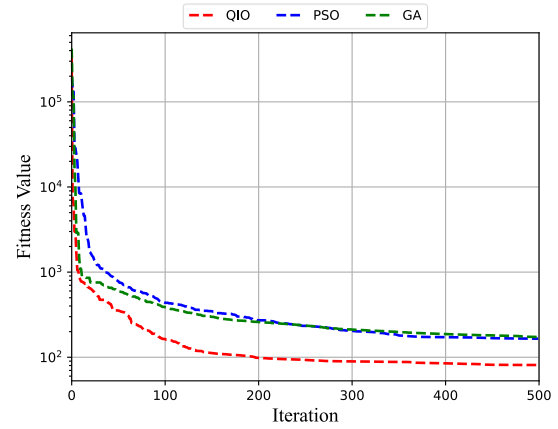


Fig. 8. Operational cost reductions achieved by algorithms.

The results demonstrate that the proposed algorithm reduced costs by 1.78% compared to the PSO and 4.88% compared to the GA in the first scenario. Furthermore, in the second scenario, this algorithm achieved a greater reduction in overall microgrid costs, decreasing them by 5.45% and 6.30% compared to PSO and GA, respectively.

VI. Conclusion

This paper has presented a comprehensive MPC framework for the optimal EM of a grid-connected, renewable-based microgrid. The primary objective was to minimize total operational costs while effectively addressing the inherent uncertainties of photovoltaic generation and load demand. The proposed strategy leverages a rolling-horizon approach, integrating real-time updated forecasts from a deep learning model to solve the day-ahead scheduling problem.

Simulation results demonstrate the efficacy of the MPC-based approach in significantly reducing operational costs and enhancing the utilization of renewable energy compared to traditional methods. Notably, the proposed MPC method achieved a forecast accuracy of 99% for predicting the next day's solar irradiance and load demand. Furthermore, numerical results indicate that the proposed QIO algorithm achieved cost reductions of 1.78% and 4.88% in the first

scenario, and 5.45% and 6.30% in the second scenario compared to PSO and GA respectively, demonstrating a significant improvement in overall microgrid operational costs. The framework's inherent robustness to forecasting errors confirms its practical applicability for ensuring reliable and economically efficient microgrid operation.

Future work will focus on integrating additional renewable energy sources, evaluating the model's performance within a multi-microgrid environment, and incorporating electric vehicles with vehicle-to-grid capability.

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Study on Multi-Phase Boost Converters for Fuel Cell Application: Power Density and Reliability Analysis

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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 04-August-2025 Received in revised form: 06-December-2025 Accepted: 28-December-2025 Published online: 22-Dec-2026 Keywords: Boost Converter, Multi-phase, Power Density, Reliability.	This paper presents a new comprehensive study on multi-phase interleaved DC-DC boost converters for fuel cell power interface. High power density DC-DC converters are usually required due to the volumetric and gravimetric constraints in the various fuel cell power system applications. On the other hand, in sensitive applications such as vehicular systems, using a reliable converter is a critical feature. Therefore, different multi-phase configurations with various phase numbers are investigated to find the configuration with the highest power density and reliability. Boost converters with single, two, and four-phase configurations are designed with considerably low input current and output voltage ripples compatible with the fuel cell application. Their performance parameters (especially their power densities) are investigated using the analytical and simulation results. To discuss their reliability, a reliability analysis based on a mathematical model and a Markov diagram is employed to calculate the converters' expected load loss values. The novelty of this paper can be found in optimizing the aforementioned goal function by choosing the best multi-phase structure. In the end, among the considered configurations, the two-phase boost converter is selected to be used in fuel cell applications due to its superior performance in terms of power density and reliability. The selected boost configuration is implemented to extract the experimental results and verify the accuracy of the calculation and simulation results.

I. Introduction

Nowadays, the fossil fuel system is dwindling due to excessive utilization and burning of fossil fuels in the vehicular application, which leads to the emission of greenhouse gases [1], [2]. The fuel cell usually has a current source characteristic with low output voltage and high output current [3]. Therefore, to solve the mismatched problem of the voltage levels between the dynamic lower voltage of the fuel cell stack and the required constant higher voltage of the DC link, a DC-DC step-up converter should be used as the power electronic interface [4]-[6].

The boost converter is considered the most widely used converter in the fuel cell application [7]. Because this converter has three main features, including simple structure, high voltage gain (even without using transformers), and low

input current ripple [8]. The research works on DC-DC boost converter in the fuel cell applications try to improve the converter performance in two different aspects: a) presenting an efficient converter model and control strategy [9]-[12] and b) improving the basis of the boost converter. Some research works in the second category are focused on enhancing the converter voltage gain using different methods such as cascading multiple basic boost converter cells [13], introducing the voltage multiplier [14], using switched capacitors [15], and series connection of the two modules at the output side [16]. Also, using the floating interleaved boost topology is a typical approach [17].

Using the interleaved multi-phase boost converters (with parallel connection of the modules) has some advantages, such as lower input current ripple, lower components' current

stress, and more output power availability in case of fault occurrence [18]-[20]. These features make multi-phase interleaved boost converters appropriate in the fuel cell application because of the usually high current amplitude and low tolerable current ripple of the fuel cell. Therefore, in this paper, the multi-phase boost converters are used as a power interface between the fuel cell and the load.

Choosing the most suitable multi-phase configuration in the fuel cell application is a challenge studied in this paper. Choosing a converter topology with high power density is a critical goal in some applications, such as fuel cell hybrid vehicles. On the other hand, in some special applications such as vehicular systems, the selected topology should have high reliability to increase the converter's fault-tolerant capability [21]. Commonly, increasing the converter power density results in lower converter reliability and vice versa. Therefore, to achieve both high power density and reliability for the converter, the approach should be carefully implemented.

Many research efforts have been made on the reliability analysis of the DC-DC converter [22]-[24]. The reliability analysis starts with extracting the components' failure rate at different operating conditions. Then a proper reliability model could be used to make it clear how the components' failure rate is related to the converter reliability [25]-[27]. Using an analytical tool to illustrate the system states in terms of reliability, such as the Markov diagram, is a useful approach to calculating the converter reliability from the obtained model [28]-[30].

In this paper, single-phase, two-phase, and four-phase interleaved boost converters, which are three common interleaved structures, are studied for the fuel cell application. The converters are designed to have a very low current ripple that is compatible with the considered fuel cell. The new study aims to choose the multi-phase configuration with the highest power density and reliability. For this purpose, the converter components' volume and total expected load loss (ELL) are calculated analytically. The reliability analysis to compute ELL has been done using a proposed reliability model and a Markov diagram for the converters. To the knowledge of the authors, analysing the power density and reliability of these structures in the same time for finding the optimum structure with maximum goal function is not presented in the literature. This paper does not aim to present novel modeling or design methods, it should find the optimum structure with the highest reliability and power density. The novelty of this paper can be found in optimizing the aforementioned goal function by choosing the best multi-phase structure. The multi-phase converter with the highest power density and reliability is simulated and implemented, and the simulation and experimental results are presented to validate the analytical results.

TABLE I Electrical characteristics of the utilized fuel cell

Oper. point	Voltage (V)	Current (A)	Oper. point	Voltage (V)	Current (A)
1	41.7	0	7	31.5	150
2	37.6	25	8	31.2	175
3*	35.4	50	9	30.8	200
4	32.7	75	10	30.4	225
5	32.3	100	11	30	250
6	31.9	125	12	29.6	275

* Nominal operation point with rated values

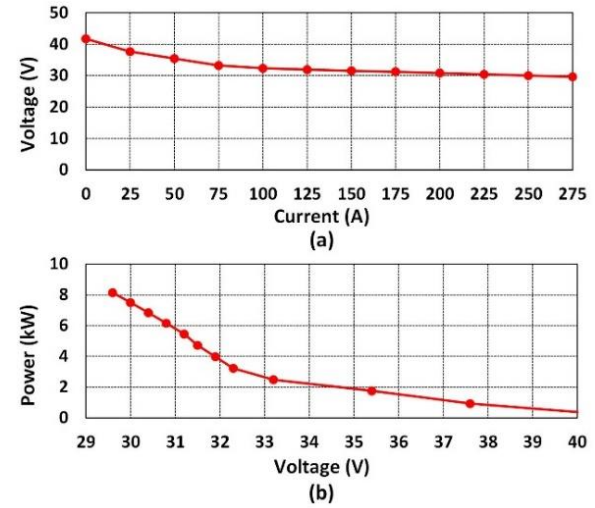


Fig. 1. Fuel cell (a) voltage-current and (b) power-voltage profiles.

TABLE II Electrical specifications of the required boost converter

Parameters	Symbol	Unit	Value
Rated input voltage	V_{i-fl}	V	35.4
Maximum input voltage	V_{i-nl}	V	41.7
Rated input current	I_{i-fl}	A	50
Rated input power	P_{i-fl}	W	1770
Maximum input current ripple	ΔI_{i-max}	A	0.5
Rated output voltage	V_o	V	72
Rated output power	P_o	W	1600
Rated output current	I_o	A	22.2
Maximum output voltage ripple	ΔV_{o-max}	V	0.4
Switching frequency	f_s	kHz	25

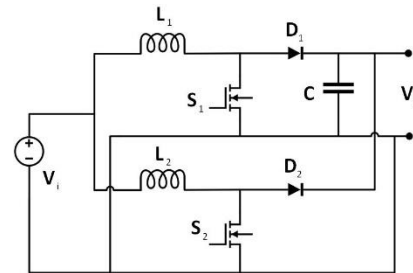


Fig. 2. Two-phase boost converter.

II. Supply power and converter specifications

The fuel cell is a power source with direct voltage and current. Technically, the characteristics of the fuel cell include parameters related to its electrical behaviour, such as rated input voltage, minimum operating voltage, maximum

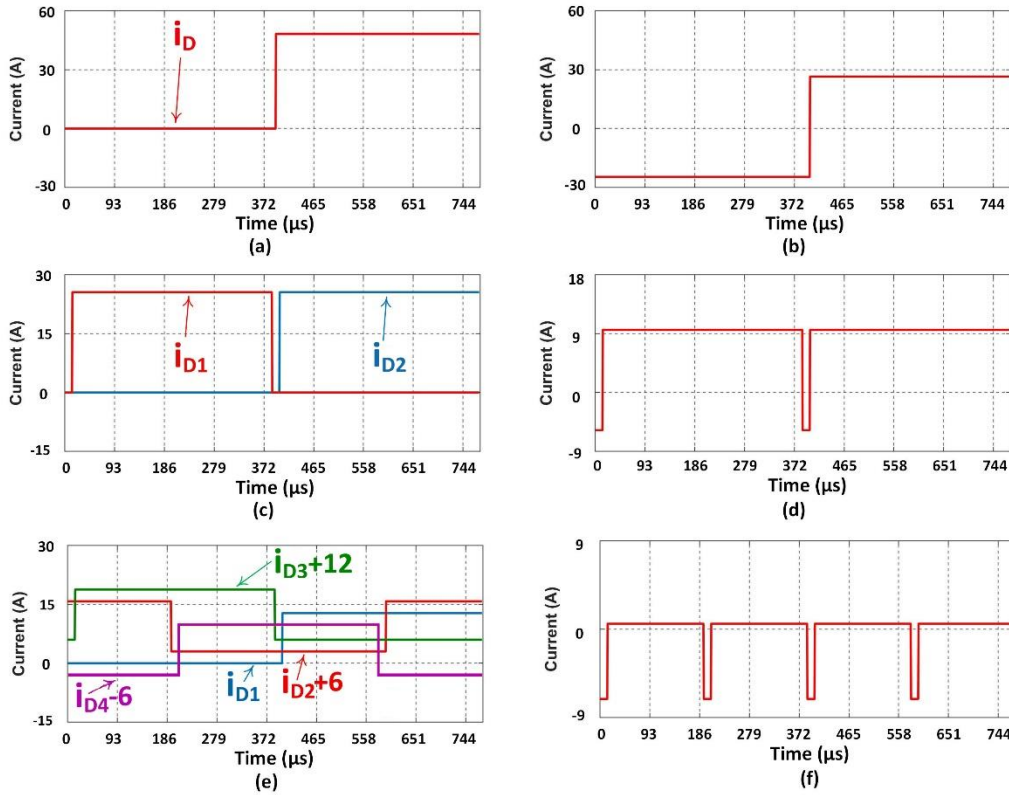


Fig. 3. Diode and capacitor currents for different designed multi-phase boost converters: (a) diode and (b) capacitor currents for single-phase, (c) diode and (d) capacitor currents for two-phase, and (e) diode and (f) capacitor currents for four-phase.

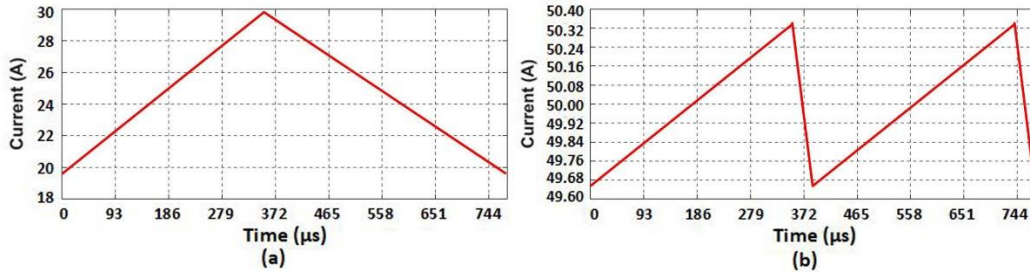


Fig. 4. The two-phase boost converter currents, (a) The phase input current, (b) The total input current.

operating voltage, rated power, and rated supply current. With the studies performed on the existing power supply system, the electrical parameters of the fuel cell are extracted and shown in Table I.

According to the reported fuel cell characteristics, the nominal input power of the designed boost converter should be equal to 1770 W at 35.4 V. Another critical fact related to the fuel cell behaviour is the voltage variation by changing the electrical load value. Using the presented fuel cell, the converter input voltage varies from 41.7 V to 35.4 V from no-load to full-load conditions, respectively. To show the load-dependent behaviour of the fuel cell, its voltage-current and power-voltage profiles are shown in Fig. 1.

The reported fuel cell has a specific feature, which is its sensitivity to the output current ripple. To protect the fuel cell, the input current ripple of the converter must be less than one

percent. Based on the aforementioned facts about the considered fuel cell as the supply voltage, the required boost converter specifications are reported in Table II.

III. Multi-phase boost

In multi-phase structures, parallel power flow paths are used with a converter in each path. Fig. 2 shows the structure of a two-phase boost converter. The current passing from the converter input to output is divided between the converter phases, which reduces the nominal currents of the components used in each phase. In other words, using a multi-phase structure increases the number of elements required and decreases their nominal values. This feature simplifies the selection of converter components for a specified required power, especially in high-power applications.

However, the multi-phase structure has another critical advantage, and that is a significant reduction in the required capacitor and inductance amounts to have an acceptable input current and output voltage ripples. The utilized phase shift angle between different phases is the main tool to achieve this feature. In the symmetrical multi-phase structures, there is a phase difference of $360^\circ/n$ degrees between different stages, where n indicates the number of phases. For example, in the two-phase structure shown in Fig. 2, the switching period of the second phase switch (S_2) starts 180 degrees after that of the switch S_1 . In this case, when S_1 in the first phase is on and the power transmission path from input to output is interrupted, S_2 can be off, and the power can be transmitted from input to output in the second phase of the converter. This feature reduces the required capacitor amount to supply the output current when the phase switch is on.

In this paper, single-phase, two-phase, and four-phase boost converters for the power interface of the fuel cell supply are investigated and compared with each other. Therefore, in this section, the operation principles of these three converters are examined. It should be noted that due to the sensitivity of the fuel cell to the output current ripple, the converters will operate in continuous conduction mode (CCM) so that their input current ripple is minimum.

It should be noted that the capacitor and inductance volumes are the two very important factors in determining the converter size. Therefore, the effect of the required output voltage ripple and the converter structure on the capacitor value and consequently, its volume is studied. Then, the same investigation to find the input current ripple and the converter structure influences on the inductance volume is presented.

A. Output capacitor of multi-phase boost

In multi-phase boost converters, the capacitor current can be calculated as [22]:

$$I_c = \sum_{j=1}^n i_{d-j} - I_o \quad (1)$$

where i_{d-j} is the diode current in phase j and I_o is the average output current of the converter. In Fig. 3, the total capacitor current and the diode current of each converter phase are presented for single, two, and four-phase converters. After calculating the area below the capacitor current waveform from zero to zero (ΔQ), the required amount of capacitor can be described as [22]:

$$C_{out} = \frac{\Delta Q}{\Delta V_o} \quad (2)$$

$$= \frac{\Delta Q}{\text{Allowable voltage ripple} = 0.006 \times V_o}$$

It should be noted that the load change transient such as load steps are not considered in this paper.

B. Current ripple

The input current ripple for each phase of the multi-phase boost converter can be calculated as follows [4]:

$$L = \frac{V_i D T_s}{\Delta I_L} \quad (3)$$

where D is the switch duty cycle and T_s is the switching period. The fuel cell current is equal to the sum of the inductance currents of the phases. Due to the phase shift angle between the inductance currents of the phases, the input current ripple is much lower than the current ripple of each inductance. For example, Fig. 4 shows the phase input current and the total input current of a two-phase boost converter.

IV. Converter size

Using the above-mentioned design procedure, single-phase, two-phase, and four-phase boost converters are designed, and the required component values are presented in Table III. It should be noted that the well-known procedure of designing a multi-phase boost converter is not presented in this paper for the sake of brevity. The total volumes of the designed components for the multi-phase boost converters are reported in Table IV.

TABLE III Design results for the required boost converter

Component	Part	N	Value	Voltage	Current
A. Single-phase boost converter					
Output cap.	Electrolyte	9	100 μ F	100 V	-
Output cap.	Electrolyte	2	47 μ F	100 V	-
Input ind.	-	1	1.47 mH	-	60 A
Switch	IRFP4227PBF	1	-	200 V ⁽¹⁾	46 A ⁽²⁾
Diode	MBR40100CT	1	-	100 V ⁽³⁾	40 A ⁽⁴⁾
B. Two-phase boost converter					
Output cap.	Electrolyte	2	100 μ F	100 V	-
Output cap.	Electrolyte	2	47 μ F	100 V	-
Input ind.	-	2	100 μ H	-	30 A
Switch	IRFP4227PBF	2	-	200 V ⁽¹⁾	46 A ⁽²⁾
Diode	MBR20150PT	2	-	150 V ⁽³⁾	20 A ⁽⁴⁾
C. Four-phase boost converter					
Output cap.	Electrolyte	1	100 μ F	100 V	-
Output cap.	Electrolyte	2	47 μ F	100 V	-
Input ind.	-	4	60 μ H	-	15 A
Switch	IRFP240	4	-	200 V ⁽¹⁾	12 A ⁽²⁾
Diode	MBR20150PT	4	-	150 V ⁽³⁾	20 A ⁽⁴⁾

⁽¹⁾ Switch drain-source minimum breakdown voltage

⁽²⁾ Switch drain-source continuous current @ 100°C

⁽³⁾ Diode peak repetitive reverse voltage

⁽⁴⁾ Diode forward current @ 100°C

The design results clearly show that the two-phase and four-phase configurations use 81.1% and 77.6% less required component volumes compared to the single-phase configuration, respectively. Therefore, from the highest to the lowest power density, the considered boost converter can be listed as the two-phase, four-phase, and single-phase

configurations. Considering the aforementioned results, using the multi-phase structure for the required boost

TABLE IV Total volume of the designed boost converters

Converter type	Component	Volume (cm ³)	Component	Volume (cm ³)
Single phase	Output cap.	51.1	Switches	3.1
	Input ind.	295.5	Diodes	1.4
	Converter total component volume: 351.1 cm ³			
Two phase	Output cap.	16.8	Switches	6.2
	Input ind.	40.2	Diodes	2.9
	Converter total component volume: 66.1 cm ³			
Four phase	Output cap.	11.9	Switches	12.5
	Input ind.	48.2	Diodes	5.8
	Converter total component volume: 78.4 cm ³			

converter significantly reduces the converter volume and consequently increases the converter power density. The reliability analysis is followed to select the best multi-phase structure among the two-phase and four-phase ones.

V. Reliability analysis

First, before starting the reliability analysis, the utilized analysis assumption can be described as:

- Without receiving a proper performance from any power components of each parallel path, the transmitting power in that phase is zero. In other words, the component failure in each phase will result in phase failure.
- If the inductor current ripple in each phase exceeds its allowable value, the protection switch corresponding to this phenomenon will trip, and the whole system will be turned off. This phenomenon is due to the low ability of the reported fuel cell to withstand the current ripple.
- In the multi-phase structure, in case of a phase failure, the other phases can continue to operate normally.
- The control system of the multi-phase structure is integrated and the fault occurrence in it disrupts the whole power delivery capability of the system.

A. Reliability analysis principles

The reliability function (probability of being healthy) is an indicator for determining the reliability of the system in terms of time, which is defined as the number of healthy units at time t divided by the number of units [25]:

$$R(t) = \frac{N_s(t)}{N_o} \quad (4)$$

The system failure probability function is also defined as the number of the failed units to the total number of the units examined [25]:

$$Q(t) = \frac{N_f(t)}{N_o} \quad (5)$$

The relationship between the reliability function and the failure probability function is as follows [25]:

$$R(t) + Q(t) = 1 \quad (6)$$

The failure rate of each element is a function of time, which is defined as the number of failures in the defined period divided by the number of elements exposed to failure [26]:

$$\lambda(t) = \frac{\text{Number of failures per unit time}}{\text{Number of components exposed to failure}} \quad (7)$$

According to the definitions, the relationship between the reliability function and the failure rate will be as follows [28]:

$$\lambda(t) = \frac{1}{N_s(t)} \cdot \frac{dN_f(t)}{dt} = \frac{-1}{R(t)} \cdot \frac{dR(t)}{dt} \rightarrow R(t) = e^{-\int \lambda(t) dt} \quad (8)$$

In many operational examples, the value of the failure rate function has a constant value over the component useful lifetime. By inserting the value of the failure rate, we will have [24]:

$$R(t) = e^{-\lambda t} \quad (9)$$

The component manufacturer can report the component failure rate by performing some reliability tests on it.

B. Component failure rate

In this section, explanations will be provided on how to calculate the failure rate of the elements used in the converter. Most systems have three parts of reliability [25]:

- Software reliability
- Hardware reliability
- Human reliability

Generally, in the reliability evaluation of power electronics systems, hardware reliability is examined and introduced as the system reliability. The most important factor determining the system's reliability is the failure rate. System failure is examined in three different periods [25]:

- Initial failure: At this time, which generally includes the construction conditions, the failure rate decreases over time.
- Lifetime failure: During the useful life of the system, the failure rate of the system can be assumed to be constant.
- Aging failure: After the useful life of the system, the failure rate of the system increases over time.

Here, the system is examined only in its useful lifetime. From now on, the reports on the failure rate of components mean their constant failure rate during their useful lifetime.

This section aims at obtaining the failure rate of the power switch, diode, capacitor, and inductance to extract the hardware reliability of the converter. To obtain the failure rate of each component, a concept called the base failure rate of that component is used. The MIL-HDBK-217 handbook states that the base failure rate can be calculated based on the following formula for components [27]:

$$\lambda_b = A \times \exp\left(\frac{N_T}{273 + T + (\Delta T)S}\right) \times \exp\left(\frac{273 + T + (\Delta T)S}{T_M}\right) \quad (10)$$

In this equation, A is the proportionality coefficient. N_T , T_M and P are the shape and structure-related coefficients. T

represents the temperature and ΔT indicates the maximum allowable temperature difference between no current and full current conditions. S is a stress-related coefficient. For some products, the base failure rate is provided by the manufacturer. By having the basic failure rate value in hand, the failure rate value of each part can be obtained by using the following equations [25]-[27].

- *Switch failure rate:*

$$\lambda_{SW} = \lambda_{b-SW} \pi_T \pi_A \pi_Q \pi_E \quad (11)$$

The coefficients used after the base failure rate, from left to right, are related to temperature, application, product quality, and environmental conditions [25].

- *Diode failure rate:*

$$\lambda_D = \lambda_{b-D} \pi_T \pi_C \pi_S \pi_Q \pi_E \quad (12)$$

In this regard, π_C is the contact structure coefficient and π_S

TABLE V Failure rate of designed components [25]

Component	Part	Value	Pack.	Failure rate (1/hour)
Capacitor	Electrolyte	100 μ F	-	10^{-7}
Capacitor	Electrolyte	47 μ F	-	10^{-7}
Inductance	-	60 μ H	-	3.8×10^{-8}
Inductance	-	100 μ H	-	3.8×10^{-8}
Inductance	-	1.47 mH	-	3.8×10^{-8}
Switch	IRFP4227PBF	-	TO247	3.8×10^{-9}
Diode	MBR20150PT	-	TO-3P	1.5×10^{-9}

is the stress coefficient [25].

- *Capacitor failure rate:*

$$\lambda_{Cap} = \lambda_{b-cap} \pi_{CV} \pi_Q \pi_E \quad (13)$$

Where π_{CV} is the capacitor value coefficient [25].

- *Inductor failure rate:*

$$\lambda_{Ind} = \lambda_{b-Ind} \pi_C \pi_Q \pi_E \quad (14)$$

Each of the above-mentioned coefficients has an experimental formula or a lookup table to be calculated. In this paper, to simplify the process of calculating reliability, we consider all experimental coefficients as one. In other words, the effect of operation and production conditions on the components' failure rates is ignored to simplify the reliability analysis procedure, which is already complicated enough. The failure rates for the components of the designed boost converter are reported in Table V.

C. Reliability modeling

Considering the operation principle of the multi-phase boost converters and the utilized assumptions for reliability analysis, the reliability model can be described as:

- The components used in each phase of the boost converter are modelled in a reliability equivalent circuit and placed next to each other in series because the failure of any phase component means that phase is out of the power delivery.
- The different phases of the boost converters are modelled in parallel in the equivalent circuit. The reason is the independence of the power delivery process in each phase compared to the other ones.

- If the input current ripple exceeds 0.5 A, the entire system will shut down.

The equivalent circuit of the system is shown in Fig.5.

Due to the series connection of the components in each phase, the phase failure rate is equal to the sum of all components' failure rates in that phase:

$$\lambda_i = \lambda_{D-i} + \lambda_{SW-i} + \lambda_{Ind-i} + \lambda_{Cap-i} \quad (15)$$

The reliability function of each phase is defined as follows:

$$R_i(t) = e^{-\lambda_i t} = e^{-(\lambda_{D-i} + \lambda_{SW-i} + \lambda_{Ind-i} + \lambda_{Cap-i})t} \quad (16)$$

Now, the failure rate and reliability function of the phases are defined. To continue the reliability calculation process of the system, it is not possible to simply consider the different phases in parallel and obtain the reliability function of the whole system. Due to the presence of the input current ripple constraint, the different phases are conditionally parallel. To obtain the relation between the failure rates of the phases and the system, two different operating conditions are examined using the Markov diagram.

- Without input current ripple protection:

In this case, due to the fact that the input current ripple protection is not utilized, the converter continues to work when each phase fails; however, it provides less output power than the nominal operating condition. Therefore, there will be three modes in the Markov diagram: healthy (converter works perfectly), partial fail (some of the phases are lost), and fail (the whole converter does not transfer power and it is not functional) [31]. The Markov diagrams for the reliability analysis of the multi-phase boost converters are shown in Fig. 6 [32]. As can be seen, the three modes are defined in this figure. In the healthy and fault modes, there is only one state considering the basic definition of these modes [31]. The partial fault mode could contain multiple states, where at least one of the phases is lost in these states, but not all of them [32]. In the presented Markov diagrams, the related failure rates for transferring from one to another state are shown. This way, the Markov diagram can show the probability of transferring from one state to another [31]. It should be noted that each phase is considered as a unit. Assuming that the units are similar, the failure rates of all units are equal. According to the Markov diagrams, the system failure rates for the multi-phase converters without using input current ripple protection can be expressed as follows for the single-phase converter:

$$\text{All possible states for } n \text{ phase: } (R(t) + Q(t))^n \quad (17)$$

$$1 \text{ phase: } \begin{cases} 1 \text{ up} \rightarrow R_1(t) = e^{-\lambda_{1s}t} \\ \text{no up} \rightarrow Q(t) = 1 - e^{-\lambda_{1s}t} \end{cases}$$

For the two-phase converter, we will have:

$$2 \text{ phase: } \begin{cases} 2 \text{ up} \rightarrow R_2(t) = e^{-2\lambda_{2s}t} \\ 1 \text{ up} \rightarrow R_1(t) = 2e^{-\lambda_{2s}t}(1 - e^{-\lambda_{2s}t}) \\ \text{no up} \rightarrow Q(t) = (1 - e^{-\lambda_{2s}t})^2 \end{cases} \quad (18)$$

where, λ_{s1} , λ_{s2} , and λ_{s4} are the failure rates of one phase in single-phase, two-phase, and four-phase structures, respectively. It should be noted that the probability of complete failure in the four-phase converter is lower than in

the other ones and this configuration has the highest availability feature. On the other hand, the single-phase boost converter has the highest probability of remaining completely healthy.

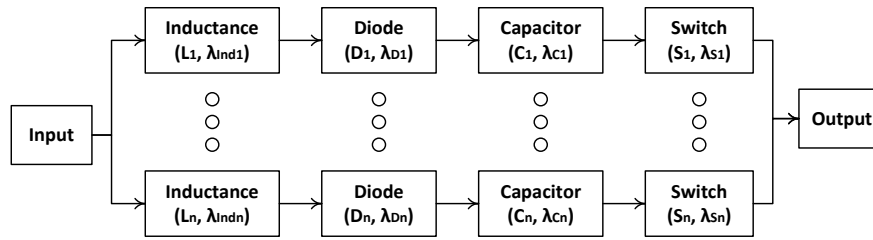


Fig. 5. . Equivalent circuit for the reliability model of multi-phase Boost converter.

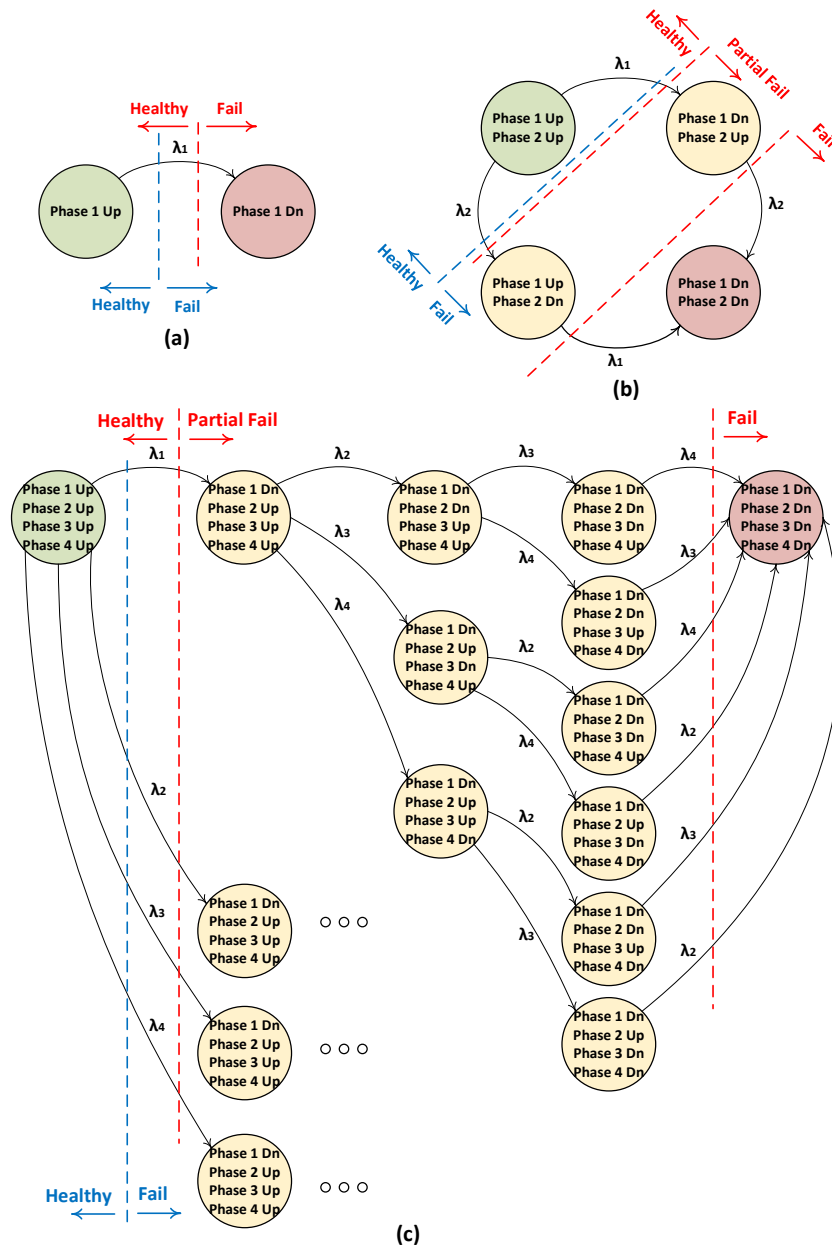


Fig. 6. Markov diagrams for the reliability analysis of the multi-phase boost converters: (a) single-phase, (b) two-phase, and (c) four-phase configurations. Modes for Case 1 (without input current ripple protection) are defined in red and modes for Case 2 (with input current ripple protection) are defined in blue.

At the end, the system failure rates for the four-phase converter can be described as:

$$2 \text{ phase: } \begin{cases} 4 \text{ up} \rightarrow R_4(t) = e^{-4\lambda_{45}t} \\ 3 \text{ up} \rightarrow R_3(t) = 4e^{-3\lambda_{45}t}(1 - e^{-\lambda_{45}t}) \\ 2 \text{ up} \rightarrow R_2(t) = 6e^{-2\lambda_{45}t}(1 - e^{-\lambda_{45}t})^2 \\ 1 \text{ up} \rightarrow R_1(t) = 4e^{-\lambda_{45}t}(1 - e^{-\lambda_{45}t})^3 \\ \text{no up} \rightarrow Q(t) = (1 - e^{-\lambda_{45}t})^4 \end{cases} \quad (19)$$

An appropriate criterion for examining and comparing the reliability of two systems with the ability to supply power partially (such as multi-phase structures) is the Expected Load Loss (ELL). This quantity is the sum of the product of the power loss and the probability of losing it. This parameter can be expressed in terms of time [25]:

$$ELL|_{t=t_0} = \sum_i P_{loss-i} R_i(t = t_0) \quad (20)$$

The ELL for the single-phase converter can be calculated as follows:

$$ELL_{1s}(t) = P_{out}(1 - e^{-\lambda_{1s}t}) \quad (21)$$

The ELL for two-phase is:

$$ELL_{2s}(t) = \frac{P_{out}}{2} 2e^{-\lambda_{2s}t}(1 - e^{-\lambda_{2s}t}) + P_{out}(1 - e^{-\lambda_{2s}t})^2 \quad (22)$$

And finally for four-phase converter, we will have:

$$ELL_{4s}(t) = \frac{P_{out}}{4} 4e^{-\lambda_{4s}t}(1 - e^{-\lambda_{4s}t}) + \frac{P_{out}}{2} 6e^{-2\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^2 + \frac{3P_{out}}{4} 4e^{-\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^3 + P_{out}(1 - e^{-\lambda_{4s}t})^4 \quad (23)$$

- With input current ripple protection:

In this case, there is a protection circuit that shuts down the converter if the input current ripple exceeds 0.5 A. Thus, although the converter may provide part of the output power in case of phase failure occurrence, the protection system turns off the converter. In other words, if each phase fails, the whole converter fails. The Markov diagram for this scenario is shown in Fig. 6. According to the presented diagram, the reliability function for the single-phase can be calculated as follows:

$$1 \text{ phase: } \begin{cases} \text{system up} \rightarrow R(t) = e^{-\lambda_{1s}t} \\ \text{system down} \rightarrow Q(t) = 1 - e^{-\lambda_{1s}t} \end{cases} \quad (24)$$

The same can be expressed for two-phase:

$$2 \text{ phase: } \begin{cases} \text{system up} \rightarrow R(t) = e^{-2\lambda_{2s}t} \\ \text{system down} \rightarrow Q(t) = 2e^{-\lambda_{2s}t}(1 - e^{-\lambda_{2s}t}) + (1 - e^{-\lambda_{2s}t})^2 \end{cases} \quad (25)$$

For the four-phase, we will have:

$$4 \text{ phase: } \begin{cases} \text{system up} \rightarrow R(t) = e^{-4\lambda_{4s}t} \\ \text{system down} \rightarrow Q(t) = 4e^{-3\lambda_{4s}t}(1 - e^{-\lambda_{4s}t}) + 6e^{-2\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^2 + 4e^{-\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^3 + (1 - e^{-\lambda_{4s}t})^4 \end{cases} \quad (26)$$

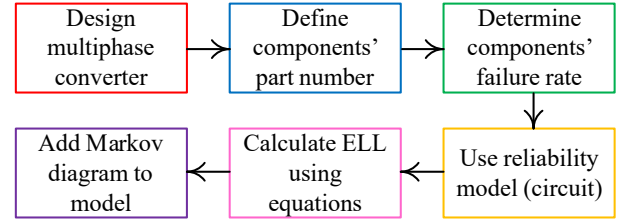


Fig. 7. ELL calculation procedure

TABLE VI Expected load loss after one million hours for different multi-phase converters

Converter	Input current ripple protection	ELL
Single-phase	Considered	$0.1306 \times P_{out}$
	Not considered	$0.1306 \times P_{out}$
Two-phase	Considered	$0.2271 \times P_{out}$
	Not considered	$0.1306 \times P_{out}$
Four-phase	Considered	$0.3434 \times P_{out}$
	Not considered	$0.1306 \times P_{out}$

TABLE VII Expected load loss after one million hours for different multi-phase converters

Phase number	Input current ripple protection	ELL (pu)*	Volume (pu)**	Objective function
1	Considered	0.1306	1	0.1306
2	Considered	0.2271	0.188	0.0427
4	Considered	0.3434	0.233	0.0800

* normalized considering the rated output power

** normalized considering the single-phase volume

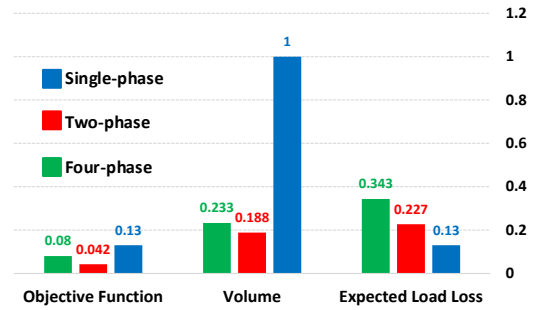


Fig. 8. ELL and volume calculation result

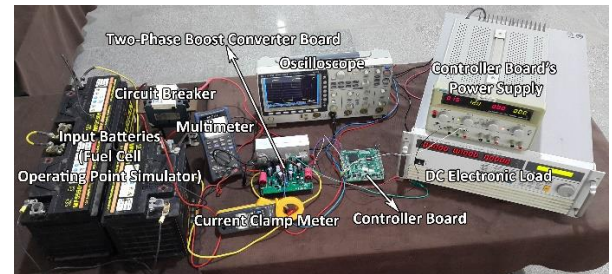


Fig. 9. Utilized test setup for the implemented two-phase boost converter.

Obviously, in this situation where the input current protection system is functional, the four-phase converter is not suitable in terms of reliability because the failure in any component results in phase failure, which is equivalent to not having access to the output power. On the other hand, the probability of any failure occurrence in the four-phase converter is higher

than that in other converters, because of its larger number of utilized components. In the presence of the input current ripple protection system, the single-phase converter is more reliable due to the small number of

components used in it. The expected load loss for the single-phase case is obtained as follows:

$$ELL_{1s}(t) = P_{out}(1 - e^{-\lambda_{1s}t}) \quad (27)$$

For the two-phase, we will have:

$$ELL_{2s}(t) = P_{out}(e^{-\lambda_{2s}t}(1 - e^{-\lambda_{2s}t}) + (1 - e^{-\lambda_{2s}t})^2) \quad (28)$$

And the ELL for the four-phase is:

$$ELL_{4s}(t) = P_{out}(e^{-3\lambda_{4s}t}(1 - e^{-\lambda_{4s}t}) + e^{-2\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^2 + e^{-\lambda_{4s}t}(1 - e^{-\lambda_{4s}t})^3 + (1 - e^{-\lambda_{4s}t})^4) \quad (29)$$

In this step of the reliability analysis, the expected load loss values (after one million hours) for different multi-phase converters are calculated (Fig. 7) in two modes with and without input current ripple protection (Table VI).

As was expected, the single-phase structure has the best performance considering the ELL value. However, the four-phase (especially when the protection system exists) has the highest ELL value, which indicates its low reliability. It should be noted that even though the four-phase structure has the lowest reliability, it has the highest availability.

VI. Comparison of multi-phase configurations

The selection of the optimal phase number will be done according to an objective function consisting of the expected load loss and the required volume of the converter:

$$\text{Objective function} = ELL \times \text{Volume} \quad (30)$$

It should be noted that the converter volume is defined as the sum of the main components' volumes (power switch, diode, inductance, and capacitor) to avoid the complexity of considering the components' arrangement in the actual circuit. On the other hand, the worst-case scenario for the converters' reliability is taken into account, where the input current ripple protection exists. The multi-phase converter with the lowest objective function value will be selected as the best option for use in fuel cell applications.

According to the results reported in Table VII and Fig. 8, the single-phase structure is more reliable than the other two options, and the two-phase structure is the best option in terms of volume in comparison with the other ones. In the end, the two-phase structure has the lowest objective function, thus, the two-phase structure is selected as the desired option. It should be noted that the results presented in Table VII are normalized using the rated output power as the normalization base for ELL and the highest converter's volume (single-phase's volume) as the normalization base for volume.

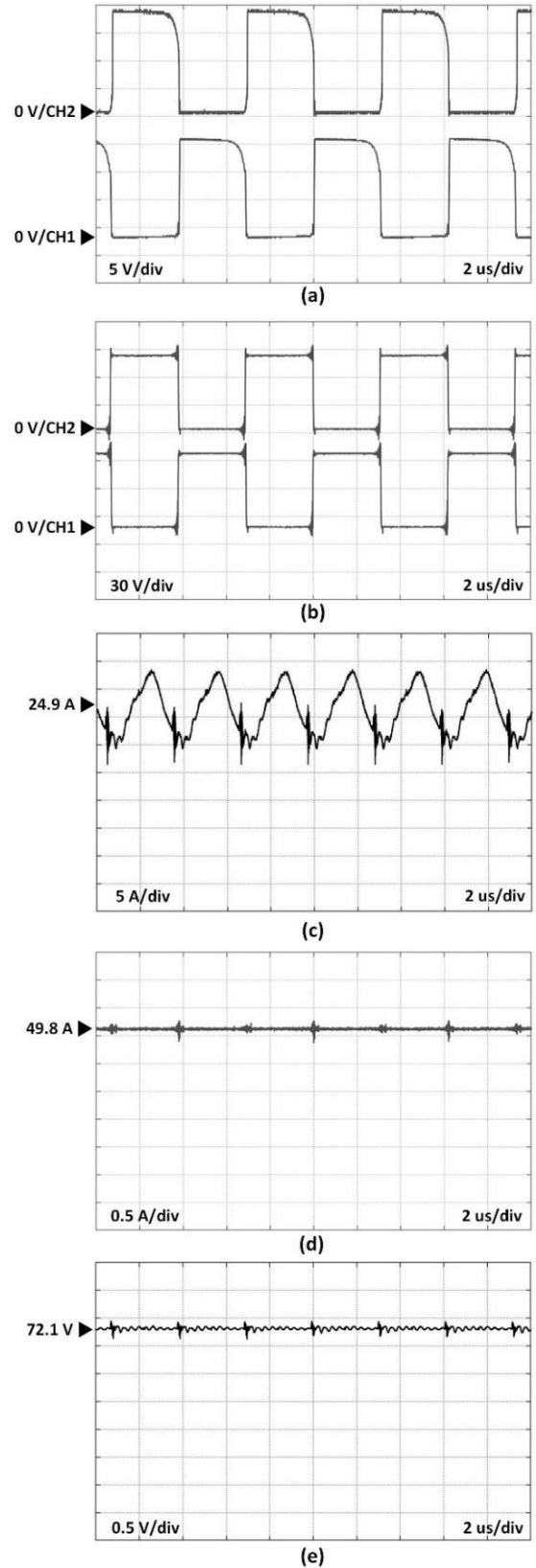


Fig. 10. Experimental results for the implemented two-phase boost converter: (a) gate-source voltage for both switches, (b) drain-source voltage for both switches, (c) phase input current, (d) converter input current, (e) output voltage.

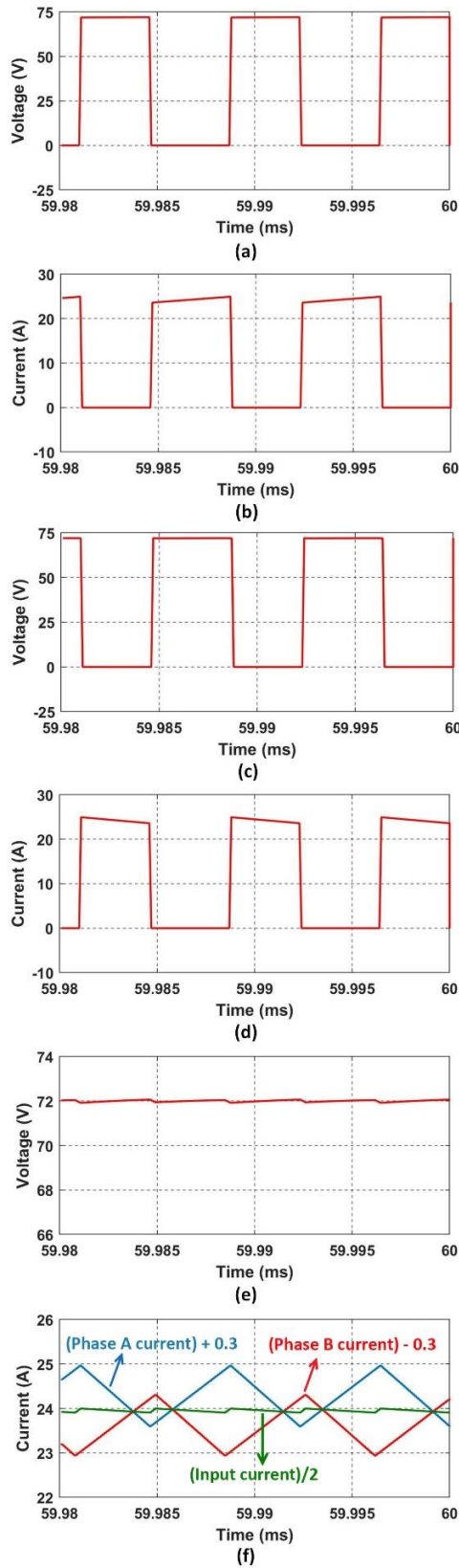


Fig. 11. Performance parameters waveforms for the designed two-phase boost converter: (a) switch voltage, (b) switch current, (c) diode voltage, (d) diode current, (e) output voltage, and (f) phases input current (in blue and red) and half of the converter input current (in green).

VII. Simulation and experimental results

A. Simulation results

The analytical results related to the design and operation of the selected two-phase boost converter should be validated using simulation results. For this purpose, the converter has

been simulated in the PSIM software environment. The important performance waveforms of the designed two-phase boost converter are shown in Fig. 9 in rated load conditions (with 22.22 A output current). The maximum value of the switch voltage is 72.1 V and it has a 49 V RMS value (Fig. 9-a). In addition, the switch has a maximum current of 24.89 A and 17.7 A RMS value (Fig. 9-b). The maximum voltage and current values of the diode are 72 V (Fig. 9-c) and 24.9 A (Fig. 9-d), respectively. Obviously, the rated values of the selected switches and diodes are completely reasonable considering the simulation results. The converter has an output voltage with a 72.06 V average value and 0.22 V ripple voltage (Fig. 9-e), which are acceptable results considering the design goals. In addition, each phase has an input current with a 24.28 A average and 1.37 A ripple values (Fig. 9-f). However, the whole input current of the converter has an average of 47.9 A and a ripple of 0.19 A (Fig. 9-f), which is the result of using a two-phase structure. Note that in Fig. 9, half of the converter input current is presented simultaneously with the phases' input currents to show the current ripple cancellation effect of the two-phase structure. According to the simulation results, the designed converter satisfies the design requirements well.

B. Experimental results

The designed converter is implemented and connected to a DC source to extract the experimental test results. A lead-acid battery pack is used to simulate the reported fuel cell behavior considering its voltage-current profile presented in Fig. 1. Considering Table I, the fuel cell nominal operating voltage is simulated by the battery voltage value. An electronic load is used as the load for the fabricated two-phase boost converter. A circuit breaker is used to turn on/off the batteries. The set used for the required measurements is shown in Fig. 10. The experimental results for different parts of the converter are shown in Fig. 11. As can be seen, the MOSFET gate receives the switching command at a frequency of 130 kHz (Fig. 11-a). The switch voltages indicate correct MOSFET switching with the desired phase shift between two phases. The maximum stress voltage on the MOSFETs used in each phase is about 76.2 V (Fig. 11-b). As can be seen, due to the two-phase converter structure, the input current ripple has been reduced to 0.32 A (Fig. 11-d). As discussed in previous chapters, using 180 degrees of phase shift between two phases greatly reduces the total input current ripple of the converter and doubles the frequency of this ripple. The output voltage is stabilized at the desired value of 72.1 V and the voltage ripple is equal to 0.35 V (Fig. 11-e). The heatsink temperature is measured 33.3 °C at its maximum. The same has been done for the switches, which are the hot spot of the

converter, with the maximum temperature of 64.8 °C. These results support the thermal performance of the converter. For evaluating the efficiency of the implemented prototype converter, its efficiency map is illustrated in Fig. 12. As can be seen, the converter has acceptable efficiency in different

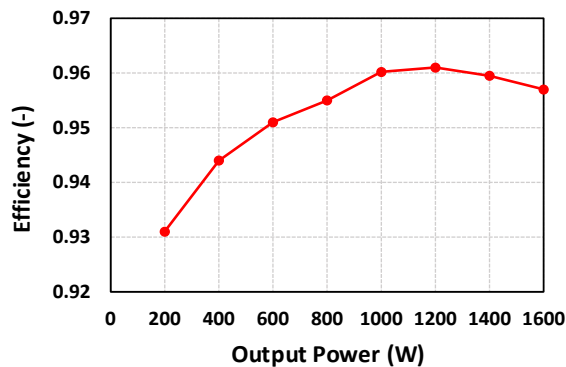


Fig. 12. Measured efficiency curve for the prototype converter.

loading conditions. According to the obtained experimental results, the accuracy of the theoretical results can be confirmed. On the other hand, these results are well-matched with the simulation results.

VIII. Conclusion

In this paper, a new study was done on the multi-phase DC-DC boost converters. The study proposed a solution to increase the power density and reliability of the boost converter for fuel cell applications because the fuel cell power systems usually require power electronics interfaces with high volumetric power density and high reliability. For this purpose, a different multi-phase boost converter was studied. According to the results presented for the main converter, using a two-phase structure, the capacitor and the input inductance volumes were reduced dramatically (67.1% and 86.4%, respectively) compared to the common single-phase structure. The reason for this reduction was using the phase shift angle between the switches command, which decreases the input current and output voltage ripples and doubles their frequency.

On the other hand, to study the converter reliability, some steps were taken in two operating conditions with and without input current ripple protection. The steps include calculation of the components' failure rates, reliability modelling (based on the Markov diagram), and ELL computation. Considering the results, the multi-phase structures from the most to the least reliable were the single-phase with $ELL=0.1306pu$, the two-phase with $ELL=0.2271pu$, and the four-phase with $ELL=0.3434pu$, respectively. In the end, the two-stage converter with a goal function ($volume \times ELL$) of 0.0427 (which was 67.3% lower than the next best option) was selected as the best multi-layer configuration. The selected two-phase converter for fuel cell application was implemented, and the obtained experimental results verified the accuracy of the analytical results.

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Comparative Evaluation of Metaheuristic MPPT Algorithms for PV Systems Under Partial Shading Conditions

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Article Info	ABSTRACT
Article type: Research Article	Enhancing the energy output of photovoltaic (PV) systems is essential due to their inherently limited efficiency. Consequently, maximum power point tracking (MPPT) techniques have become a crucial component in photovoltaic systems for improving energy harvesting efficiency. However, conventional MPPT methods often require accurate mathematical modeling of PV systems, which remains a significant challenge due to their highly nonlinear behavior. To address this issue, researchers have proposed various MPPT strategies. The complex nonlinear behavior of PV systems makes pattern extraction difficult, often leading to simplified assumptions that may reduce tracking accuracy and overall system performance. This study investigates the performance of five metaheuristic optimization algorithms for MPPT under partial shading conditions (PSC) to improve PV system efficiency. The considered algorithms include Particle Swarm Optimization Algorithm (PSOA), Grey Wolf Optimization Algorithm (GWOA), Cuckoo Search Optimization Algorithm (CSOA), Genetic Algorithm (GA), and Quantum-Inspired Evolutionary Algorithm (QIEA). Although several studies have investigated metaheuristic MPPT techniques under partial shading conditions, relatively limited research has provided a comprehensive comparative evaluation of swarm-based, evolutionary-based, and physics-inspired optimization approaches considering tracking efficiency and dynamic response characteristics. The results demonstrate that the physics-inspired QIEA achieves a superior balance between exploration and exploitation, thereby enhancing its ability to accurately identify the global maximum power point.
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I. Introduction

Despite the limited availability of conventional energy resources, global electricity demand continues to increase steadily. Consequently, the demand for renewable and environmentally friendly energy sources has grown significantly, with photovoltaic systems emerging as one of the most attractive solutions due to the clean, inexhaustible, and non-polluting nature of solar energy. In addition, PV systems are considered cost-effective and sustainable alternatives for future energy generation. However, the efficient utilization of PV systems remains challenging because their output characteristics are highly nonlinear and strongly affected by environmental conditions such as solar irradiance and temperature variations. These fluctuations

often result in multiple power peaks, particularly under PSC. Therefore, maximizing the performance of photovoltaic modules requires continuous tracking of the maximum power point (MPP) under varying operating conditions. Maximum power point tracking techniques are specifically designed to ensure PV systems operate consistently at their optimal power point. Several MPPT techniques have been proposed in the literature, including the Hill Climbing Technique (HCT), the Incremental Conductance method, the Fractional Open-Circuit Voltage method, and the Fractional Short-Circuit Current method. Generally, MPPT systems consist of a switch-mode power converter and a control algorithm that tracks the MPP. Developing robust energy conversion techniques with effective MPPT capability is essential for

maintaining voltage stability and improving energy extraction under rapid environmental changes and varying load conditions. Several modifications have been introduced to improve the performance of conventional MPPT techniques, particularly under rapidly changing irradiance conditions. For instance, the method proposed in [1] employs a simple mathematical relationship between the load line and the I–V characteristic curve to achieve fast MPP tracking. Similarly, the approach presented in [2] introduces a dynamic MPPT controller capable of handling rapid irradiance variations and PSC by utilizing a scanning-based technique to identify the global maximum power point. Nevertheless, conventional MPPT techniques face considerable challenges when dealing with nonlinear objective functions and multiple decision variables. As a result, researchers have increasingly focused on heuristic and metaheuristic optimization approaches to address these limitations [3]. Metaheuristic algorithms employ iterative and intelligent search strategies inspired by natural or physical phenomena. Their effectiveness relies on achieving a suitable balance between exploration and exploitation processes [4]. Exploration refers to the global search capability for identifying promising regions within the search space, whereas exploitation focuses on refining solutions around the best candidate points. A proper balance between these two mechanisms is essential for obtaining an optimal solution. Traditional gradient-based MPPT methods, such as Perturb and Observe (P&O), HCT, and InC, often exhibit poor performance under PSC because they are unable to distinguish between local maximum power points (LMPPs) and the global maximum power point [5]. Therefore, numerous studies have investigated the application of heuristic optimization algorithms for improving MPPT performance. Effective MPPT methods play a critical role in regulating PV system output power and reducing power fluctuations caused by changes in solar irradiance, temperature, and shading [6,7]. By continuously tracking the MPP, these algorithms enhance the stability, reliability, and overall efficiency of PV power generation systems [8,9]. Among the proposed methods, several studies have demonstrated the effectiveness of Particle Swarm Optimization Algorithm (PSOA)-based approaches. In [10], a hybrid MPPT method combining PSOA and the Incremental Conductance technique was proposed to improve GMPP detection performance. In [11], PSOA was integrated with the Genetic Algorithm to reduce computational complexity. Furthermore, [12] presented an improved PSOA approach that reduces steady-state oscillations. Additional investigations into PSOA under PSC were conducted in [13], where a nonlinear inertia weight strategy was employed to enhance tracking performance. In [14], the authors evaluated the performance of the Cuckoo Search Optimization Algorithm and compared it with the conventional P&O method. The results showed that the CSOA achieved faster tracking performance, requiring only 22 ms compared to 45

ms for the P&O method. Moreover, the CSOA required only voltage and current sensors, thereby reducing implementation cost. In [15], a hybrid CSOA-Golden Section Search (GSS) method was proposed to improve convergence speed and tracking accuracy. In this approach, CSOA first identifies the region around the MPP, and the GSS algorithm then precisely determines the GMPP. Recent studies have focused on improving MPPT performance under partial shading conditions using intelligent and hybrid optimization techniques. Wang et al. [16] proposed a hybrid POA&PO approach with high tracking efficiency and rapid convergence; however, the hybrid structure increased the computational complexity. [17] Comprehensively reviewed recent intelligent MPPT techniques and highlighted the effectiveness of machine-learning-driven metaheuristic strategies, although limited comparative dynamic analyses were provided. In [18], comparative evaluations of PSO-, GA-, and ACO-based MPPT methods demonstrated improved tracking performance under PSC, but only a limited number of shading scenarios were investigated. Moreover, [19] introduced a heap-based optimization method with high tracking accuracy and reduced settling time, although implementation complexity remained a challenge. These limitations motivate further comparative investigations of different metaheuristic optimization approaches for MPPT applications under PSC. Motivated by these advancements, this study investigates the performance of five MPPT optimization algorithms under PSC, namely QIEA, GWOA, PSOA, CSOA, and GA. These algorithms are comparatively evaluated from the perspectives of swarm-based, evolutionary-based, and physics-inspired optimization approaches to improve PV system efficiency and enhance dynamic response performance under partial shading conditions. As a result, the main contributions of this study can be summarized as follows:

- A comprehensive comparative evaluation of different metaheuristic optimization categories, including swarm-based, evolutionary-based, and physics-inspired algorithms, for MPPT applications under partial shading conditions.
- Investigation of the tracking performance of PSOA, GWOA, CSOA, GA, and QIEA in terms of convergence behavior, oscillation characteristics, and global maximum power point tracking capability under identical operating conditions.
- Comparative assessment of the dynamic response performance of the investigated MPPT techniques, including tracking stability, overshoot, undershoot, and settling time characteristics under partial shading conditions.

The remainder of this paper is organized as follows. Section 2 presents the system description. Section 3 describes the proposed cost function. Section 4 explains the methodologies of the investigated optimization algorithms. Section 5 discusses the simulation results and comparative analysis, while Section 6 concludes the paper.

II. PV description

A photovoltaic (PV) system consists of multiple solar cells connected in series and parallel configurations to achieve the desired voltage and current levels. Series connections increase the output voltage, whereas parallel connections increase the output current. Under partial shading conditions, the presence of bypass diodes results in multiple peaks in the power–voltage (P–V) characteristics, including several local maximum power points (LMPPs) and a single global maximum power point. To mitigate hot-spot formation during PSC, bypass diodes are connected in parallel with each PV module. Hot spots occur when shaded modules consume power rather than generate it, potentially leading to overheating and damage to the PV system. The incorporation of bypass diodes improves system reliability and protects the PV modules under non-uniform irradiance conditions. Figure 1 illustrates the overall structure of the proposed heuristic-based MPPT system. In addition, Figure 2 presents the PSC scenario along with the corresponding P–V characteristic curve. The following equation represents the single-diode model of a PV cell.

$$I = I_{pv} - I_0 \left[\exp\left(\frac{V + IR_s}{V_t} - 1\right) \right] - \frac{V + IR_s}{R_p} \quad (1)$$

I_{pv} denotes the current generated by incoming light;

$\left[\exp\left(\frac{V + IR_s}{nV_t} - 1\right) \right]$ represents the Schottky diode current;

I_0 signifies the leakage current;

V_t indicates the thermal voltage of the diode;

R_s refers to the series resistance in the analogous model of the photovoltaic cell;

I is the corresponding current;

R_p Denotes the model's parallel resistance

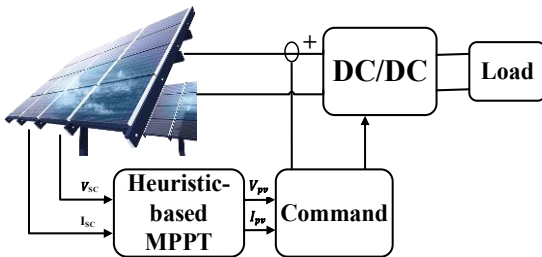


Fig. 1. Schematic of heuristic-based MPPT system.

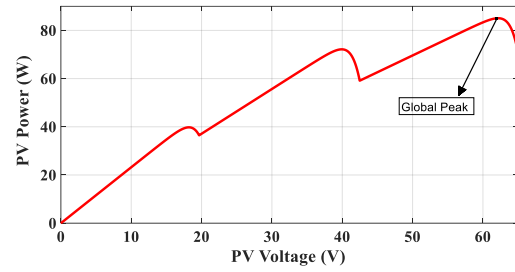


Fig. 2. P–V curves under PSC.

III. Cost Function

The cost function for the suggested issue may be described as Eq. (2). The cost function presented in Eq. (2) was formulated to achieve a balanced trade-off between maximum power extraction and dynamic response performance under partial shading conditions. The first term maximizes the extracted output power relative to the PV system's rated power, while the remaining terms penalize undesirable transient characteristics, including overshoot, undershoot, and settling time. The weighting coefficients w_1 , w_2 , and w_3 were selected empirically through repeated simulation trials to provide an appropriate trade-off between tracking accuracy and dynamic response stability. Larger values of w_1 prioritize maximizing output power, whereas increasing w_2 and w_3 place greater emphasis on reducing transient oscillations and improving settling characteristics. Although the selected weighting coefficients provided satisfactory performance under the investigated PSC scenario, a more comprehensive sensitivity analysis considering different weighting combinations and operating conditions is recommended for future investigation.

$$v_i^{k+1} = \omega \times v_i^k + c_1 r_1 (p_{local\ best, i}^k - p_i^k) + c_2 r_2 (p_{global\ best}^k - p_i^k) \quad (2)$$

Minimization

$$J = \omega_1 \left(1 - \frac{P_{max}}{P_{rated}} \right) + \omega_2 (O_v + U_s) + \omega_3 T_s \quad (3)$$

where,

J is the aggregate cost function designated for minimization;

P_{rated} is the nominal power output of the PV cell under conventional test circumstances;

P_{max} signifies the maximum power output under PSC;

The weighting variables ($\omega_1, \omega_2, \omega_3$) serve to equilibrate the relative significance of the different terms.

O_v is the highest overshoot in output power during disturbances;

U_s is the maximum undershoot;

T_s is the settling time of the output power during disturbances.

IV. Evaluation of the presented method

As stated above, this study examines the use of five algorithms—PSOA, GWOA, CSOA, GA, and QIEA—for efficient MPPT in solar cells under partial shading conditions, while accounting for PQ.

A. PSOA-based MPPT Technique (Swarm-based Technique)

The behavior of bird flocking is analogous to PSOA, which is also inspired by biological phenomena. To achieve optimal results, it operates with minimal assumptions. The fundamental concept of this bio-inspired technique is to view each PV array or module as a molecule, with the MPP functioning as the target to be monitored. Each photovoltaic module functions as a subordinate to a master module within this structure, interacting with its master controller to monitor the requisite MPP [24]. Initially, PSOA produces a population of individual swarms, which signify probable solutions, and these "particles" traverse the search space. They observe the paths of adjacent particles alongside their own. The position of a particle is dictated by the most advantageous solution recognized by the whole population $P_{Local\ best}^i$ and the optimum particle in its proximity $P_{Local\ best}^k$. The particle's position is modified based on the best particle nearby and the ideal solution identified by the whole population of particles. Eq.(3) is executed to alter the location of a particle. The GMPP can be easily monitored because the PSOA strategy is based on the search method. When the MPP point is followed, the enhanced PSOA technique can reduce steady-state oscillations, as explained in specific literature. To prevent superfluous and excessive searching, particles can be effectively initialized around the MPP, ensuring that the swarm efficiently explores the area and rapidly converges to the true MPP. Parameter control is a critical attribute of this methodology. Figure 3 depicts the flowchart of the proposed PSOA-based MPPT algorithm.

where,

w	is the inertia weight;
c_1, c_2	are the rising speed coefficients;
r_1, r_2	is the particle index;
k	is the iteration index;
v_i^k	is the velocity of the i th particle at the k th iteration;
$P_{Local\ best}^k$	is the local best;
P_i^k	represents the position of the i th particle at the k th iteration;
$P_{global\ best}^k$	represents the best position experienced by all particles up to the k th iteration;

B. GWOA-based MPPT Technique as a swarm-based technique

The GWOA emulates the leadership structure and hunting strategies of grey wolves in the wild, as presented in [22]. Grey wolves are apex predators that live communally in

packs. The program models the social order via four classifications of grey wolves: alpha (α), beta (β), delta (δ), and omega (ω). In the GWOA design, the optimal solution is designated as alpha (α), while the second and third best options are referred to as beta (β) and delta (δ), respectively. The residual candidate solutions are designated as omega (ω). Figure 4 delineates the three primary phases of the GWOA, including searching for, pursuing, and monitoring prey, surrounding prey, and assaulting prey. These stages are essential for executing optimization. In the course of a hunt, grey wolves surround their quarry, a behavior represented by the following equations:

$$\vec{D} = |\vec{C} \cdot \vec{X}_p(t) - \vec{X}_p(t)| \quad (4)$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (5)$$

In this context, t signifies the current iteration, D , A , and C imply coefficient vectors, $\vec{X}_p(t)$ represents the position vector of the prey, and $\vec{X}$ indicates the position vector of the grey wolf. The vectors A and C are computed in the following manner:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (6)$$

$$\vec{C} = 2 \cdot \vec{r}_2 \quad (7)$$

The components decline linearly from 2 to 0 during the repetitions, while $\vec{r}_1, \vec{r}_2$ are random vectors within the interval $[0, 1]$. Alpha wolves often lead the hunt, with beta and delta wolves following closely behind. Delta and omega wolves attend to the injured members of the pack. The alpha wolf, with superior knowledge of the prey's position, directs the hunt, culminating in an assault when the prey ceases to move. The controller monitors V_{pv} and I_{pv} via sensors for various duty ratios of grey wolves, and calculates the output power. Under PSC, the P–V curve shows many peaks, including several local peaks (LPs) and a single global peak (GP). When wolves identify the MPP, their respective coefficient vectors move toward zero. The proposed method combines GWOA with direct duty-cycle control. At the MPP, the duty cycle remains unchanged, reducing the steady fluctuations common in traditional MPPT systems and decreasing power losses caused by these fluctuations, thereby enhancing system efficiency. In the GWOA-based MPPT implementation, the duty cycle D is described as a Gray wolf. Therefore, Eq. (5) can be revised as follows:

$$D_i(k+1) = D_i(k) - A \cdot D \quad (8)$$

Consequently, the fitness function of the GWOA is articulated as:

$$P(d_i^k) > P(d_i^{k-1}) \quad (9)$$

where;

p denotes power

d signifies the duty cycle

i indicates the quantity of current gray wolves

k represents the number of repeats

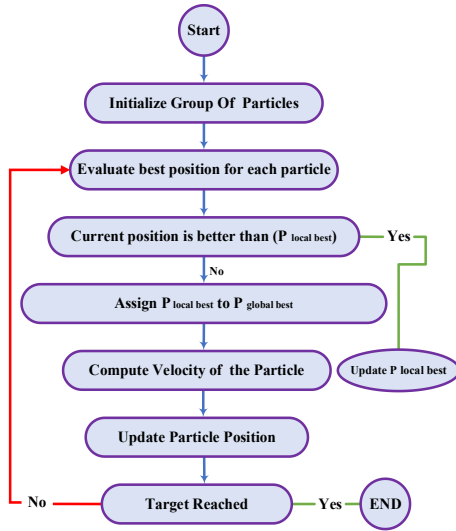


Fig. 3. Flowchart of the PSO algorithm.

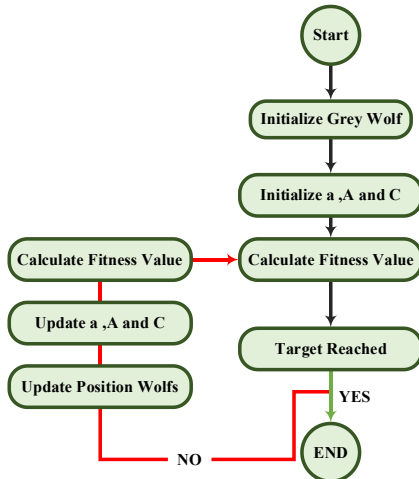


Fig. 4. Flowchart of the GWOA.

C. CSOA-based MPPT technique as a swarm-based method

This method uses the characteristics of the cuckoo bird as a metaphor for finding the best solution in MPPT. This concept, explained in [23], shows the tracking technique. The cuckoo's egg represents the best new solution, while the eggs already in the nest stand for the existing solutions. By applying an appropriate fitness function, suboptimal solutions (old eggs) are gradually replaced by the optimal solution (cuckoo's egg). Figure 5 illustrates the planned CSOA-based MPPT methodology. The cuckoo search method is based on three principles.

- Each cuckoo successively deposits eggs in randomly selected nests.
- The nests preserve the highest quality eggs for subsequent generations.

- The quantity of accessible host nests is constant, while the likelihood of a host bird detecting a cuckoo's egg ranges from 0 to 1. In this situation, the host bird may either eliminate the egg or forsake the nest to build a new one.

To simplify, this last assumption is represented by substituting nests with new ones (exhibiting random configurations) if a bird identifies an egg that does not belong to it, resulting in its destruction via rapid flight. [24] offers a comprehensive elucidation of this technique. The fitness function (F) has two parameters, namely the operating voltage of the PV array (V) and the step size α for optimizing the GMPP, which is illustrated in Eq.(10):

$$F = f(v) \text{ and } V_i^{t+1} = V_i + \alpha \oplus \text{levy}(\lambda) \quad (10)$$

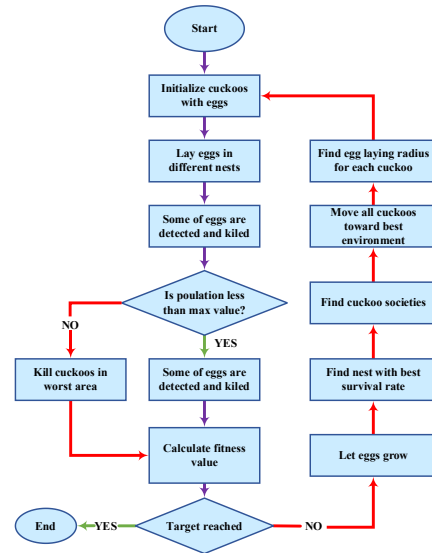


Fig. 5. Flowchart of the CSOA algorithm

D. GA based MPPT Technique as an Evolutionary Method

The GA remains the most effective method among evolutionary approaches; therefore, it has been selected for comparison. Following previous methodologies, the GA begins with an initial population (initial solution). The controller measures V_{pv} and I_{pv} calculates the output power. The GA then applies a series of genetic operators, including selection, crossover, and mutation, on the population to generate a new set of individuals. The selection operator chooses parents based on their fitness, while the crossover operator combines the genetic material of the selected parents to create new offspring. The mutation operator introduces random variations in the offspring to explore different areas of the search space. This iterative process continues until the convergence criterion is met. The final solution, representing the optimal values of the control variables V_{pv} and I_{pv} , aligns with the maximum power point (MPP) of the solar cell. The genetic algorithm effectively solves the maximum power point tracking problem by exploring the search space and converging on the optimal solution. This method is especially

useful when the power output function is complex or nonlinear, as the evolutionary algorithm can handle such situations effectively. Figure 6 shows the flowchart of the proposed genetic algorithm-based maximum power point tracking method.

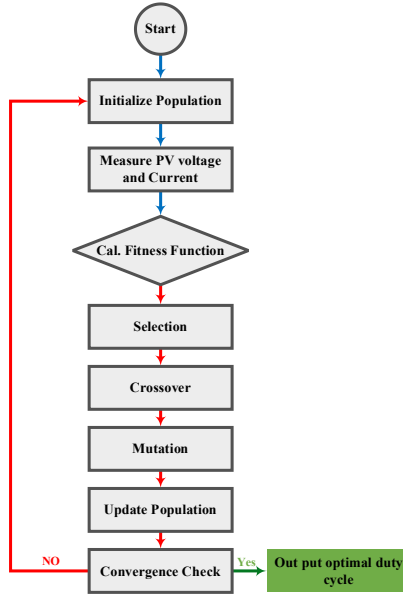


Fig. 6. Flowchart of the GA algorithm

E. QIEA-based MPPT Technique as Physics-based Method

The Quantum-Inspired Evolutionary Algorithm is an innovative evolutionary algorithm inspired by concepts from quantum computing [25], [26]. Quantum Evolutionary Algorithms (QIEA) use a probabilistic representation based on the quantum bit, or qubit, unlike conventional evolutionary algorithms that use binary, arithmetic, or symbolic representations. The qubit serves as the essential unit of information in quantum computing, capable of being in a superposition of states. The qubit-based system enables QIEA to present a wide range of possible solutions within its group, thereby enhancing its ability to explore options. QIEA utilizes an innovative Q-bit format, characterized as a kind of probabilistic encoding. A qubit is the fundamental unit of information in quantum error analysis (QIEA) and is represented by a pair of integers (α, β) , satisfying the condition $|\alpha|^2 + |\beta|^2 = 1$. Here, $|\alpha|^2$ denotes the likelihood of the qubit being in the '0' state, while $|\beta|^2$ signifies the probability of it being in the '1' state. A qubit may occupy the '0' state, the '1' state, or a linear superposition of both states. A single Q-bit encoded as a sequence of m Q-bits is denoted as:

$$\begin{bmatrix} \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \dots & \alpha_m & \beta_m \end{bmatrix}^T \quad (11)$$

$$|\alpha|^2 + |\beta|^2 = 1, i = 1, 2, \dots, m$$

The Q-bit format has the benefit of probabilistically depicting a linear superposition of states. A three-qubit system with three pairs of amplitudes may represent information from

eight unique states. This depiction illustrates a greater variety of populations compared to other forms. QIEA is a probabilistic algorithm, similar to several others. Nonetheless, QIEA distinctly sustains a population of Q-bit individuals denoted as $Q(t) = \{q_1^t, q_2^t, \dots, q_n^t\}$ at generation t , where n signifies the population size. Each Q-bit entity q_j^t for $j = 1, 2, \dots, n$ is delineated in (10). Figure 7 illustrates the comprehensive structure of QIEA.

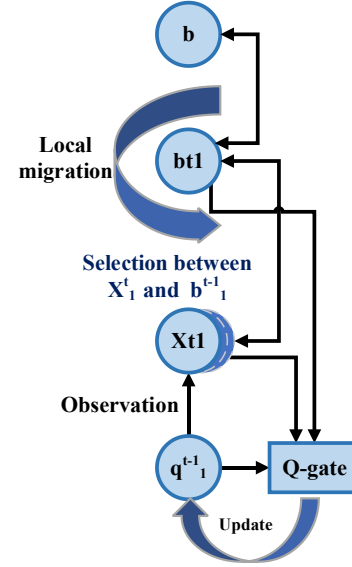


Fig. 7. Structure of QIEA algorithm.

- Initialization of $Q(t)$: In this phase, the values of α_i^0, β_i^0 for all $q_j^0, j=1, 2, \dots, n$ are set to $\sqrt{2}^{-1}$. This implies that each Q-bit individual, q_j^0 , represents a variety of phases of the energy output of the photovoltaic cell.
- Binary Solution Generation: Binary solutions in $P(0)$ are produced by analyzing the states of $Q(0)$, where $P(0) = \{x_1^0, x_2^0, \dots, x_n^0\}$ at generation $t = 0$. Each binary solution x_j^0 is a binary string of length m , created by choosing either 0 or 1 for each bit according to the probability $|\alpha_i^0|^2$ or $|\beta_i^0|^2$.
- Fitness Assessment: Each binary solution x_j^0 is assessed according to the power output of the solar cell.
- Choosing the First Best Solutions: The best solutions from $P(0)$ are found and written down in $B(0)$, where $B(0) = \{b_1^0, b_2^0, \dots, b_n^0\}$ at the start of the generation.
- Picking of starting Optimal Values: The optimal solutions from $P(0)$ are chosen and recorded in $B(0)$, where $B(0) = \{b_1^0, b_2^0, \dots, b_n^0\}$ and b_j^0 matches x_j^0 at the beginning of the iteration.
- While Loop: In every repetition of the while loop, binary solutions in $P(t)$ are formed by watching the states of $Q(t-1)$ as in step 2, and each binary solution is assessed

for fitness. Notice that x_j^t in $P(t)$ can come from many evaluations of q_j^{t-1} in $Q(t-1)$.

- **Q-bit Update:** The Q-bit entities in $Q(t)$ are modified utilizing Q-gates, which function as variation operators in QIEA. The revised Q-bits must adhere to the normalization requirement, $|\alpha'|^2 + |\beta'|^2 = 1$, where α' and β' represent the updated values. The subsequent rotation gate serves as a fundamental Q-gate in QIEA:

$$U(\Delta\theta_i) = \begin{bmatrix} \cos(\Delta\theta_i) & -\sin(\Delta\theta_i) \\ \sin(\Delta\theta_i) & \cos(\Delta\theta_i) \end{bmatrix} \quad (12)$$

In this context $\Delta\theta_i$ for $i = 1, 2, \dots, m$, represents the rotation angle for each Q-bit, dictated by its sign and customized for the particular application.

- **Best Solution Selection:** The best solutions from $B(t-1)$ and $P(t)$ are selected and stored in $B(t)$. If the best solution in $B(t)$ outperforms the current best solution b , the stored solution b is updated.
- **Global Migration:** Upon meeting the global migration criterion, the optimal solution b is disseminated to $B(t)$ on a worldwide scale.
- **Local Migration:** When the local migration criterion is met, the optimal resolution for a local group inside $B(t)$ is to relocate to other members of the same group.
- By adhering to these procedures, QIEA may proficiently monitor the maximum power point of a solar cell and enhance operational circumstances to get optimal power generation.

V. Simulation Results

The simulation studies were conducted using a MATLAB/Simulink-based photovoltaic system model under predefined irradiance and partial-shading conditions. The investigated PSC scenarios were designed to provide a controlled, consistent environment for the comparative evaluation of MPPT algorithms under identical operating conditions. The simulations were conducted using synthetic irradiance profiles rather than experimentally measured datasets. In this study, a representative PSC scenario with rapid irradiance variation from 0.6 to 0.85 kW/m² was considered to provide a controlled comparative evaluation environment for the investigated MPPT algorithms. The selected scenario was intended to facilitate consistent analysis of convergence behavior, oscillation characteristics, and tracking performance under identical operating conditions. Although the adopted PSC scenario provided useful comparative insight, further investigations involving multiple stochastic shading patterns, noisy operating conditions, and experimentally measured irradiance profiles are recommended for future research. Figure 8 depicts the MPP tracking curve for the suggested system, showing a rapid rise in irradiance from 0.6 to 0.85 kW/m². This curve illustrates the irradiance pattern used to assess the proposed MPPT and the algorithms' effectiveness in pinpointing optimal locations.

The red areas in Figure 8 indicate the optimal locations accurately identified by the PSOA, GWOA, and QIEA algorithms. The results for the initial irradiation situation of 0.6 kW/m² are further studied, and the effectiveness of all algorithms is evaluated. Figures 9 to 14 illustrate the outcomes of voltage, current, and output power derived from the five algorithms: PSOA, GWOA, CSOA, GA, and QIEA. Figures 9 and 10 illustrate the current scores achieved by the algorithms. The QIEA demonstrated superior tracking of current points with fewer oscillations relative to the other algorithms. This method reduces the mean overshoot and undershoot, as well as the settling time. Enhanced tracking speed, reduced oscillations, and increased precision will yield improved dynamic response performance. On the other hand, the control system produced the least satisfactory results and was unable to monitor the maximum power accurately. In the cases this study examined, the immediate and authentic implementation of shade modifications was the specific cause. Implementing the scenarios separately would have resulted in varied outcomes. Figure 10 also presents the outcomes of all algorithms for enhanced comparison in the steady-state mode. The QIEA attained more advantageous outcomes. A 17% undershoot is noted at the commencement of operation in the 0.4th second. During this interval, the GWOA improved by 10%. Nevertheless, the CSOA produced the most significant undershoot of around 26% at the 0.4-second mark. The settling time denotes the duration required to achieve 95% of the steady-state condition, characterized by variations of less than 3%.

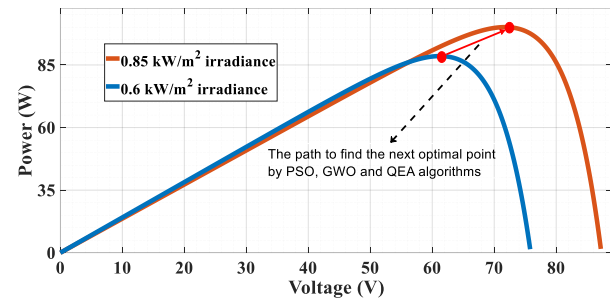


Fig. 8. Tracking curve for a sudden increase in irradiance.

It is important to note that although the GWOA seems to outperform the CSOA, and the CSOA appears inferior to the GWOA, there are occasions when this assessment is not totally correct. For instance, GWOA had a longer maximum settling time than CSOA at a specific point during operation. On average, the GWOA outperformed CSOA. Under typical conditions, average performance is generally the foremost consideration; nevertheless, in exceptional situations, such as with sensitive loads, the maximum and lowest values become critical. According to Figs. 10 and 12, the output current was comparatively lower while using the GA; however, the output voltage was greater with the GA in contrast to the other algorithms. The observed phenomena

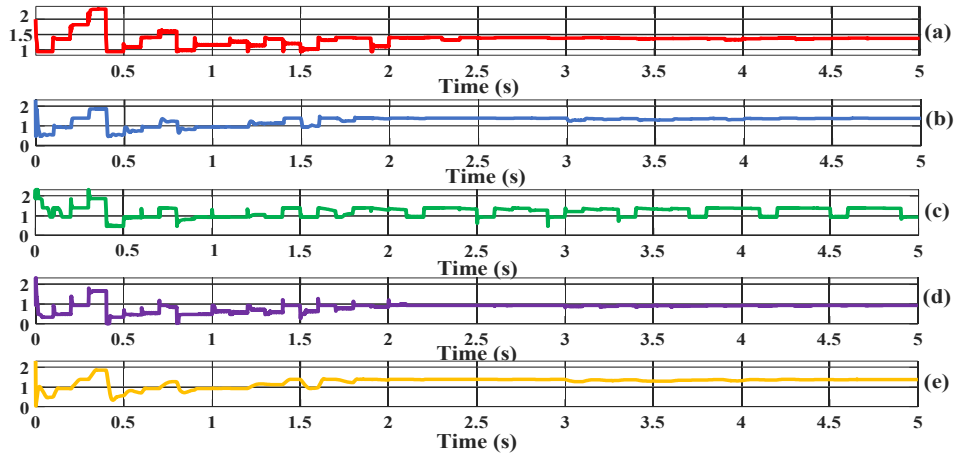


Fig. 9. Output current related to algorithms, (a) PSOA, (b) GWOA, (c) CSOA, (d) GA, (e) QIEA.

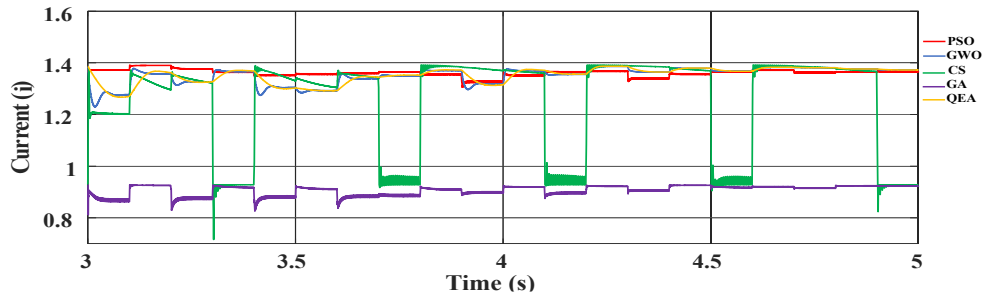


Fig. 10. Steady state of output current in all five algorithms.

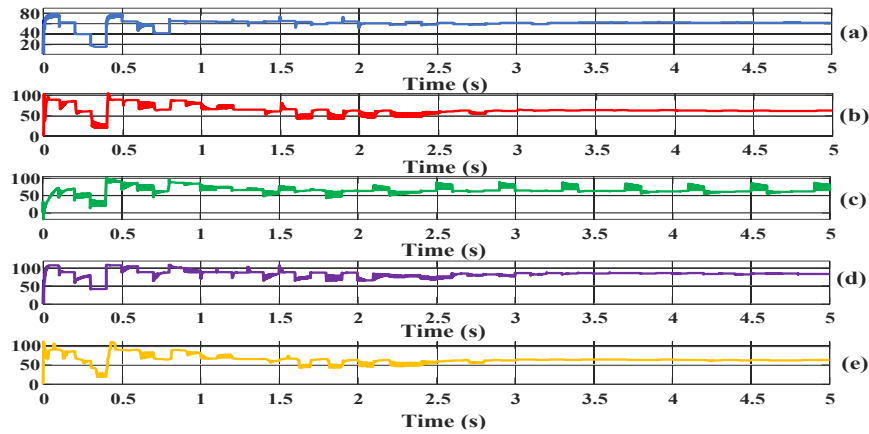


Fig. 11. Output voltage related to algorithms, a: PSOA, b: GWOA, c: CSOA, d: GA, e: QIEA

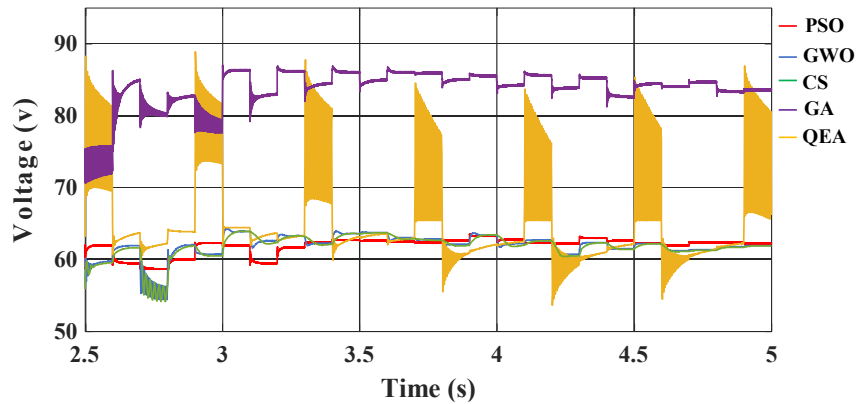


Fig. 12. Steady state of output voltage in all five algorithms.

may stem from the GA algorithm's inability to efficiently optimize the current control and maximize output current, as the optimization appears to have prioritized voltage regulation, resulting in elevated output voltage but diminished output current. Figures 11 and 12 illustrate the output voltage generated by the algorithms. These outputs are similar to the output current in terms of oscillations and transient response characteristics. The performance of QIEA surpasses that of the other algorithms, whereas CSOA exhibits the worst performance. The maximum overrun in the QIEA algorithm is 15%, but in the CSOA algorithm it is 26%. The QIEA algorithm demonstrates superior performance for average overshoots, undershoots, and settling time. The findings concerning the output power are illustrated in Figures 13 and 14. Consistent with prior findings regarding voltage and current, it is anticipated that the power output associated with the QIEA surpassed that of the other algorithms, while the results pertaining to the CSOA algorithm were the least favorable. The power output from the QIEA exhibits minimal oscillation. This is entirely undesirable in the CSOA algorithm. As shown in Figs. 13 and 14, starting at 2.5 seconds, while the other algorithms show little change, CSOA has caused large swings in the output power.

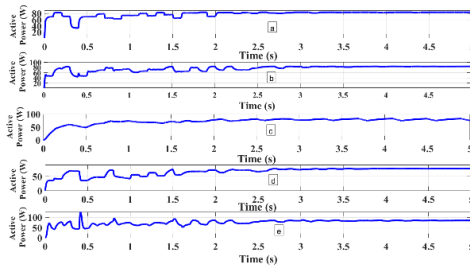


Fig. 13. Output power related to algorithms, a: PSOA, b: GWOA, c: CSOA, d: GA, e: QIEA

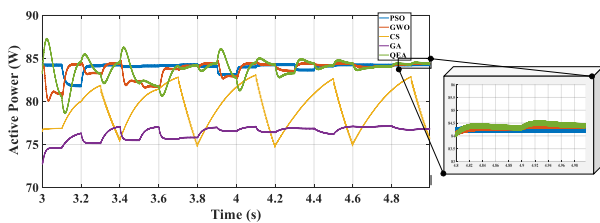


Fig. 14. Steady state of output power in all five algorithms.

Table 1 shows that when conditions are stable, the QIEA performs better than earlier methods in both following the current and producing power, reaching a maximum current of 1.37 A and the best output power of 84.8 W. The GWOA and PSOA exhibited commendable performance, with GWOA achieving 1.36 A and PSOA attaining 1.35 A, but yielding marginally lower power outputs. Conversely, the GA demonstrated the poorest performance in current tracking

(0.95 A) and achieved an output power of 80.37 W. The CSOA did not converge, underscoring its ineffectiveness in this application. The QIEA and PSOA are endorsed for their exceptional efficacy in optimizing power production in shadowed environments. The physics-based Quantum-inspired Evolutionary Algorithm demonstrated superior performance in maximizing the power output of a solar cell, specifically by identifying the maximum power point under shadowed conditions and improving the stability and transient response of the output power. The Grey Wolf Optimizer algorithm, a technique based on swarm intelligence, succeeded the QIEA. The PSOA and GA ranked next, while the CSOA had the poorest performance. The intrinsic attributes and advantages of the various algorithmic methodologies elucidate the observed ranking. QIEA efficiently modeled the intricate non-linear behavior of the solar cell and precisely tracked the maximum power point, particularly under PSC. This arises from the algorithm's basis in quantum physics concepts, which enhanced comprehension of the system dynamics. Swarm-based algorithms, like GWOA and PSOA, are better at searching for solutions than the evolutionary GA method, which leads to better optimization results. The physics-based methodology of the QIEA finally achieved the optimal equilibrium between exploration and exploitation, leading to its enhanced performance in the maximum power point tracking test under shadowed conditions. Table 2 encapsulates the efficacy of the five methods. The presented comparative evaluation was performed under identical operating conditions to ensure a consistent performance assessment of the investigated MPPT algorithms. Although the obtained results demonstrate clear differences in convergence behavior and tracking performance, additional statistical investigations involving multiple independent simulation runs, mean performance values, standard deviation analysis, and detailed convergence curves would provide deeper insight into the robustness and repeatability of the investigated methods. These aspects are recommended for future investigation.

VI. Conclusion

This study examined the efficacy of five distinct algorithms—PSOA, GWOA, CSOA, GA, and QIEA—in optimizing the power output of a solar cell under partial shading conditions and enhancing the quality of the output power. The study's results indicated that among the five algorithms evaluated, the QIEA demonstrated superior performance. The QIEA demonstrated superior tracking of current points with reduced oscillations, less average overshoot and undershoot, and abbreviated settling periods relative to the other algorithms. These features led to the QIEA attaining the most advantageous output power results, accompanied by minimal oscillations. The GWOA algorithm outperformed the QIEA. PSOA and GA followed in the rankings. The CSOA algorithm

had the poorest performance among the five algorithms evaluated. The QIEA's exceptional performance is due to its physics-based foundation, enabling it to predict the intricate

non-linear behavior of the solar cell system and correctly monitor the highest power point, particularly under PSS conditions.

TABLE I Results under steady-state conditions.

Algorithm	Output Current (A)	Output Voltage (V)	Output Power (W)
PSOA	1.35	62.7	84.64
GWOA	1.36	62.0	84.32
CSOA	Not converged	Not converged	Not converged
GA	0.95	84.6	80.37
QIEA	1.37	61.9	84.8

TABLE II Comparative performance evaluation of the investigated MPPT algorithms under PSC

	Swarm-based methods			Evolutionary method	Physics-based method
	PSOA	GWOA	CSOA	GA	QIEA
Tracking speed	Moderate	Moderate	Low	Low	Fast
Tracking accuracy	Accurate	Accurate	Low	Low	Highly accurate
Convergence to global peak	Yes	Yes	No	No	Yes
Steady state oscillations	Low	Low	Very high	Moderate	Low
Implementation complexity	Low	Medium	Medium	Low	Medium

Swarm-based algorithms such as GWOA and PSOA show superior efficacy in exploring the solution space compared to GA; nevertheless, QIEA achieved the optimal equilibrium of exploration, exploitation, and comprehension of the solar cell system. Conversely, the CSOA algorithm failed to adequately represent the intricate non-linear attributes of the solar cell system when compared to alternative approaches, leading to its suboptimal performance. The obtained results were further compared with recently published MPPT techniques reported in the literature. Compared with conventional PSO- and GA-based methods, the QIEA demonstrated improved convergence characteristics and lower steady-state oscillations under partial shading conditions. The superior performance of the QIEA can be attributed to its effective balance between exploration and exploitation processes, which enables more accurate identification of the global maximum power point (GMPP). Moreover, the GWOA exhibited competitive tracking accuracy and relatively low oscillations compared with the CSOA and GA methods. Although the CSOA achieved acceptable convergence speed in some operating conditions, it suffered from significant oscillations and convergence instability under rapidly changing irradiance levels. In contrast, the GA showed relatively lower tracking accuracy due to premature convergence behavior and limited global search capability. Compared with recently published hybrid and intelligent MPPT approaches, the investigated algorithms demonstrated comparable tracking performance while maintaining relatively moderate implementation complexity. However, further investigation under stochastic shading conditions and noisy operating environments remains an important topic for future research.

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A Bio-Inspired Optimization: A New Meta-heuristic Based on Animal Adaptability



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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 15 July 2025 Received in revised form: 30 September 2025 Accepted: 16 October 2025 Published online: 22-Dec-2026 Keywords: Animals' adaptability algorithm, meta-heuristic algorithm, optimization, Decision-making.	Organisms continuously adapt to changing resources and environmental conditions to ensure survival. Such adaptations may involve shifts in diet, habitat, or hunting strategies. Species that fail to adapt often face extinction, as evidenced by many species that no longer exist. Adaptation can generally be understood as a two-stage process. The first stage encompasses phenotypic adjustments, which occur in response to factors such as climate change, food availability, or the development of adaptive behaviors. These short-term changes increase the chances of survival for individuals by favoring the fittest under immediate conditions. The second stage involves genetic changes, where offspring chromosomes are formed through crossover and mutation of parental chromosomes. These genetic variations may either enhance the survival of the next generation or reduce it, thereby influencing long-term evolutionary success. In this study, a novel metaheuristic algorithm is introduced, inspired by the adaptability of socially living animals. The proposed algorithm was evaluated on a set of standard benchmark functions and compared with several well-established optimization methods. The results indicate that the proposed algorithm outperforms others in terms of convergence speed, solution accuracy, and robustness. Finally, to demonstrate its practical applicability, the algorithm was applied to an engineering problem involving air-conditioner load scheduling, where it achieved superior performance.

NOMENCLATURE			
$T_{in}(^{\circ}C)$	Tripping time of the i -th relay.	$P_{cool}(W)$	electrical power consumption of the AC system
$C(J/^{\circ}C)$	thermal capacity of the room, accounting for heat storage within the room's air and surfaces.	COP	Coefficient Of Performance of the AC, which measures its efficiency in converting electrical energy into cooling capacity.
$Q_{cool}(w)$	cooling power supplied by the AC system.	Cost	Total electricity cost over the optimization period.
$Q_{loss}(w)$	heat losses due to thermal conduction and convection through walls and windows.	N	Number of time intervals considered.
$U(W/m^2^{\circ}C)$	heat transfer coefficient of walls and windows, indicating their insulation efficiency.	$T \circ max_{min}/$	the acceptable range for indoor temperature, ensuring thermal comfort
$A(m^2)$	the total surface area of walls and windows exposed to external conditions.	$Q_{cool,min/max}$	specify the operational limits of the AC cooling power.
APP_i^{new}	outdoor temperature.	Price(t)	the electricity price at time t.

I. Introduction

Metaheuristic algorithms have become essential tools for solving complex optimization problems in engineering, computer science, and resource management. These algorithms provide effective strategies for exploring large and

complex search spaces, offering solutions where conventional optimization methods often fail. Their flexibility and wide applicability have led to extensive adoption in real-world problems, including network design, scheduling, and industrial optimization. Although many metaheuristics are

inspired by natural or social phenomena, their effectiveness ultimately depends on maintaining an appropriate

trade-off between global exploration and local refinement within the search space.

Over the past decades, numerous metaheuristic algorithms have been proposed, each addressing specific optimization challenges. For example, Genetic Algorithms (GA) provide robust global search capabilities but may exhibit slow convergence [1-4]. Similarly, Ant Colony Optimization (ACO) effectively tackles routing and combinatorial problems, though at a high computational cost [5-6]. In contrast, Particle Swarm Optimization (PSO) and Differential Evolution (DE) perform well in continuous optimization; however, PSO can be trapped in local optima, while DE may converge slowly in highly multi-modal landscapes [7-10]. Swarm-based approaches, such as the Artificial Bee Colony (ABC) algorithm, are simple and require few parameters, yet they often struggle with more complex problems [11-12]. The Firefly Algorithm (FA) shows strong performance in multi-objective optimization but tends to converge slowly [13]. In Contrast, Cuckoo Search (CS) can escape local optima but is highly sensitive to problem scaling [14]. Likewise, the Bat Algorithm (BA) offers a reasonable balance between diversification and intensification, though it may underperform in large-scale scenarios [15]. Teaching-Learning-Based Optimization (TLBO) is parameter-free and straightforward approach, but its convergence rate slows considerably in high-dimensional problems [16]. More recent algorithms, including the Grey Wolf Optimizer (GWO), Moth-Flame Optimization (MFO), and Whale Optimization Algorithm (WOA), aim to enhance the exploration–exploitation balance and improve global search capability. Nevertheless, issues such as premature convergence, parameter sensitivity, and inconsistent performance in complex or high-dimensional problems remain [17-21]. Collectively, these methods highlight substantial progress in metaheuristic optimization while revealing persistent limitations.

Despite these advances, convergence speed and solution accuracy remain critical challenges. These issues are further exacerbated as the dimensionality of the problem increases, with many existing algorithms unable to maintain both high-quality solutions and rapid convergence in large-scale, complex scenarios. Therefore, there is a clear need for algorithms that can overcome these limitations while preserving an effective balance between exploration and refinement.

Thus, despite the significant progress in metaheuristic design, there is still a lack of algorithms that simultaneously ensure fast convergence and high-quality solutions in large-scale, complex problems. This gap motivates the development of the proposed Animal Adaptability Algorithm(AdA).

In this study, inspired by the adaptability of animals and their responses to changing environments, the Animal Adaptability Algorithm(AdA) is introduced. AdA leverages mechanisms inspired by territorial and social behaviors to achieve faster convergence, higher accuracy, and greater robustness in complex optimization problems. By simulating natural processes such as search space refinement, competition among individuals, and the generation of new offspring, AdA enables effective diversification and intensification of solutions simultaneously, thereby making it particularly suitable for a wide range of challenging optimization tasks in engineering, resource management, and network design.

II. Methodology

A. Fundamental Concept of the Animal Adaptability Algorithm

Some animal species, including cheetahs, tigers, wild boars, and songbirds, establish specific territories for survival and reproduction. This territorial behavior is a natural phenomenon from the perspective of behavioral biology. Moreover, it can serve as an inspiring model for designing search and optimization algorithms in the field of artificial intelligence. Animals with greater physical strength and health are capable of claiming richer territories. In algorithmic terms, this is analogous to stronger agents or better solutions occupying more promising regions in the search space. Consequently, access to abundant resources increases the likelihood of survival and, ultimately, the probability of transmitting favorable traits to the next generation.

The foraging behavior within a territory can also be compared to the search process in metaheuristic algorithms. Instead of conducting a random and dispersed search, animals utilize experience and knowledge acquired from their parents to narrow their search domain, thereby finding higher-quality resources. Similarly, in algorithms, the search usually begins in broad areas, but as the process progresses and memory and learning are incorporated, the focus gradually shifts toward more promising regions. In the AdA algorithm, this natural behavior is modeled using a distance-halving method, whereby the search space is progressively reduced to facilitate access to more promising points.

Furthermore, the competition among animals to maintain their territories and repel same-sex rivals is analogous to the selection operation in evolutionary algorithms, where only the strongest agents survive and contribute more to the formation of the next generation. Subsequently, the process of generating new individuals begins, which corresponds to the crossover and mutation operations in evolutionary algorithms. These processes enable the transmission of genetic information from parents to offspring and introduce diversity into the population.

It should be noted that the distance-halving method is not restricted to unimodal functions; it primarily accelerates convergence near promising solutions, while other operators—such as crossover, mutation and absorption/repulsion—continue to explore the broader search space for global optima.

B. Implementation of Adaptability Algorithms

The mathematical modeling of the proposed adaptability algorithm is based on the following hypotheses:

- Animals that locate superior food sources are more likely to overcome their competitors.
- Both the behavioral and genetic characteristics of animals evolve when they encounter novel or challenging circumstances.
- It is feasible to assess the simultaneous effects of rivalry and individual performance in a given environment.
- The superior traits of earlier generations are maintained and will persist if no more adaptable individuals are produced in subsequent generations.

B.1 Initialization of the Population

The optimization process begins when species, such as birds, mammals, or aquatic animals, colonize a new environment. To model this scenario, an initial population of potential solutions is generated within a feasible search space. Each solution in the population represents a distinct characteristic of the animals. Formally, the initial population can be denoted as:

$$r_{i,j} = |APP_j^{old} - APP_i^{old}|$$

$$APP_i^{new} = APP_i^{old} + e^{\frac{-1}{r_{i,j}}} (APP_j^{old} - APP_i^{old}) \quad (1)$$

$$APP_k^{new} = APP_j^{old} - e^{\frac{-1}{r_{j,k}}} (APP_j^{old} - APP_k^{old}) \quad [APP_1, APP_2, \dots, APP_N], \quad NP = \text{Population size}$$

, representative of distinctive feature of the animal i.

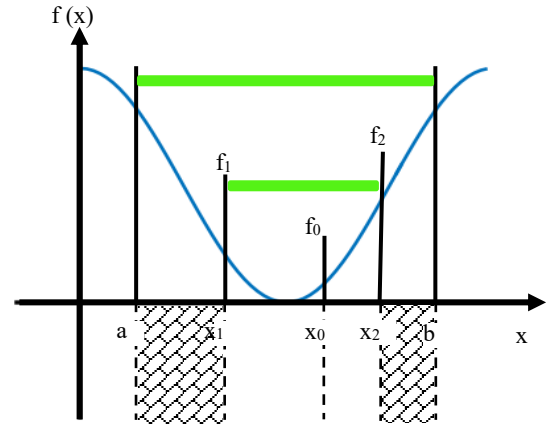
B.2 Modeling Animal Decision-Making

Animals often face numerous challenges and threats in their environment, prompting them to make decisions such as selecting food sources or relocating to new habitats. These decisions can either improve or impair their survival and well-being. To model this process, a unimodal function is used, as the goal is for the animal to find the optimal solution. If the animal's decision is correct, it progresses toward its goal, but an incorrect decision may lead to weakening or even death.

The probability of succeeding through random decision-making without prior knowledge is low, and the consequences may be irreversible. Therefore, animals tend to rely on the knowledge and experiences of other group members. This social learning behavior is common in species such as whales, dolphins, and others. In this study, the interval halving method is employed to model the decision-making process. As

depicted in Figure 1(a), the method eliminates 50% of the information from group members whose results are weaker.

For example, if a disease spreads in the population, animals avoid high-risk areas and move toward safer regions $[a, x_0]$. In the interval halving approach, information from the group is gathered, and decisions are made within the search space. Given that decision-making assumes the unimodal nature of the goal function, 50% of the search space is discarded. Ignoring the current situation or an animal's prior experience would be irrational and may lead to suboptimal decisions. As a result, the animal's own experiences are integrated into the search space, reducing the search space to the region $[x_1, x_0]$, as shown in Figure 1(b).



(a)

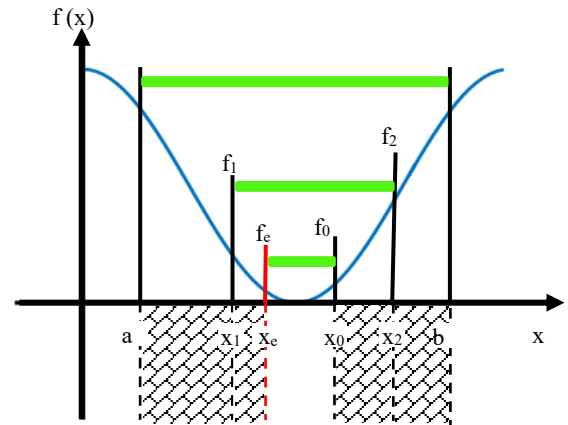


Figure 1: (a) Interval halving method, (b) Integration of the animal's experience with collective knowledge from the group

When the gathered information is accurate, new decisions lead to improved conditions. However, if the information is incorrect, the decision-maker may either recognize and discard it, or it may result in erroneous decisions. When the collected information contradicts the unimodal function, the information will be eliminated, and the decision-making area will be updated.

Once the new search space has been identified, the decision-maker proceeds with the search for an optimal solution. If the decision improves the animal's condition, it will reach its goal. However, an incorrect decision will push the animal away from the optimal solution and create opportunity for competitors to capitalize on the mistake.

B.3 Repulsion and Attraction

Male animals, considering their physical condition, engage in competition to define and defend territories. The strongest animals control larger territories, and as a result, they attract more mates, thereby playing a significant role in the formation of the next generation. This competitive behavior is modeled by the following equation:

$$\begin{aligned} r_{i,j} &= \left| \frac{APP_j^{old} - APP_i^{old}}{APP_i^{new}} \right| \\ &= APP_i^{old} + e^{\frac{-1}{r_{i,j}^2}} (APP_j^{old} - APP_i^{old}) \\ &= APP_j^{old} - e^{\frac{-1}{r_{j,k}^2}} (APP_j^{old} - APP_k^{old}) \end{aligned} \quad (2)$$

In Equation (2), the variable APP_i^{old} on the right-hand side represents the previous position of agent i , while the variable APP_i^{new} on the left-hand side denotes the updated (new) position of agent and $r_{i,j}$ shows Euclidean distance of the two animals APP_i^{old} and APP_j^{old} .

B.4 Creating new generation and teaching them

After forming territories, parents meet each other and create new generations. And the new generation will inherit genetic characteristics from their parents. In order to model this section, mutation and crossover were used.

B.5 Crossover

As it was mentioned earlier, new generations inherit their parents characteristics. the simplest methods choose one or more points in the chromosome to mark as the crossover points. Then the variables between these points are merely swapped between two parents. Consider the two parents to be:

$$\begin{aligned} APP_i &= [APP_{i,1}, APP_{i,2}, \dots, APP_{i,h}, \dots, APP_{i,N}] \\ APP_j &= [APP_{j,1}, APP_{j,2}, \dots, APP_{j,h}, \dots, APP_{j,N}] \end{aligned} \quad (3)$$

In Equation (3), $APP_{i,h}$ represents h -th dimension of the i -th agent position or equivalently h -th decision variable of the i -th individual and N indicates number of decision variables of the optimization problem.

In this method, parents are selected according to roulette wheel mechanism.

Crossover points are randomly selected, and the variables in crossover point present:

$$\begin{aligned} APP_{i,h}^{new} &= APP_{i,h} - \beta (APP_{i,h} - APP_{j,h}) \\ APP_{j,h}^{new} &= APP_{j,h} + \beta (APP_{i,h} - APP_{j,h}) \end{aligned} \quad (4)$$

β is a random number; It should be noted that the range of the β parameter in the intersection mechanism is limited to $[0, 1]$, and a boundary correction strategy is applied to ensure that generated offspring remain within the feasible solution space.

The new generation will be formed as follows:

$$\begin{aligned} APP_i^{new} &= [APP_{i,1}, \dots, APP_{i,h-1}, APP_{i,h}^{new}, APP_{j,h+1}, \dots, \\ &APP_{j,N}] \\ APP_j^{new} &= [APP_{j,1}, \dots, APP_{j,h-1}, APP_{j,h}^{new}, APP_{i,h+1}, \dots, APP_{i,N}] \end{aligned} \quad (5)$$

B.6 Mutation

During the formation of a new generation, genetic mutations may introduce variations between offspring and their parents. These mutations not only account for random changes in inherited traits but also enable the acquisition of new skills. Unlike their parents, the new generation benefits both from inherited experiences and trial-and-error learning, allowing them to develop capabilities that were not present in previous generations. This process can be effectively modeled using the mutation operator.

$$\begin{aligned} APP_i^j &= (var \ h \ i_{APP_i}^j - var \ l \ o_{APP_i}^j) \\ &* rand + var \ l \ o_{APP_i}^j \end{aligned} \quad (6)$$

Flowchart of the algorithm is shown in Figure 2.

The following pseudo-code summarizes all main operations of the algorithm.

TABLE I : pseudo-code for ada algorithm

Line1:	Initialize agents randomly.
Line2:	Evaluate the fitness
Line3:	BestSolution = best agent.
Line4:	for iter = 1 to MaxIter do
Line5:	Reduce the search space using distance-halving.
Line6:	Apply attraction-repulsion adjustment.
Line7:	Generate offspring via crossover.
Line8:	Apply mutation to offspring.
Line9:	Evaluate updated fitness.
Line10:	Update BestSolution if improved.
Line11:	end for
Line12:	Return BestSolution.

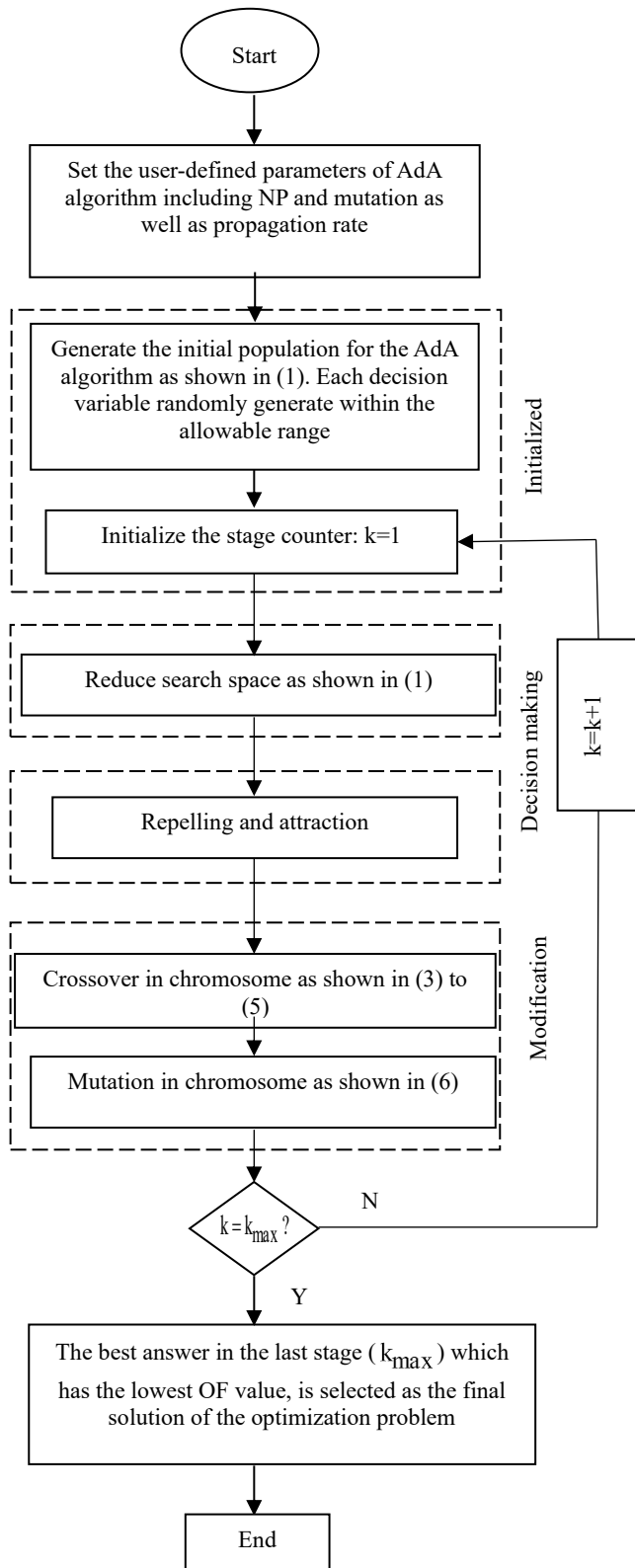


Figure 2: Flowchart of the algorithm

C. Test function

To evaluate the accuracy and efficiency of the proposed method, a comprehensive set of standard benchmark functions, as outlined in Table 2, has been employed.

Furthermore, the proposed algorithm has been tested in a real-world scenario involving the optimal scheduling of air conditioning systems to minimize energy consumption and associated costs. The results of this practical evaluation highlight the algorithm's effectiveness in addressing complex real-world optimization problems, demonstrating its strong potential for deployment in practical applications.

In the following, a dynamic model[23] is presented for optimizing electricity costs and regulating indoor temperature (T_{in}), aiming to balance air conditioner (AC) energy consumption with thermal comfort. This model incorporates cooling power, heat losses, and internal heat sources, enabling intelligent analysis and management of HVAC systems. The indoor temperature dynamically evolves according to Equation (7):

$$C \frac{dT_{in}}{dt} = Q_{cool} - Q_{loss} \quad (7)$$

Heat losses through walls and windows are modeled using Equation (8):

$$Q_{loss} = U.A.(T_{in} - T_{out}) \quad (8)$$

The AC power consumption is determined based on its cooling capacity and coefficient of performance (COP), as expressed in Equation (9):

$$P_{cool} = \frac{Q_{cool}}{COP} \quad (9)$$

The objective function for minimizing electricity costs is formulated in Equation (10):

$$Cost = \sum_{t=1}^N P_{cool}(t) \cdot At. Price(t) \quad (10)$$

s.t.

$$T_{min} \leq T_{in} \leq T_{max}$$

$$Q_{cool,min} \leq Q_{cool} \leq Q_{cool,max}$$

III. Test Results

The comparative performance of the proposed Animal Adaptability Algorithm (AdA) and ten well-established metaheuristics on four standard benchmark functions (Rastrigin, Griewangk, Schaffer N.2, and Bohachevsky N.1) is reported in Table 3. Each experiment was performed over 100 independent runs, and the minimum (Min), mean (Mean), and standard deviation (Std) of the obtained solutions were calculated to ensure statistical robustness and to assess the algorithm's accuracy and stability.

A detailed analysis of the results reveals that AdA consistently achieved the global optimum across all benchmark functions, with both the minimum and mean values approaching the optimal solution. Moreover, the standard deviation of AdA remained significantly lower than that of most competing algorithms, indicating not only high accuracy but also strong stability and repeatability of the results. For instance, in the Rastrigin and Griewangk test

TABLE II : test functions

No	Benchmark Function	Function	Absolute minimum
1	Rastrigin Benchmark Function	$f_1(x) = 20$ $+ \sum_{k=1}^2 (x_k^2 - 10 \cdot \cos(2 \cdot \pi \cdot x_k))$ $-20 \leq x_k \leq 20$	0
2	Griewangk Benchmark Function	$f_2(x) = \sum_{k=1}^2 \frac{x_k^2}{4000} - \prod_{k=1}^2 \cos\left(\frac{x_k}{\sqrt{k}}\right) + 1$ $-600 \leq x_k \leq 600$	0
3	Schaffer N.2 Benchmark Function	$f_3(x_1, x_2) = 0.5$ $+ \frac{\sin^2(x_1^2 - x_2^2) - 0.5}{(1 + 0.001(x_1^2 - x_2^2))^2}$ $-100 \leq x_k \leq 100, k = 1, 2$	0
4	Bohachevsky N.1 Benchmark Function	$f_4(x_1, x_2) = x_1^2 + 2x_2^2$ $- 0.3 \cos(3 \cdot \pi \cdot x_1)$ $- 0.4 \cos(4 \cdot \pi \cdot x_2) + 0.7$ $-100 \leq x_k \leq 100, k = 1, 2$	0
5	Ackley Benchmark Function	$f_5(x) = -\alpha \exp\left(-b \sqrt{\frac{1}{d} \sum_{i=1}^d x_i^2}\right)$ $- \exp\left(\frac{1}{d} \sum_{i=1}^d \cos(cx_i)\right) + \alpha$ $+ e$ $-32.768 \leq x_k \leq 32.768$	0
6	Levy Benchmark Function	$f_6(x) = \sin^2(\pi \omega_1)$ $+ \sum_{i=1}^{d-1} (\omega_i - 1) [1 + 10 \sin^2(\pi \omega_i + 1)]$ $+ (\omega_d - 1)^2 [1 + \sin^2(2\pi \omega_d)]$ $\omega_i = 1 + \frac{x_i - 1}{4}$ $-10 \leq x_k \leq 10$	0

functions (f1 and f2), AdA outperformed algorithms such as BA, BFO, and HS by several orders of magnitude in terms of solution accuracy and robustness. Another notable feature of AdA is its computational efficiency. The average execution time of AdA over 100 runs ranged from 6.4 to 8.7 seconds, which is substantially faster than algorithms such as HS (over 300 seconds), SA (about 150–200 seconds), and even classical methods like PSO (20–40 seconds). This fast convergence highlights the algorithm's effective balance between exploration and exploitation. When compared to strong competitors such as CS, BBO, and GSA, AdA demonstrated comparable or superior accuracy but achieved convergence in fewer iterations and with much lower computational cost. While algorithms like CS and BBO also reached optimal

solutions, they required significantly more time, which limits their practicality for real-world large-scale problems.

To further assess the scalability and robustness of the proposed AdA algorithm, additional experiments were performed on four benchmark functions: Rastrigin (f1), Griewangk (f2), Ackley (f5), and Levy (f6), with 10, 50, 100, and 500 decision variables. Each configuration was executed 100 times independently, and the minimum (Min), mean (Mean), standard deviation (Std), and execution time were recorded, as summarized in Table 4. The results clearly demonstrate that AdA consistently achieves near-optimal or optimal solutions across all tested functions and dimensions. Even as the dimensionality increases, the algorithm maintains low variation among runs, indicating strong stability and repeatability. For instance, in the 500-dimensional Rastrigin and Griewangk test function, AdA successfully converged to high-quality solutions without a significant increase in computational error, while maintaining reasonable execution times.

In the continuation, the performance of the proposed Adaptive Algorithm (AdA) in optimizing energy consumption in air conditioning systems is evaluated. To assess the effectiveness of this algorithm, a comparison is made with other metaheuristic algorithms, including PSO, GA, and BBO. The simulation results, presented in Figure 3, demonstrate the superiority of the AdA algorithm in terms of convergence speed and accuracy in reaching the optimal solution.

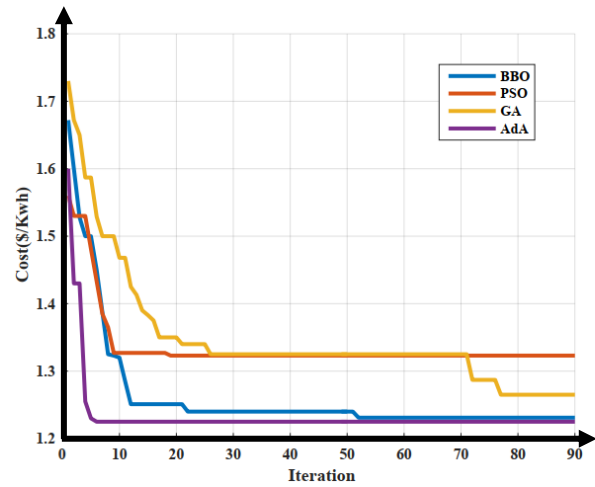


Figure3: Rate of convergence of algorithms

IV. Limitations and Future Work

AdA demonstrates robust and efficient performance across the tested benchmark functions. However, certain aspects present limitations, which also highlight some of the algorithm's strengths.

- I. **Parameter Adaptability:** AdA requires tuning of certain control parameters; however, this flexibility allows the algorithm to be effectively adapted to a wide range of optimization problems.

Table 3: Simulation results

Function	Statistic	Algorithms				
		GSA	HS	PSO	SA	AdA
f1	Min	0	8.42E-08	0	4.19E-05	0
	Mean	1.41E-01	4.62E-05	4.30E-11	5.32E-03	0
	Std	3.17E-01	7.37E-05	4.30E-10	7.65E-03	0
	Time(s)	102.4	345.8	39.6	208	6.4
f2	Min	0	4.44E-10	0	1.12E-08	0
	Mean	9.04E-03	4.22E-03	4.01E-03	1.25E-05	0
	Std	6.22E-03	3.68E-03	3.85E-03	1.82E-05	0
	Time(s)	107.4	360.8	41.7	192.6	8.7
f3	Min	0	1.11E-12	0	2.96E-10	0
	Mean	4.61E-05	1.02E-03	5.97E-08	7.71E-08	0
	Std	2.89E-04	1.71E-03	5.69E-07	1.65E-07	0
	Time(s)	73.7	341.2	20.2	101.9	6.7
f4	Min	0	4.53E-08	0	1.71E-06	0
	Mean	2.89E-03	5.14E-06	4.16E-07	9.50E-04	0
	Std	2.89E-02	1.12E-05	4.16E-06	2.49E-03	0
	Time(s)	73.7	337.3	23.4	159.8	6.7

Table 4: AdA performance for 10–500 variables (100 runs)

Function	Statistic	Dimension (D)			
		10	50	100	500
F1	Min	0	0	0	0
	Mean	0	0	0	0
	Std	0	0	0	0
	Time(s)	11.1	14.5	17.58	112.4
F2	Min	0	0	0	0
	Mean	0	0	0	0
	Std	0	0	0	0
	Time(s)	12.6	19.1	30	199.5
F5	Min	8.8E-16	8.8E-16	8.8E-16	8.8E-16
	Mean	8.8E-16	8.8E-16	8.8E-16	8.8E-16
	Std	8.8E-16	8.8E-16	8.8E-16	8.8E-16
	Time(s)	12.9	17.1	23.1	145
F6	Min	1.9E-4	1.8E-3	2.1E-3	2.1E-2
	Mean	8.3E-3	2.7	5.46	33.9
	Std	7E-3	1.6	3.36	14.2
	Time(s)	27.2	50.3	104.1	618

- I. Computational Effort: The computational cost may increase for very large-scale problems, yet this is due to its thorough exploration and exploitation mechanisms, which enhance solution quality.
- II. No Guarantee of Global Optimum: Like most metaheuristics, AdA does not guarantee finding the global optimum for all problem types. Nevertheless, it consistently achieves high-quality solutions across diverse benchmarks, demonstrating its robustness and reliability.

For future studies, extending AdA to handle multi-objective and dynamic optimization problems can broaden its applicability. Furthermore, developing hybrid versions that integrate AdA with classical optimization methods or machine learning techniques may further enhance its performance. In addition, applying AdA to real-world engineering problems such as energy systems, scheduling, and network design can provide deeper insights into its practical effectiveness.

Table 3 (continued): Simulation results

Function	Statistic	Algorithms					
		ABC	CS	BA	BBO	CA	GA
f1	Min	3.43E-08	0	3.99E-09	0	0	9.63E-05
	Mean	6.19E-03	5.54E-15	1.47E-05	4.57E-06	1.79E-01	7.39E-01
	Std	1.53E-02	1.21E-14	1.77E-05	2.29E-05	4.09E-01	7.32E-01
	Time(s)	108.3	104.8	123	41	36.8	15.9
f2	Min	1.37E-05	3.33E-15	1.38E-10	0	0	1.18E-05
	Mean	3.63E-03	2.24E-04	5.61E-08	3.25E-03	1.31E-02	5.10E-03
	Std	3.10E-03	7.69E-04	5.90E-08	3.69E-03	1.15E-02	4.10E-03
	Time(s)	111.9	140.6	130.8	44.8	39.3	21
f3	Min	2.62E-08	9.33E-15	1.40E-14	0	0	1.56E-07
	Mean	7.22E-04	1.81E-05	2.12E-07	4.01E-04	3.35E-06	4.70E-03
	Std	1.30E-03	4.48E-05	1.57E-06	8.79E-04	3.35E-05	4.80E-03
	Time(s)	102.8	119.3	97.5	38.3	35.5	13.3
f4	Min	0	0	3.59E-09	0	0	1.32E-06
	Mean	1.77E-07	5.03E-15	1.91E-06	5.42E-08	1.31E-14	8.58E-02
	Std	2.28E-06	2.99E-15	2.25E-06	2.91E-07	8.65E-14	0.1436
	Time(s)	97.35	64.8	99.9	38.1	35.6	13.3

V. Conclusions

In recent decades, optimization problems in various fields such as engineering have become increasingly complex. Meta-heuristic algorithms, particularly those inspired by nature and swarm intelligence, have emerged as effective approaches to tackle such problems. Their flexibility allows them to handle complex scenarios and provide high-quality solutions. In this study, a new meta-heuristic algorithm based on animals' adaptability was proposed. The algorithm consists of three stages: decision making, repelling or attraction, and reproduction. In the first stage, the animals' decision-making process in different situations was modeled using the interval halving method. In the second stage, competition among members to define territories was represented, followed by reproduction in the final stage.

The proposed algorithm was first tested on small-scale benchmark problems, where it outperformed other algorithms such as BBO, GA, PSO, and CA in terms of convergence speed and solution stability. To further evaluate its robustness, the number of decision variables was increased up to 500, and

the algorithm continued to provide reliable and stable results. Furthermore, the algorithm was applied to a practical problem of air conditioner load scheduling, where it again demonstrated superior performance compared to the other methods in terms of solution quality and stability.

The algorithm was implemented in MATLAB and compared with other well-known methods. The results demonstrated that the proposed approach is capable of reaching the global optimum with faster convergence and reduced computational time compared to existing algorithms. Overall, these findings highlight the flexibility, robustness, and effectiveness of the proposed method for both benchmark and real-world optimization problems.

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Adhesion Force Estimation of Railway Vehicles using Intelligent Square Root Cubature Kalman Filter

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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 11 March 2025 Received in revised form: 26 May 2025 Accepted: 06 October 2025 Published online: 22-Dec-2026 Keywords: Adhesion force, Square root cubature Kalman filter, Railway vehicles, Differential evolution (DE)	Railway traction systems rely on the adhesion force between wheels and rails to effectively transmit mechanical power. Accurate estimation of this force is crucial to avoid wheel slip, improve safety, and enhance traction and braking performance. However, due to the influence of numerous uncertain factors, adhesion force estimation is a complex task. This paper proposes an Intelligent Square Root Cubature Kalman Filter (ISRCKF) for the estimation of wheel–rail adhesion force, without requiring prior knowledge of noise statistics. The proposed method integrates the Differential Evolution (DE) algorithm to adaptively tune the process and measurement noise covariance matrices Q and R, enhancing estimation accuracy and robustness under unknown and time-varying disturbances. In addition, the square-root formulation ensures that all covariance matrices remain positive semi-definite, thereby improving numerical stability. The proposed approach actively contributes to improving traction utilization and reducing wheel wear by minimizing excessive creepage. Simulation results indicate that the proposed ISRCKF achieves a 20% reduction in speed RMSE and a 50% reduction in load torque RMSE compared to the Unscented Kalman Filter (UKF). Additionally, the ISRCKF exhibits a faster convergence rate, enabling rapid stabilization of adhesion force estimates under varying wheel–rail conditions

I. Introduction

Estimating the adhesion conditions at the wheel-rail contact interface during railway operations is a crucial undertaking for the railway industry. The adhesion force encountered in this interface is not only difficult to measure but also influenced by numerous uncertain parameters [1]. Accurately estimating the adhesion force is crucial for understanding the braking and traction capabilities, which play a vital role in performance and safety considerations [2-3]. However, determining the adhesion force is a challenging task due to the simultaneous consideration of various operational and environmental factors, including weather conditions and load conditions [4]. In addition, when the adhesion is underestimated, unexpected occurrences of wear as well as noise at the wheel-rail interface may be detected during operation [4-5]. Researchers focus on estimation of adhesion force.

In the past few years, there have been various proposed models, such as the Kalker model, Oldrich Polach model, and Beam model, aimed at estimating the adhesion force [6-8]. However, these models cannot accurately determine the

adhesion characteristics in practical use. Maximum likelihood estimation identifies the adhesion model but is time-consuming and not easy to do on-line [4],[9]. Sliding mode observers estimate adhesion force with a simple structure and strong robustness, but the disadvantage of which is the inevitable chattering phenomenon [10].

A novel approach has been proposed in [11-12] to estimate the adhesion force of the traction AC motor. This approach utilizes the stator voltage, current, and speed, and employs the Extended Kalman filter (EKF) for accurate estimation. Another method is multi-rate EKF state identification to detect slip velocity [13-14]. The Kalman-Bucy filtering technique is employed to estimate the forces causing creep on the rear wheelset of the front bogie. This estimation is based on a non-linear contact mechanics model of the half vehicle [15-16]. To effectively address the wheel slip/slide phenomenon and estimate the adhesion restriction, a multiple version estimation method is utilized. This method involves employing a bank of Kalman filters at specific operation points to accurately identify the adhesion levels [17-19]. Nevertheless, the EKF employs a simplified representation of the machine through the computation of the

Jacobian matrix. Unfortunately, this approach not only introduces additional intricacy and hardware implementation challenges but also proves inadequate for highly nonlinear model systems.

In situations where the system exhibits significant nonlinearity, the EKF may not provide sufficiently accurate results, thereby leading to the possibility of divergence. Furthermore, in order to linearize the system and acquire the Jacobian matrices, it is necessary for the system to possess continuous differentiability. Consequently, due to the computational complexity involved in calculating the Jacobian matrices, the EKF incurs substantial computational costs. In order to address these challenges, the unscented Kalman filter (UKF) is utilized to estimate the adhesion force.

The UKF is a recursive linear estimator. In this method, a set of sigma points are used to calculate the mean and the covariance estimation. Computational complexity is reduced when these sigma points are used in the UKF. In [20-21], UKF is used to estimate the adhesion force from traction motor behaviours. Using the traction motor behaviors, UKF is used to calculate the binding force. Under various contact conditions, UKF gave accurate friction estimation, but the estimated friction coefficient is unreliable and also affected by the traction load when it is very low. These issues are the result of inaccurate observations and a flawed system dynamic model. However, it can be challenging to change the UKF parameters in a variety of applications to get the intended performance. Furthermore, the effectiveness of the filter in higher dimensions could be diminished as a consequence of a specific parameter in the UKF becoming negative.

To address UKF drawbacks, a cubature Kalman filter (CKF) is presented [22]. The CKF overcomes the difficulties associated with evaluating the Jacobian matrix, as experienced in the EKF. Moreover, it eliminates the need for parameter tuning, which is necessary in the case of UKF[23-24]. A limitation CKF for estimation of the adhesion force is that a priori statistics of the stochastic errors in both measurement and process models are assumed to be available. It is intractable to obtain an accurate estimation with the unknown statistical characteristics of noise. Therefore, the development and application of CKF method have been limited. However, accurate noise statistics data are difficult to obtain, and they are subject to change over time. Consequently, the set of unknown time-various statistical parameters of noise needs to be simultaneously estimated with the system state and the error covariance. The covariance matrix additionally needs to be symmetric and positive definite in order to carry out an accurate state estimation and set up the convergence condition for CKF. In many instances, the covariance matrix turns negative definite as a result of computational inaccuracy and numerical errors. Recent studies have aimed to enhance the robustness and adaptability of Kalman filter variants in complex railway environments.

For example, the STF-SCKF-NE algorithm proposed in [25] addresses nonlinear adhesion estimation but still relies on fixed noise covariance matrices, limiting its adaptability to time-varying conditions. In [26], a high-order Cubature Kalman Filter (CKF) is presented for joint estimation of vehicle dynamics and road adhesion coefficient, offering improved nonlinear performance. However, this approach lacks a mechanism for online noise adaptation, which is critical in practical traction systems.

Furthermore, an Interacting Multiple Model (IMM)-based SRCKF is employed in [27] to increase robustness in vehicle state estimation under dynamic operating scenarios. While this technique improves estimation accuracy, it comes at the cost of increased computational complexity, making real-time implementation more challenging.

In this study, we propose a novel approach based on an Intelligent Square Root Cubature Kalman Filter for estimating the wheel-rail adhesion force. The proposed method models the dynamics of a traction system comprising wheels, gearbox, and motor, with contact dynamics described using the Polach model. Instead of propagating full covariance matrices, our method computes and propagates their Cholesky factors, inherently ensuring positive semi-definiteness and improving numerical stability. Moreover, to overcome the limitations posed by unknown and time-varying noise statistics, we integrate DE algorithm for online optimization of noise covariance matrices. This adaptation enhances estimation robustness under varying operating conditions. The estimated adhesion-creep curve is then used to identify optimal traction operating points. The methodology is evaluated under multiple friction scenarios. The key contributions of this paper are summarized as follows:

1. Numerical Stability: The SRCKF propagates the square root (Cholesky factor) of the covariance matrix rather than the covariance matrix itself, ensuring numerical stability and maintaining positive semi-definiteness throughout the filtering process. This property is critical in long-term real-time applications, preventing filter divergence and improving robustness.

2. Adaptive Noise Covariance Tuning: Our method enhances the SRCKF by integrating a Differential Evolution (DE) algorithm that adaptively tunes the process and measurement noise covariance matrices online. This adaptive feature addresses the practical challenge of unknown and time-varying noise statistics in railway environments, improving estimation accuracy and robustness without increasing algorithmic complexity.

3. Computational Efficiency: Compared to more complex filtering techniques such as interacting multiple model filters or particle filters, the SRCKF combined with DE maintains a low computational burden, making it suitable for embedded systems with limited processing capabilities.

4. Improved Nonlinear Estimation: The SRCKF uses a third-degree spherical-radial cubature rule for Gaussian integrals, providing higher accuracy than EKF and avoiding the parameter tuning difficulties of the UKF.

The remainder of this paper is organized as follows: Section 2 presents the problem formulation. Section 3 describes the proposed ISRCKF-based estimation methodology. Section 4 discusses the experimental results. Finally, Section 5 concludes the paper.

II. Modelling of System

The wheel-rail system and the wheel-rail adhesion force dynamic model are explained in this section. The model comprises three essential components: a traction motor, wheels and rails, and a gearbox[2]. The main purpose of the traction system is to transfer the rotational force generated by the traction motor to the wheelset via the gearbox, enabling the wheels to rotate at a speed known as v_d . It is important to note that when traction is engaged, the wheel speed v_c is consistently higher than v_t . The difference in speed between the vehicle and the wheels during traction operation is commonly referred to as v_s . The locomotive is propelled in a forward direction due to the adhesion force $F\mu$, which arises from the creeping phenomenon occurring between the train's wheel and the railway track.

A. Model of Adherence Force

An adhesion force occurs between the rails and wheels only when there is a certain amount of velocity difference. This velocity difference, known as slip velocity, is defined as [1-2]:

$$v_s = v_c - v_t \quad (1)$$

A term slip instead of the slip velocity is used more often. The slip ξ is defined as follows:

$$\xi = \frac{v_c - v_t}{v_t} \quad (2)$$

The adhesion force $F\mu$ is modelled in this paper using Polach's technique [4-5]:

$$F\mu = \frac{2F_N\mu}{\pi} \left(\frac{k_A\epsilon}{1+(k_A\epsilon)^2} + \arctan(k_S\epsilon) \right), k_S \leq k_A \leq 1 \quad (3)$$

where μ represents the coefficient of adhesion, and F_N denotes the normal force exerted on the wheel. In a static state, the F_N is a constant value and in a dynamic state the F_N changes. In the case of the longitudinal force ϵ , the following relation holds [6]:

$$\epsilon = \frac{G\pi ab C_{11}}{4F_N\mu} \xi \quad (4)$$

where G is the modulus of rigidity of the wheel and rail, ξ is the creepage between the wheel and rail. Then coefficient μ depends on the slip velocity (ξv_t) and is as follows [8]:

$$\mu = \mu_0((1-D)e^{-B\xi V} + D) \quad (6)$$

where D and B represent reduction factors associated with varying friction coefficients.

B. Modelling of Traction System

The present system utilizes an AC induction motor and a gearbox to facilitate the movement of a wheel set comprising two wheels. The interaction between the wheels and the rail generates creep forces, enabling the railway vehicle to propel forward. The system's motion is governed by dynamic formulae, as outlined in [5-6].

$$\frac{d\omega_m}{dt} = \frac{T_m - T_L}{J_{eqv}} \quad (7)$$

$$\omega_c = \frac{\omega_m}{n_i} \quad (8)$$

The given equation represents the relationship between various parameters in a mechanical system. In this equation, ω_c refers to the angular velocity of the wheel, ω_m represents the angular velocity of the motor, n_i signifies the gear reduction ratio, T_m denotes the torque generated by the motor, and T_L and J_{eqv} correspond to the load torque and the equivalent moment of inertia, respectively. These variables are interconnected as described in a scholarly reference [2-3].

$$T_L = \frac{2rFa}{n_i} \quad (9)$$

$$J_{eqv} = J_m + \frac{J_g + J_x + J_{wR} + J_{wL}}{n_i^2} \quad (10)$$

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \dot{\varphi}_{r\alpha} \\ \dot{\varphi}_{r\beta} \\ \dot{w}_m \\ \dot{T}_L \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{L_\sigma} + \frac{R_r L_m^2}{L_r^2 L_\sigma}\right) & 0 & \frac{R_r L_m}{L_r^2 L_\sigma} & \frac{L_m}{L_r L_\sigma} \rho_p \omega_m & 0 & 0 \\ 0 & \left(\frac{R_s}{L_\sigma} + \frac{R_r L_m^2}{L_r^2 L_\sigma}\right) & -\frac{L_m}{L_r L_\sigma} \rho_p \omega_m & \frac{R_r L_m}{L_r^2 L_\sigma} & 0 & 0 \\ \frac{R_r}{L_r} L_m & 0 & -\frac{R_r}{L_r} & -\rho_p \omega_m & 0 & 0 \\ 0 & \frac{R_r}{L_r} L_m & \rho_p \omega_m & -\frac{R_r}{L_r} & 0 & 0 \\ -\frac{3\rho_p L_m}{2J_t L_r} \varphi_{r\beta} & \frac{3\rho_p L_m}{2J_t L_r} \varphi_{r\alpha} & 0 & 0 & 1 & -\frac{1}{J_t} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \varphi_{r\alpha} \\ \varphi_{r\beta} \\ w_m \\ T_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L_\sigma} \\ \frac{1}{L_\sigma} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} \quad (11)$$

We have several inertial moments to consider. These moments are represented by J_g , for the gearbox, J_x for the

wheelset axle, J_{wR} for the right wheel, and J_{wL} for the left wheel. Additionally, we have F_a , which signifies the adhesion

force exerted by a single wheel. With two measurement-ready input variables and three state variables, a system of fifth-order differential equations can accurately describe an induction motor as Eq.(11).

The measurement of the model can be written as:

$$z = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \quad (12)$$

where ρ_p is the pole pairs, J_t is the total inertia of IM and load, R_s is the stator resistance and L_s is the stator inductance, R_r is the rotor resistance and L_r is the rotor inductance, $L_\sigma = \sigma L_s$ is the stator transient inductance ($\sigma = 1 - \frac{L_m^2}{L_r L_s}$), L_m is the mutual inductance, $v_{s\alpha}$ and $v_{s\beta}$ are the measured stator stationary axis components of stator voltages.

III. Estimation of the Wheel-Rail Adhesion Force using Intelligent SRCKF

The design phase of the proposed adhesion force estimator is detailed in this section. The procedure begins with the estimation of the load torque using a traction motor load torque estimator. The estimated load torque is subsequently utilized to compute the wheel-rail adhesion force. In this

study, a sixth-order rotor flux-based induction motor (IM) model is adopted and integrated into the framework of the ISRCKF to enable accurate estimation of various internal states of the motor. To implement the ISRCKF, the continuous-time state-space model of the system is formulated as

$$\dot{X} = f(X) + \omega$$

where X is the state vector and ω is the process noise. This continuous-time model is discretised using a sampling period T , resulting in a discrete-time model suitable for real-time implementation:

$$X_k = (I + A * T)X_{k-1} + GTu_k = A(X_{k-1})X_{k-1} + GTu_k$$

where T is the sampling period. The proposed discrete-time IM model, expressed in the stator stationary reference frame and extended to accommodate the estimation algorithm, is presented in Equation (13). The matrices and vectors comprising this extended model are defined to include mechanical dynamics, electromagnetic torque generation, and rotor flux evolution, all of which are crucial for accurately estimating the load torque and consequently the adhesion force.

$$\begin{aligned} X_k &= A(X_{k-1})X_{k-1} + Bu_k + w_{k-1} \\ z_k &= h(X_k) + v_k = HX_k + v_k \end{aligned} \quad (13)$$

$$\begin{aligned} \underbrace{\begin{bmatrix} i_{s\alpha}(k) \\ i_{s\beta}(k) \\ \varphi_{r\alpha}(k) \\ \varphi_{r\beta}(k) \\ w_m(k) \\ T_L(k) \end{bmatrix}}_{X_k} &= \underbrace{\begin{bmatrix} 1 - (R_s + \frac{R_r L_m^2}{L_r^2}) \frac{T}{L_\sigma} & 0 & \frac{R_r L_m}{L_r^2 L_\sigma} T_2 & \frac{L_m}{L_r L_\sigma} \rho_p T \omega_m(k-1) & 0 & 0 \\ 0 & 1 - (R_s + \frac{R_r L_m^2}{L_r^2}) \frac{T}{L_\sigma} & -\frac{L_m}{L_r L_\sigma} \rho_p T \omega_m(k-1) & \frac{R_r L_m}{L_r^2 L_\sigma} T & 0 & 0 \\ \frac{R_r}{L_r} L_m T & 0 & 1 - \frac{R_r}{L_r} T & -\rho_p T \omega_m(k-1) & 0 & 0 \\ 0 & \frac{R_r}{L_r} L_m T & \rho_p T \omega_m(k-1) & 1 - \frac{R_r}{L_r} T & 0 & 0 \\ -\frac{3 \rho_p L_m}{2 J_t} \frac{T}{L_r} \varphi_{r\beta}(k-1) & \frac{3 \rho_p L_m}{2 J_t} \frac{T}{L_r} \varphi_{r\alpha}(k-1) & 0 & 0 & 1 & -c_B \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{A(X_{k-1})} \underbrace{\begin{bmatrix} i_{s\alpha}(k-1) \\ i_{s\beta}(k-1) \\ \varphi_{r\alpha}(k-1) \\ \varphi_{r\beta}(k-1) \\ w_m(k-1) \\ T_L(k-1) \end{bmatrix}}_{X_{k-1}} + \\ & \underbrace{\begin{bmatrix} \frac{T}{L_\sigma} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{T}{L_\sigma} & 0 & 0 & 0 & 0 \end{bmatrix}}_B \underbrace{\begin{bmatrix} v_{s\alpha}(k) \\ v_{s\beta}(k) \end{bmatrix}}_{u_k} + w_{k-1} \\ \underbrace{\begin{bmatrix} i_{s\alpha}(k) \\ i_{s\beta}(k) \end{bmatrix}}_{Z_k} &= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_H X_k + v_k \end{aligned} \quad (14)$$

where X_k is the state vector at time step k , u_k is the input (control) vector at time step k , w_k is the process noise vector at time step k , Z_k is the measurement vector at time step k , v_k is the measurement noise vector at time step k . Assume at time $k-1$, $S_{K-1|k-1}$ is the square root of $P_{K-1|k-1}$, the current state cubature points are computed as follows:

$$\begin{aligned} \chi_{k-1|k-1}^i &= S_{K-1|k-1} I(i) + \hat{x}_{k-1|k-1} \quad i \\ &= 1, \dots, 2n \end{aligned} \quad (15)$$

where $P_{K-1|k-1}$ represents the state estimation error covariance matrix at time step $k-1$, $I(i)$ is as

$$I(i) = \begin{cases} \sqrt{n} 1_i & i=1, \dots, n \\ -\sqrt{n} 1_{i-n} & i=n+1, \dots, 2n \end{cases} \quad (16)$$

$$P_{K-1|k-1} = S_{K-1|k-1} S_{K-1|k-1}^T$$

where 1_i is the i -th column of the identity matrix I . The cubature points are then transmitted into the state equation: $\chi_{k|k-1}^i = f(\chi_{k-1|k-1}^i)$

The predicted mean $\hat{x}_{k|k-1}$ and square root of the covariance matrix $S_{k|k-1}$ are calculated as:

$$\hat{x}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \chi_{k|k-1}^{i*} \quad (17)$$

$$S_{k|k-1} = \text{Tria}\{X_{k|k-1}, \sqrt{Q_k}\} \quad (18)$$

where $X_{k|k-1}$ is:

$$X_{k|k-1} = \frac{1}{\sqrt{2n_x}} \quad (19)$$

$$[\chi_{k|k-1}^{1*} - \hat{x}_{k|k-1} \quad \chi_{k|k-1}^{2*} - \hat{x}_{k|k-1} \quad \dots \quad \chi_{k|k-1}^{2n*} - \hat{x}_{k|k-1}]$$

In Equations (18), (23), and (27), the operator $\text{Tria}()$ denotes the upper triangular matrix obtained via the QR decomposition of the input matrix. This operation is commonly used in square root filtering to preserve the numerical stability and ensure positive semi-definiteness of the covariance matrices. Formally, for a matrix MM , we compute $[Q,R]=\text{qr}(M)$, and define:

$$\text{Tria}(M) \triangleq R$$

where R is an upper triangular matrix. We have updated the equations accordingly and clarified this definition in the main text. When measurement is revisited, the cubature point set is calculated as follows:

$$\chi_{k|k-1}^i = s_{k|k-1} I(i) + \hat{x}_{k|k-1} \quad i = 1, \dots, 2n \quad (20)$$

the transformed cubature points are transmitted in measurement equation:

$$\chi_{k|k-1}^{i**} = h(\chi_{k|k-1}^i) \quad (21)$$

The mean values and square-root of the covariance matrix of predicted measurement points are estimated:

$$\hat{z}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \chi_{k|k-1}^{i**} \quad (22)$$

$$S_{zz,k|k-1} = \text{Tria}\{Z_{k|k-1}, S_R\} \quad (23)$$

where $Z_{k|k-1}$ is as:

$$Z_{k|k-1} = \frac{1}{\sqrt{2n}} \quad (24)$$

$$[\chi_{k|k-1}^{1**} - \hat{z}_{k|k-1} \quad \chi_{k|k-1}^{2**} - \hat{z}_{k|k-1} \quad \dots \quad \chi_{k|k-1}^{2n**} - \hat{z}_{k|k-1}]$$

$$S_{xz,k|k-1} = X_{k|k-1} Z_{k|k-1}^T$$

$$X_{k|k-1} = \frac{1}{\sqrt{2n_x}} \quad (25)$$

$$[\chi_{k|k-1}^1 - \hat{x}_{k|k-1} \quad \chi_{k|k-1}^2 - \hat{x}_{k|k-1} \quad \dots \quad \chi_{k|k-1}^{2n} - \hat{x}_{k|k-1}]$$

The updated state $\hat{x}_{k|k}$ and the square-root of covariance $S_{k|k}$ are obtained as follows:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1}) \quad (26)$$

$$S_{k|k} = \text{Tria}([X_{k|k-1} - K_k Y_{k|k-1}, K_k \sqrt{R_k}]) \quad (27)$$

where K_k is as:

$$K_k = \frac{\begin{pmatrix} S_{xz,k|k-1} \\ S_{zz,k|k-1}^T \end{pmatrix}}{S_{zz,k|k-1}} \quad (28)$$

To determine the approximate adhesion force, one can employ the estimated load torque information by performing the following calculation:

$$\hat{T}_L = \frac{2r}{n_i} \hat{F}_a \quad (29)$$

While the square-root cubature Kalman filter (SRCKF) provides a computationally efficient and stable framework for nonlinear estimation, its conventional form assumes prior knowledge of noise statistics (Q and R), which is often impractical in railway applications. Reconsidering the equations of $S_{k|k-1}$ and $S_{k|k}$ in (18) and (28), respectively, it can be concluded that the process covariance matrices Q_k and measurement covariance matrices R_k have a critical effect on their values. As a result, the choice of elements of Q_k and R_k is important in the design of SRCKF since it effects the convergence, performance, and stability of the system. However, obtaining precise noise statistics can be a challenging task, as these statistics can vary with time. In previous studies, researchers have manually adjusted these matrices through trial and error, which is a time-consuming process [26]. To overcome this challenge and eliminate the need for trial and error methods, this paper proposes considering the covariance matrices as adjustable parameters. Our Intelligent SRCKF (ISRCKF) addresses this limitation by integrating a differential evolution algorithm to adaptively tune Q and R . This hybridization retains the numerical stability of SRCKF while adding robustness against uncertain and time-varying noise conditions. The DE optimization ensures that the filter dynamically adjusts to real-world operational challenges, such as varying adhesion coefficients and sensor inaccuracies, without requiring offline calibration. Compared to recent adaptive filters, the ISRCKF achieves comparable accuracy with lower computational complexity.

In proposed method, an alternative for the tuning and optimization of Q_k and R_k based on the DE algorithm is proposed. In this method, set the Q_k and R_k dimensions as 6×6 and 2×2 , which are assumed as:

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 \end{bmatrix} \quad (24)$$

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \quad (25)$$

The aim is to enhance the covariance matrices by modifying the components of Q_k and R_k according to a fitness function. The fitness function serves to minimize the disparity between the estimated and measured values of the stator current.

$$J = \frac{1}{N} \sum_{k=0}^N [(i_{s\alpha}(k) - \hat{i}_{s\alpha}(k))^2 + (i_{s\beta}(k) - \hat{i}_{s\beta}(k))^2] \quad (30)$$

where N represents the number of samples considered. The variable i_s denotes the actual stator current, while $\hat{i}_s$

represents the estimated stator current and $\|\cdot\|_2$ is the Euclidean distance. Stator current is typically measured using current sensors, and a smaller disparity between the estimated current and its measurement signifies a more precise estimation of the speed of the induction motor.

Suppose $C^k = [c_1^k \dots c_n^k]^T$ represents the current population of DE at iteration k , which $c_i^k = \{q_{i1}^k, q_{i2}^k, q_{i3}^k, q_{i4}^k, q_{i5}^k, q_{i6}^k, r_{i1}^k, r_{i2}^k\}$ is each candidate solution to the problem of optimization at iteration k . To generate a new population, the crossover, mutation, and selection operators are applied. The crossover and mutation operators are used to generate the trial vectors. A critical stage in the DE process is the creation of the trial vector. To select the best trial vector, selection operator is used in the next generation. In the suggested approach, the DE mutation operator creates a trail vector for each member of the current population. For each c_i^k in the population, trail vector v_i^k is generated as follows:

$$v_i^k = c_i^k + B(c_i^{k,gbest} - c_i^k) \quad (31)$$

where B is the mutation scale factor, c_i^k is the target vector to be perturbed at iteration k , $c_i^{k,gbest}$ is the target vector in the present population with the smallest value of the objective function. A crossover mechanism is used to increase the diversity of the new population of vectors. The crossover operation is applied to every pair of the target vector c_i^k and its corresponding mutant vector v_i^k to generate a trial vector $c_i'^k$. The binomial crossover stands as the most frequently utilized crossover method in DE, and it is precisely defined as follows:

$$c_i'^k = \begin{cases} v_i^k & \text{if } j \in J \\ c_i^k & \text{otherwise} \end{cases} \quad (32)$$

where J is the set of crossover points. Following the crossover, to obtain the fitness function, the trial vectors are evaluated and selection operations are performed. The value of fitness function of each trial vector $J(c_i'^k)$ is compared with its corresponding target vector $J(c_i^k)$. In order to determine whether the trial vector $c_i'^k$ should be included in the next generation $k + 1$, it is compared to c_i^k . If the trial vector $c_i'^k$ produces a superior result for the objective function compared to c_i^k will be replaced by $c_i'^k$ in the upcoming generation. Alternatively, if the previous value c_i^k proves to be more favorable, it will be retained for the subsequent generation.

$$c_i^{k+1} = \begin{cases} c_i'^k & \text{if } J(c_i'^k) < J(c_i^k) \\ c_i^k & \text{otherwise} \end{cases} \quad (33)$$

The process of mutation, selection and crossover is repeated after the new population is propagated until the ideal state is reached. When the best fitness value hits a predetermined value, the iteration is terminated. Based on fitness function values, DE optimizes the unknown parameters of Q_k and R_k by updating the particle solutions to provide particles better sets. The new solutions and updated Q_k and R_k are then used for the adaptation of SRCKF for the next iteration until a predetermined number of iterations has been achieved. Subsequently, the most favorable values of Q_k and R_k are acquired. Ultimately, the optimized values of Q_k and R_k are incorporated into the online running SRCKF estimator. It is

important to highlight that the intelligent SRCKF needs to be executed multiple times to enable the fine-tuning of the Q_k and R_k parameters based on each individual measurement.

Algorithm 2. Intelligent SRCKF

- 1: Initialize the particles consisting of the components from Q and R
 - 2: **Iterate the following steps:**
 - 3: Reconstruct Q and R
 - 4: Run SRCKF
 - 5: Evaluate the fitness function particles
 - 7: Update the velocities and positions of the particles using Equations (28) to (29).
 - 9: **Continue until reaching the maximum iteration**
 - 10: Output the optimal particles for reconstructing the matrices Q and R .
-

After a predetermined number of iterations of the DE algorithm, we are able to acquire the optimal parameter values for the diagonal elements of matrices Q and R . Subsequently, these optimal values are incorporated into the proposed method, resulting in the estimation of the state variables. The intelligent SRCKF is developed in Algorithm 2 by leveraging the principles of the SRCKF and DE.

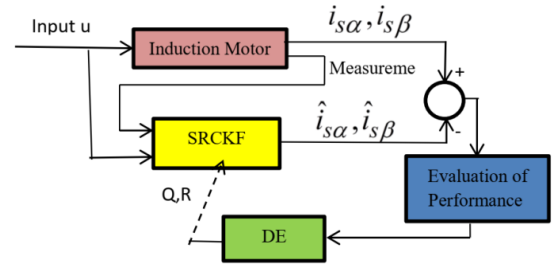


Fig. 1. Block diagram of Intelligent SRCKF

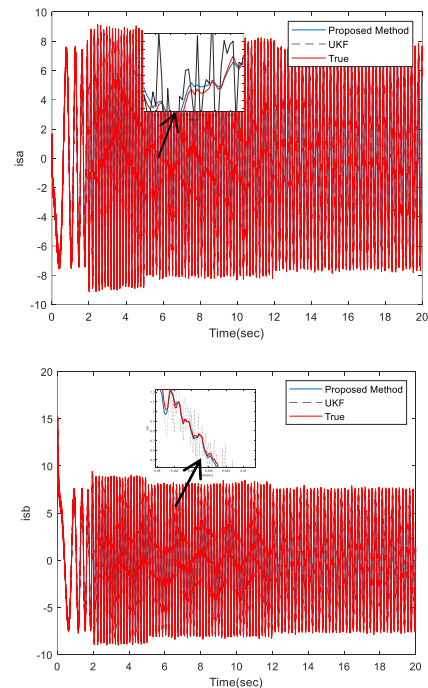


Fig. 2. Estimated currents

IV. Result

In this section, the effectiveness of the proposed approach for adhesion force is evaluated and compares with UKF.

TABLE I parameters of the IM

$N_p(r/min)$	$T_r(N.m)$	ρ_s	$R_s(\Omega)$	$R_r(\Omega)$	$L_s(H)$	$L_r(H)$	$L_m(H)$	$J(kg.m^2)$	$B(N.m/rad/s)$
9	2	2	3	2	0	0	0	0.	0.
50	2		.03	.5	.14	.15	.13	055	001
							5		

Table I presents the parameters pertaining to the traction system employed in the simulation. The covariance matrices used in methods are considered as:

$$Q = \text{diag}\{10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}\}$$

$$R = \text{diag}\{10^{-1}, 10^{-1}\}$$

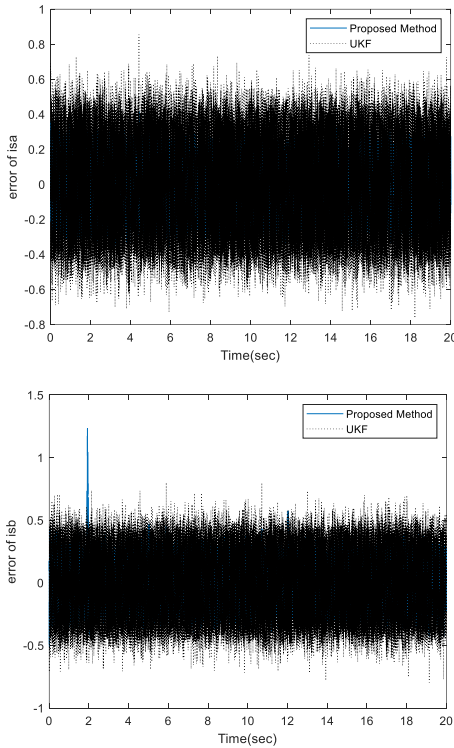


Fig. 3. Current errors

In all conducted experiments, the sampling time was set to $T=0.0001s$ in all conducted experiments to ensure accurate tracking of the high-frequency dynamics of the induction motor. This fine resolution is particularly important for capturing the rapid changes in stator currents and rotor flux, and for maintaining numerical stability in the Euler-discretized model used by the SRCKF algorithm. Additionally, the initial conditions for all estimated states in both algorithms were assumed to be zero. The parameters used in these experiments include the semi-axes of the contact ellipse ($a = 1.5 \times 10^{-3} m$, $b = 7.5 \times 10^{-3} m$), the normal force

at the wheel-rail contact ($F_N = 50 kN$), the longitudinal Kalker's coefficient ($C11 = 4.12$), and the shear modulus ($G = 8.4 \times 10^{10} N/m^2$). Fig.2 and 3 depict the trajectory of the estimated motor currents and current errors, respectively, using both the suggested technique and UKF. Fig.4 and 5 exhibit the path traced by the estimated rotor fluxes and the corresponding flux errors, respectively.

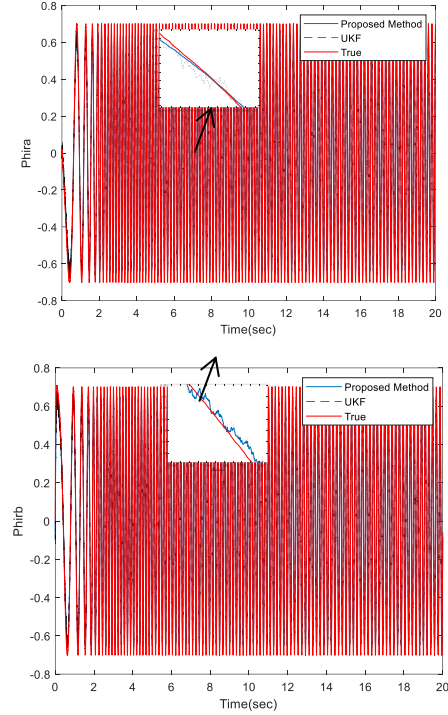


Fig. 4. Estimated fluxes

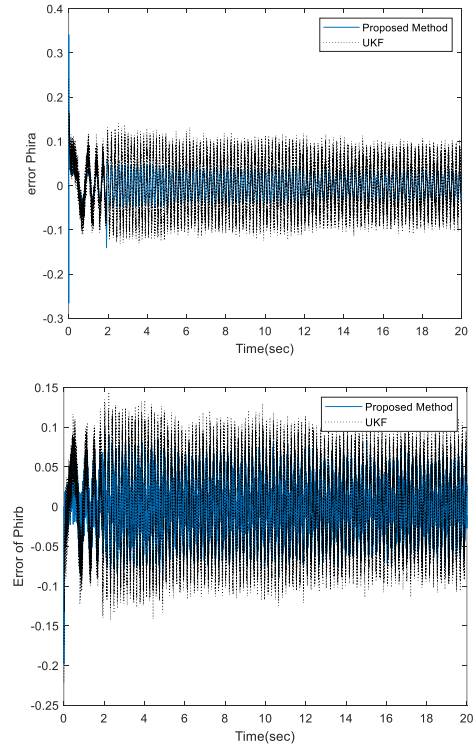


Fig. 5. Flux errors

Fig.6 illustrates the path of both the real and predicted motor speed, along with the speed error. The sudden error spikes observed in the standard Kalman filter at approximately 2 and 5 seconds (Fig. 6) correspond to abrupt changes in the adhesion force, likely caused by rapid variations in slip or external disturbances. During these intervals, the fixed noise covariance matrices used by the standard filter do not accurately capture the system's stochastic behavior, leading to a temporary degradation in estimation accuracy. The proposed method overcomes this limitation by adaptively updating the noise statistics, thereby maintaining robust performance even under sudden dynamic changes. Extensive evidence confirms that the estimator swiftly converges and accurately follows the speed trajectory, maintaining an impressively minimal margin of error. On the

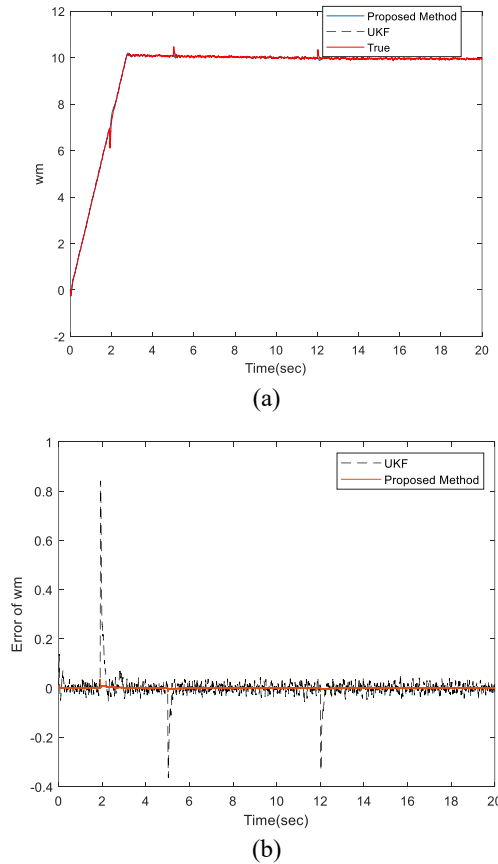


Fig. 6. (a) True and estimated motor speed (b) Speed error

other hand, Fig.7 exhibits the path of the estimated load torque and the associated load torque errors. the trajectory of the estimated adhesion force (F_a) is determined by utilizing the estimated load torque data, denoted as $TL = 2rF_a$. This information is extracted from the relevant section (1) and visually depicted in Fig.8.

To further evaluate the accuracy of the estimations provided by different approaches, the root mean square error (RMSE) for speed and load torque estimations is presented in Figs 9-10. These figures illustrate the RMSE obtained over 40 Monte Carlo runs. Fig 9 depict the RMSE of estimations over time,

while Fig 10 display the mean and variance of the RMSE. Each bar in Fig 10 represents the mean and variance of the RMSE. The results indicate that the proposed method outperforms the Unscented Kalman Filter (UKF) in terms of both the mean and variance of the RMSE. The RMSE associated with the proposed method is consistently smaller than that of the other methods. This suggests that the speed and load torque estimated by the proposed method are closer to their actual values. Consequently, the adhesion force estimated using the proposed method is more accurate.

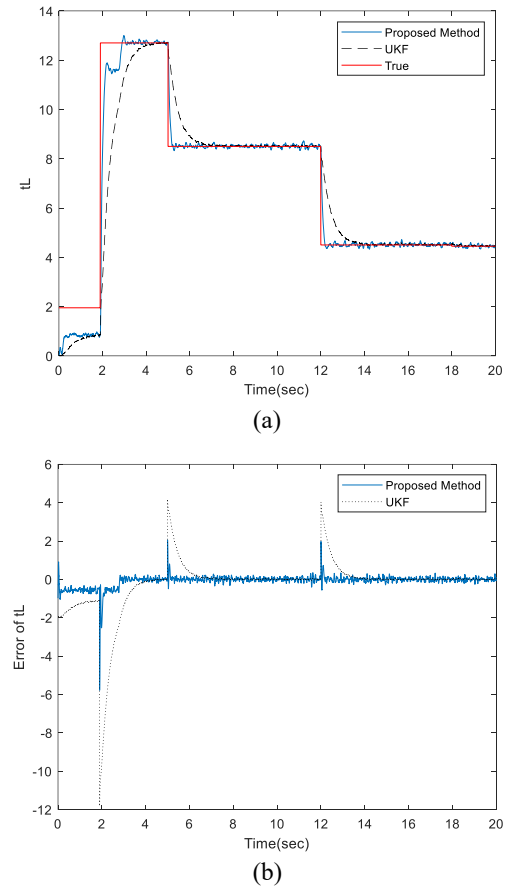


Fig. 7. (a) Estimated load torque (b) Load torque error

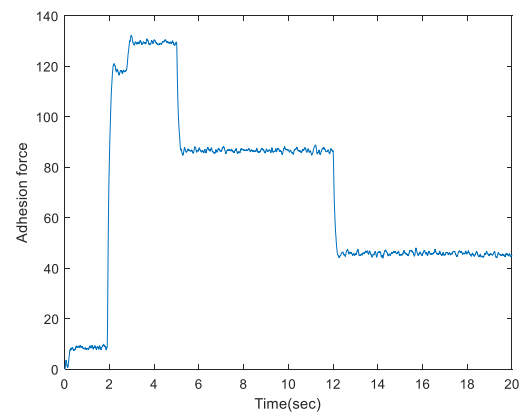


Fig. 8. Estimated adhesion force

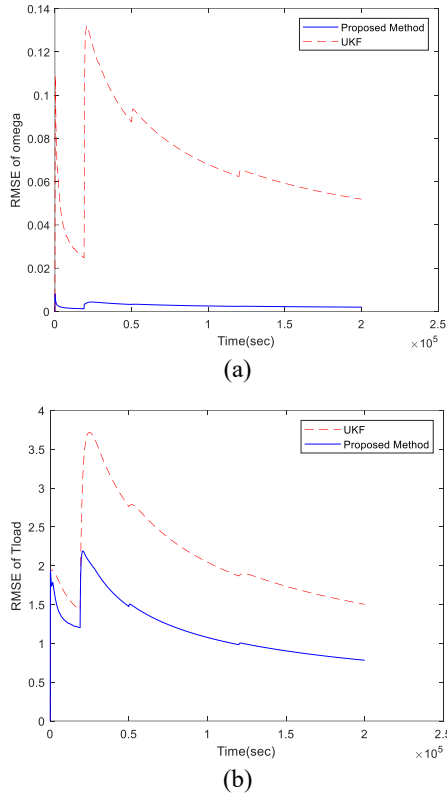


Fig. 9. RMSE over time (a) Omega (b) load torque

superior accuracy and adaptability of the proposed method. Specifically, the proposed method exhibits strong performance in handling time-varying noise and nonlinearities inherent in modern railway systems. This enhanced robustness, coupled with a competitive computational cost, highlights the practical advantages of the proposed method for real-time traction control applications.

The computational cost of the proposed method was evaluated through MATLAB simulations conducted on an Intel® Core™ i5-4590 CPU @ 3.3 GHz. The performance comparison of different algorithms is summarized in Table III. To clearly highlight the differences and facilitate a better understanding of the proposed method's efficiency, the computational costs are presented alongside the estimation accuracy metrics.

The results show that the running time of the proposed Intelligent SRCKF method is comparable to that of other state-of-the-art filters. Specifically, the proposed method requires approximately 1.1 seconds per run. However, this additional computational effort is justified by significant improvements in estimation performance, including a 20% reduction in speed RMSE and a 50% reduction in load torque RMSE. Moreover, the integration of DE for optimizing the noise covariance matrices ensures that the method remains feasible for real-time online operation. This balance between computational cost and enhanced accuracy confirms the practical applicability and superiority of the proposed

approach for adhesion force estimation and induction motor parameter identification in industrial railway systems.

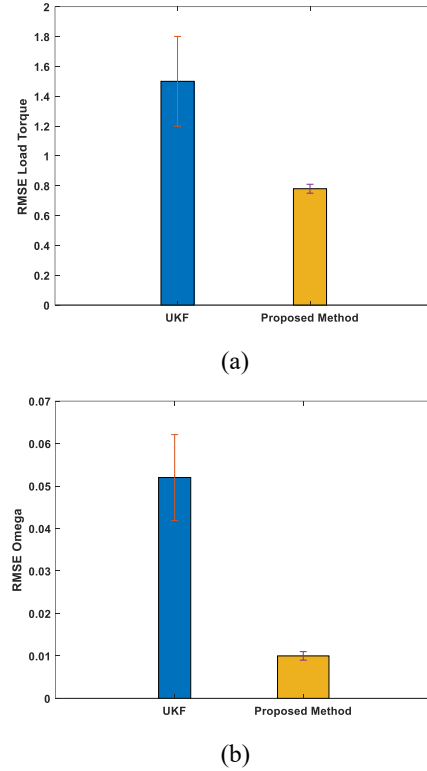


Fig. 10. RMSE (a) load torque (b) Omega

TABLE II Comparison of the Proposed Method with Recent Work

Method	Processing time (sec)
UKF	2.2
CKF[26]	2.1
Proposed Method	3.2

TABLE III Computational cost of algorithms

Method	RMSE	
	Omega	Load Torque
UKF	0.052	1.5
CKF[26]	0.041	1.2
Proposed Method	0.01	0.78

In order to assess the impact of unknown static noises on the estimation performance of our proposed method, we conducted a study. Specifically, we examined a scenario where process noise is mistakenly considered. In this scenario, both our proposed method and the EKF method, which utilize the same R matrices, were simultaneously applied at high and low speeds. The estimation performances and errors of both algorithms were visualized in Fig.11. Upon analyzing the estimation performances at high and low speeds, it became evident that our proposed method outperforms the EKF method. This superiority can be

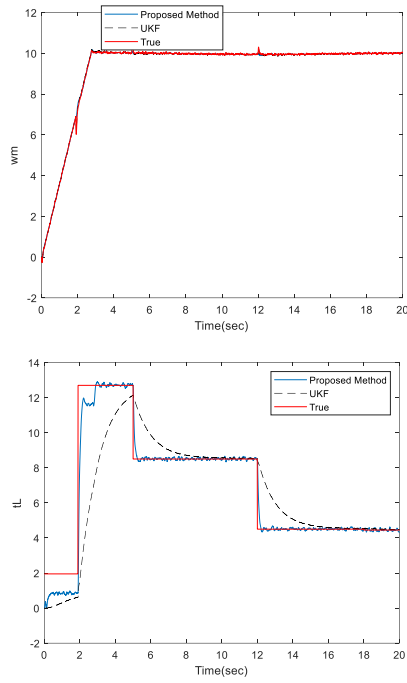


Fig. 11. Performance of EKF and proposed method under unknown static noises

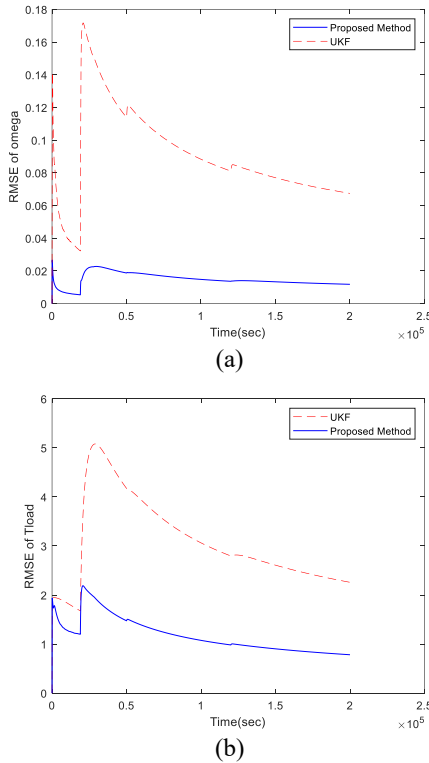


Fig. 12. RMSE over time (a) Omega (b) load torque

attributed to the fact that the pre-determined Q matrix in the EKF method, determined through a trial-and-error approach, is not suitable for low and zero speeds. Consequently, it negatively affects the estimation performance of the EKF method. Furthermore, determining a Q matrix that consistently delivers satisfactory estimation performance across the entire speed range is challenging

using the trial-and-error method. In contrast, our proposed method relies on the presumption that the noises are constrained to a specific energy level and does not necessitate any prior knowledge of these noises. Additionally, our method dynamically adjusts the process covariance, ensuring adaptability to varying conditions. The RMSE of speed and load torque estimations are presented in Figs 12-13.

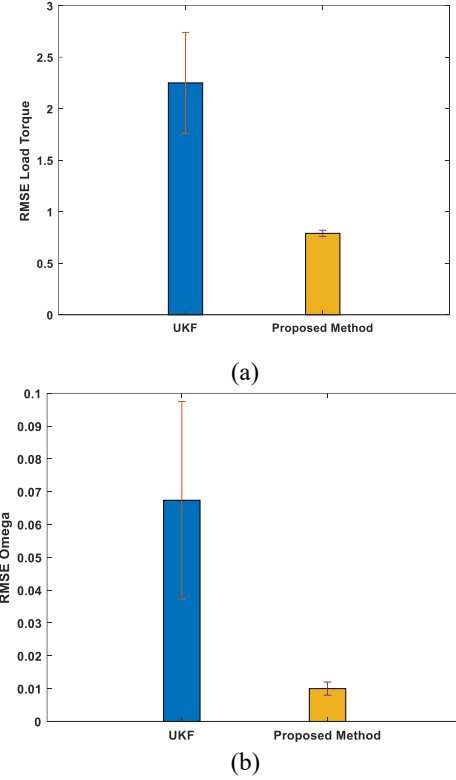


Fig. 13. RMSE (a) load torque (b) Omega

TABLE IV Comparison of the Proposed Method with Recent Work

Method	RMSE	
	Omega	Load Torque
UKF	0.071	2.25
CKF[26]	0.053	1.99
Proposed Method	0.015	0.79

Fig 12 show the RMSE of estimations over time, while Fig 13 display the mean and variance of these RMSE values. It is evident that the proposed method outperforms the UKF in terms of both the mean and variance of the RMSE. Additionally, the RMSE of the proposed method is smaller than that of the UKF, indicating that the speed and load torque estimations by the proposed method are closer to their actual values. Consequently, the adhesion force estimated using proposed method is more accurate.

The comparisons presented in Table IV include state-of-the-art methods, such as the approach proposed in [26], which addresses adhesion force estimation under dynamic and uncertain conditions with unknown static noises. Similar to the previous case, the results clearly demonstrate the superior accuracy and adaptability of the proposed method.

Specifically, the proposed ISRCKF exhibits strong performance in handling time-varying noise and nonlinearities inherent in modern railway systems. This enhanced robustness, coupled with a competitive computational cost, highlights the practical advantages of the proposed method for real-time traction control applications.

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Adaptive Disturbance Rejection Sliding Mode Control for Robots via an Orthogonal Functions-Based Estimator


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Article Info	ABSTRACT
Article type: Research Article	This paper presents an innovative adaptive sliding mode controller with disturbance rejection for robotic manipulators. The proposed approach employs orthogonal functions-based estimation to handle system uncertainties, while an adaptive mechanism is introduced to estimate the unknown upper bounds of external disturbances. Moreover, the dynamics of the robot actuators, namely the motors, are explicitly considered in the control law design. Three adaptive laws are proposed in this work. The first addresses the estimation of the parameters of orthogonal functions, the second deals with the approximation error, and the third concerns the estimation of the upper bound of external disturbances. Furthermore, a robust control term is proposed to compensate for the approximation error. Stability of the closed-loop system is ensured using Lyapunov theory. The performance of the proposed controller is evaluated through simulations on a SCARA robotic manipulator and compared with conventional and dynamic sliding mode control schemes in terms of tracking accuracy, control effort, and disturbance rejection.
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Keywords: Uncertainty Estimation, Orthogonal Functions, Adaptive Sliding Mode Control, Trajectory Tracking, Robot Manipulators.	

NOMENCLATURE			
q	Joint position vector	K_m	Motor torque constant matrix
$D(q)$	Robot's inertia matrix	R	Motor resistance
$C(q, \dot{q})\dot{q}$	Centrifugal force vector	L	Motor inductance
$G(q)$	Gravitational force vector	I_a	Motor current
τ_l	Joint torque vector		
J_m	Motor inertias		
B_m	Damping coefficients		
r	Gear ratios		

I. Introduction

Over the past few decades, sliding mode control (SMC) has gained significant attention in the control community due to its inherent robustness against model uncertainties and external disturbances, its capability for finite-time convergence, and its relatively straightforward design procedure [1-8]. SMC has been successfully implemented in a wide range of dynamic systems, including but not limited to industrial robotic manipulators, autonomous underwater

vehicles, spacecraft attitude control systems, power converters, and electric drive systems [9-15].

The methodology of SMC is typically divided into two distinct phases: the reaching phase and the sliding phase. During the reaching phase, the control law ensures that the system trajectories are driven toward a predefined sliding surface within a finite time interval. Once the trajectories intersect this surface, the system enters the sliding phase, where the states evolve along the surface and asymptotically approach the origin. Traditional SMC schemes often employ

linear sliding surfaces, which lead to asymptotic convergence of the system states during the sliding motion [16, 17].

Robotic manipulators have found diverse applications across several domains, including agriculture, medicine, sports, automotive manufacturing, as well as marine and aerospace industries [18-28]. For instance, agricultural robotic arms are categorized using various classification schemes depending on specific criteria. Based on their operational functions in the field, these robotic manipulators can be grouped into categories such as harvesting, weeding, seeding, disease and pest detection, spraying, and plant management systems [18]. Moreover, according to the employed harvesting and cultivation techniques, robotic arms are further classified into Cartesian manipulators, articulated jointed arms, high-degree-of-freedom hybrid arms, and multi-arm systems [29]. Another classification approach considers the environmental context in which the robots operate, dividing them into greenhouse, dry field, paddy field, and orchard robotic arms. Over the past years, comprehensive research on these diverse types of agricultural robotic arms has yielded significant advancements, establishing a robust technical foundation essential for the advancement of precision agriculture [18]. Therefore, the design of an effective control system plays a critical role in ensuring the precise and reliable operation of robotic arms.

The design procedure of sliding mode control (SMC) is relatively simple, and the resulting controller exhibits strong robustness against external disturbances, parameter variations, and unmodeled dynamics. However, two major drawbacks are associated with conventional SMC. The first and most critical issue is the chattering phenomenon, which arises due to the discontinuous nature of the control law. The second limitation is the requirement for a known upper bound on system uncertainties. To address these issues, this paper proposes an approach in which system uncertainties are approximated using Legendre polynomials, and the upper bound of external disturbances is estimated through an adaptive law.

In recent years, the estimation and compensation of system uncertainties in adaptive control using orthogonal functions such as Legendre polynomials have attracted considerable research interest [30, 31]. In [32], an adaptive controller was introduced for robotic systems. However, the proposed control framework in [32] does not include any uncertainty estimator. In contrast, the proposed approach in this study employs a Legendre polynomial-based uncertainty estimator to address uncertainties. Moreover, the uncertainties in [32] are assumed to be constant over time, while in the present study, the uncertainties are considered to be time-varying.

The main contributions of this paper can be summarized as follows: (i) Design of an adaptive sliding mode control framework that takes into account the dynamics of the robot actuators, (ii) Estimation of system uncertainties using orthogonal functions due to their simple structure, (iii) Design

of three adaptive laws for estimating the parameters, approximation errors, and bounds of disturbances, (iv) Achievement of Lyapunov-based stability and robust performance against uncertainties and disturbances, and (v) Design of a robust control term to compensate for approximation errors.

The structure of this paper is as follows: In Section II, a mathematical description of robotic systems, including manipulators and actuators, is presented. Section III details the design of the proposed adaptive sliding mode controller incorporating an orthogonal functions-based estimator. A Lyapunov-based stability analysis is presented in Section IV. Section V presents the simulation results. Section VI discusses the limitations of the proposed method, and Section VII outlines potential future research directions. Finally, Section VIII concludes the paper.

II. Modeling of Robotic Systems Including Manipulators and Actuators

The dynamics of the robotic system, including both the manipulator and the actuators, are formulated as follows [33-35]:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau_l \quad (1)$$

$$J_m r^{-1} \ddot{q} + B_m r^{-1} \dot{q} + r \tau_l = K_m I_a \quad (2)$$

$$R I_a + K_m r^{-1} \dot{q} + L \dot{I}_a + d = U \quad (3)$$

In the above equations, U and d represent the motor voltage and external disturbance vectors, respectively.

Substituting equation (1) into (2) gives:

$$I_a = K_m^{-1}((J_m r^{-1} + rD)\ddot{q} + (B_m r^{-1} + rC)\dot{q} + rG) \quad (4)$$

By substituting the above equation into (3), we obtain:

$$U = RK_m^{-1}((J_m r^{-1} + rD)\ddot{q} + (B_m r^{-1} + rC)\dot{q} + rG) + K_m r^{-1} \dot{q} + L \dot{I}_a + d \quad (5)$$

It can be reformulated as:

$$\begin{aligned} U &= \bar{D}\ddot{q} + \bar{C}\dot{q} + \bar{G} + L\dot{I}_a + d \\ \bar{D} &= RK_m^{-1}(J_m r^{-1} + rD) \\ \bar{C} &= RK_m^{-1}(B_m r^{-1} + rC) + K_m r^{-1} \\ \bar{G} &= RK_m^{-1}rG \end{aligned} \quad (6)$$

Using (6), we have:

$$\begin{aligned} U &= \ddot{q} + Y + d \\ Y &= \bar{D}\ddot{q} + \bar{C}\dot{q} + \bar{G} + L\dot{I}_a - \ddot{q} \end{aligned} \quad (7)$$

Here, Y denotes the uncertainties and nonlinearities of the system.

III. Design of the Proposed Adaptive Sliding Mode Controller Incorporating an Orthogonal Functions-Based Estimator

The tracking error and its derivatives, along with a sliding surface, are defined as follows:

$$e = q_d - q \rightarrow \dot{e} = \dot{q}_d - \dot{q} \rightarrow \ddot{e} = \ddot{q}_d - \ddot{q} \quad (8)$$

$$s = \dot{e} + \lambda e \quad (9)$$

By substituting (7) into (8), the following equation is obtained:

$$\ddot{e} = \ddot{q}_d - \ddot{q} = \ddot{q}_d + \Upsilon + d - U \quad (10)$$

By differentiating the sliding surface and substituting (10) into it, we obtain:

$$\dot{s}(t) = \ddot{e} + \lambda \dot{e} = \ddot{q}_d + \Upsilon + d - U + \lambda \dot{e} \quad (11)$$

According to (11), the proposed control law is designed as follows:

$$U = \ddot{q}_d + \hat{\Upsilon} + \hat{d} + \lambda \dot{e} + U_r \quad (12)$$

where $\hat{\Upsilon}$ and $\hat{d}$ denote the estimations of system uncertainties and external disturbances, respectively, and U_r represents the robust control term.

Substituting (12) into (11) yields:

$$\dot{s}(t) = \Upsilon - \hat{\Upsilon} + d - \hat{d} - U_r \quad (13)$$

The equations for an orthogonal function, such as Legendre polynomials, are defined as follows [30, 36]:

$$(i+1)\zeta_{i+1}(x) = (2i+1)x\zeta_i(x) - i\zeta_{i-1}(x) \quad (14)$$

$$i = 1, \dots, m-1$$

$$\zeta_0(x) = 1, \zeta_1(x) = x, \zeta_2(x) = 0.5(3x^2 - 1)$$

Since the uncertainty functions in control systems are generally time-dependent, we introduce x as a mapping that transforms the time variable, ranging from zero to infinity, into the bounded interval $[-1, 1]$. In this study, x is defined as $x = \sin(\omega_0 t)$.

At this point, the system uncertainty Υ is approximated using Legendre polynomials:

$$\Upsilon = P^T \Psi + \varepsilon, \quad (15)$$

$$P = [p_0 \ p_1 \ \dots \ p_m]^T,$$

$$\Psi = [\zeta_0 \ \zeta_1 \ \dots \ \zeta_m]^T$$

$$\hat{\Upsilon} = \hat{P}^T \Psi \quad (16)$$

Here, P and Ψ denote the vector of Legendre parameters and the vector of Legendre basis functions, respectively, while ε represents the approximation error.

Applying (15) and (16) yields:

$$\begin{aligned} \tilde{\Upsilon} &= \Upsilon - \hat{\Upsilon} = P^T \Psi + \varepsilon - \hat{P}^T \Psi \\ &= (P - \hat{P})^T \Psi + \varepsilon \\ &= \tilde{P}^T \Psi + \varepsilon \end{aligned} \quad (17)$$

Assuming that the external disturbance is bounded by $|d| \leq \rho$, the disturbance is estimated as follows:

$$\hat{d} = \hat{\rho} \text{sign}(s) \quad (18)$$

By substituting (17) and (18) into (13), the following expression is obtained:

$$\dot{s}(t) = \tilde{P}^T \Psi + \varepsilon + d - \hat{\rho} \text{sign}(s) - U_r \quad (19)$$

IV. Lyapunov-Based Stability Analysis

A Lyapunov candidate function is defined as

$$V = \frac{1}{2} s^2 + \frac{1}{2\eta} \tilde{\rho}^2 + \frac{1}{2\theta} \tilde{P}^T \tilde{P} + \frac{1}{2\vartheta} \tilde{\varepsilon}^2 \quad (20)$$

Taking the time derivative of the Lyapunov function yields:

$$\dot{V} = s\dot{s} - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} - \frac{1}{\theta} \tilde{P}^T \dot{\tilde{P}} - \frac{1}{\vartheta} \tilde{\varepsilon} \dot{\tilde{\varepsilon}} \quad (21)$$

Employing (19), the following is derived:

$$\begin{aligned} \dot{V} &= s(\tilde{P}^T \Psi + \varepsilon + d - \hat{\rho} \text{sign}(s) - U_r) - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \\ &\quad - \frac{1}{\theta} \tilde{P}^T \dot{\tilde{P}} - \frac{1}{\vartheta} \tilde{\varepsilon} \dot{\tilde{\varepsilon}} \end{aligned} \quad (22)$$

The first adaptation law is proposed as follows:

$$\dot{\hat{\rho}} = \theta s \Psi \quad (23)$$

By applying this adaptation law, equation (22) becomes:

$$\dot{V} = s(\varepsilon + d - \hat{\rho} \text{sign}(s) - U_r) - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} - \frac{1}{\vartheta} \tilde{\varepsilon} \dot{\tilde{\varepsilon}} \quad (24)$$

Now, the robust control term is designed as follows:

$$U_r = \hat{\varepsilon} + ks \quad (25)$$

Inserting (25) into (24) leads to:

$$\dot{V} = s(\tilde{\varepsilon} + d - \hat{\rho} \text{sign}(s) - ks) - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} - \frac{1}{\vartheta} \tilde{\varepsilon} \dot{\tilde{\varepsilon}} \quad (26)$$

where $\tilde{\varepsilon} = \varepsilon - \hat{\varepsilon}$. Subsequently, the second adaptation law is proposed as follows:

$$\dot{\hat{\varepsilon}} = \vartheta s \quad (27)$$

By applying this adaptation law, equation (24) is obtained as follows:

$$\begin{aligned} \dot{V} &= s(d - \hat{\rho} \text{sign}(s) - ks) - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \\ &= sd - \hat{\rho}|s| - ks^2 - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \\ &\leq |s||d| - \hat{\rho}|s| - ks^2 - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \\ &\leq \rho|s| - \hat{\rho}|s| - ks^2 - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \\ &= \tilde{\rho}|s| - ks^2 - \frac{1}{\eta} \tilde{\rho} \dot{\tilde{\rho}} \end{aligned} \quad (28)$$

The third adaptation law is now introduced as:

$$\dot{\hat{\rho}} = \eta|s| \quad (29)$$

Substituting the above adaptation law into (28) yields:

$$\dot{V} \leq -ks^2 \quad (30)$$

Hence, based on $\dot{V} \leq 0$ and by invoking Barbalat's Lemma [37], the stability of the closed-loop system is guaranteed.

Remark 1: This is a fact that time delays are inherent in many physical systems. Incorporating such delays would necessitate a fundamental restructuring of the mathematical proofs—specifically through the use of augmented subsystems, Pade approximations, or integral inequality lemmas to handle the delayed states [38-40]. Given the complexity of these derivations, a comprehensive analysis of the system's robustness against time-varying delays is identified as a significant and necessary direction for future research.

V. Simulation Results

Consider a SCARA-type robotic manipulator with the configuration illustrated in Fig. 1. The maximum permissible input voltage for each actuator is limited to $U_{max} = 40$ V. The parameters of the robotic system are selected as $m_1 =$

11.94Kg, $m_2 = 37.973\text{Kg}$, $m_3 = 0.263\text{Kg}$, $a_1 = 0.33\text{m}$, $a_2 = 0.27\text{m}$, $R = 1.26$, $L = 0.001$, $K_m = 0.26$, $r = 0.01$, $J_m = 0.0002$, and $B_m = 0.001$, while the parameters of the proposed adaptive controller are set to $\lambda = 10$, $\eta = 0.001$, $\theta = 750$, $\vartheta = 1$, $k = 1$ and $\omega_0 = \pi/14$. The initial conditions for the positions of the joints are chosen as $q(0) = (0.04 - 0.05 \ 0.06)^T$.

The reference trajectories for the robot joints are defined as follows:

$$q_d = 0.6(1 - \cos(\pi t/14)) \quad (31)$$

The reference trajectory is illustrated in Fig. 2. The external disturbance considered for the simulation is a pulse signal, as depicted in Fig. 3.

The tracking errors and control signals are depicted in Figs. 4 and 5, respectively. As shown in Fig. 4, the tracking errors converge appropriately to zero, demonstrating accurate trajectory tracking. In addition, the control signals presented in Fig. 5 remain within permissible ranges and do not exhibit chattering behavior, confirming the appropriateness of the control inputs. The estimated Legendre polynomial coefficients are shown in Fig. 6, where all parameters remain bounded. Collectively, Figs. 4 to 6 confirm the effectiveness of the proposed controller in both trajectory tracking and disturbance rejection.

In order to assess the effectiveness of the proposed controller and its ability to suppress chattering, its performance is compared with that of a conventional sliding mode controller (SMC) [32], whose control signal is provided below.

$$U_{SMC} = \ddot{q}_d + \lambda \dot{e} + \Gamma_{smc} \text{sign}(s), \quad (32)$$

$$|Y + d| \leq \Gamma_{smc}$$

where Γ is an upper bound on the aggregate of external disturbances and system uncertainties. Here, the values of λ and Γ are selected as 10 and 10, respectively.

It should be emphasized that, in practical applications, the use of a conventional sliding mode controller faces difficulties due to the unavailability of the upper bounds of external disturbances and system uncertainties.

Figures 7 and 8 respectively illustrate the tracking errors and control inputs corresponding to the conventional sliding mode controller.

Although the tracking errors are satisfactory, Fig. 8 reveals that the control signals exhibit noticeable chattering. Hence, this method is not suitable for controlling the robotic system under consideration.

As a second comparison study, the proposed approach is evaluated against dynamic sliding mode control (DSMC) [41]. Owing to the integral action in dynamic sliding mode control, the chattering effect can be considerably attenuated. The control signal in this method is designed as follows.

$$\dot{U}_{DSMC} = \ddot{q}_d + \delta_1 \Lambda + \delta_2 \dot{e} + \delta_3 e + \delta_4 \int_0^t e(\tau) d\tau \quad (33)$$

$$+ \mathcal{L}_{dsmc} \text{sign}(\sigma_2),$$

$$U_{DSMC} = \int_0^t \dot{U}_{DSMC}(\tau) d\tau,$$

$$\sigma_1 = \dot{e} + q_1 e + q_2 \int_0^t e(\tau) d\tau,$$

$$\sigma_2 = \dot{\sigma}_1 + b_1 \sigma_1 + b_2 \int_0^t \sigma_1(\tau) d\tau$$

where

$$\begin{aligned} \delta_1 &= q_1 + b_1, \delta_2 = q_2 + b_2 + q_1 b_1, \delta_3 \\ &= q_1 b_2 + q_2 b_1, \delta_4 = q_2 b_2; \Lambda \\ &= \ddot{q}_d - U_{DSMC}; \Phi \\ &= Y + d; |\delta_1 \Phi + \dot{\Phi}| \\ &\leq \mathcal{L}_{dsmc} \end{aligned} \quad (34)$$

In the above equations, σ_1 and σ_2 denote the primary and secondary sliding surfaces in the DSMC method. The controller parameters are chosen as follows: $q_1 = 30$, $q_2 = 200$, $b_1 = 60$, $b_2 = 400$, $\mathcal{L}_{dsmc} = 10$.

The corresponding tracking errors and control inputs are depicted in Figs. 9 and 10, respectively. In comparison with conventional sliding mode control, a substantial reduction in control chattering is observed, resulting in a more desirable control signal profile. Subsequently, the tracking performance of this method is compared with that of the proposed controller. As can be seen from Figs. 4 and 10, the proposed method exhibits better tracking performance in both transient and steady-state phases. In contrast, the dynamic sliding mode control achieves tracking in an oscillatory manner.

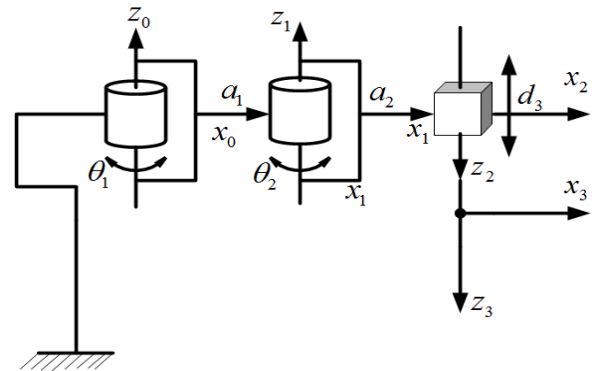


Fig. 1. Configuration of the SCARA robot [42]

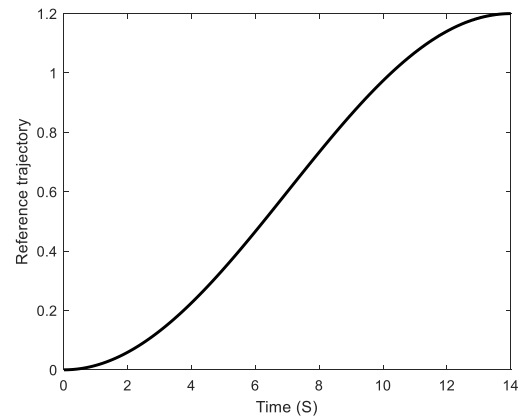


Fig. 2. Desired trajectory profile

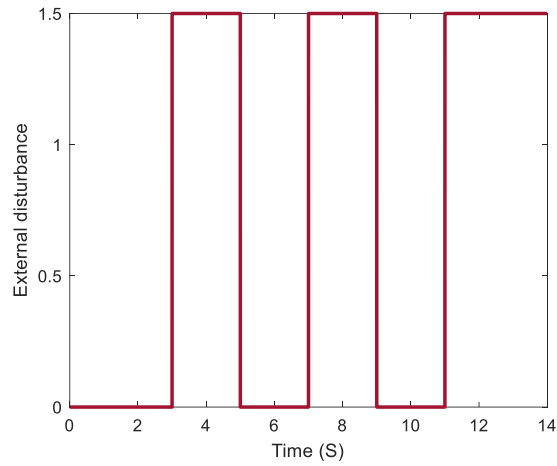


Fig. 3. External disturbance profile

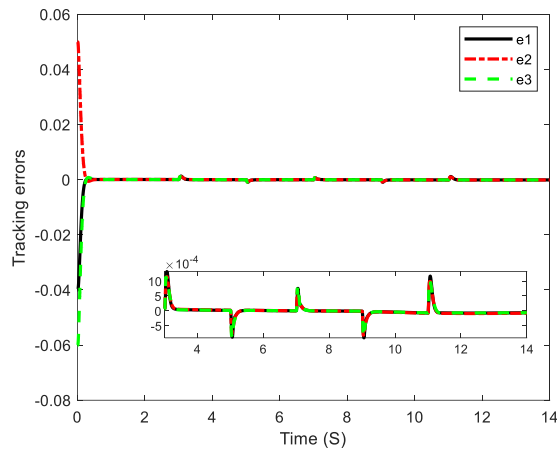


Fig. 4. Tracking errors under the proposed adaptive controller

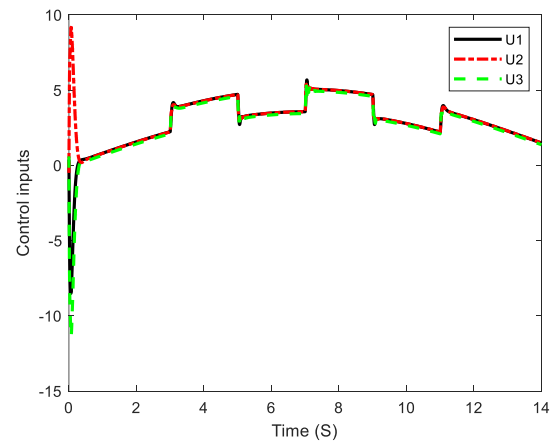


Fig. 5. Control signals under the proposed adaptive controller

VI. Limitations

The proposed adaptive sliding mode controller assumes the availability of velocity signals; in practical implementations, observer-based estimation may be required. The number of Legendre polynomial terms introduces a trade-off between approximation accuracy and computational complexity.

Additionally, the controller parameters are tuned empirically, which could be improved in future studies using intelligent optimization algorithms.

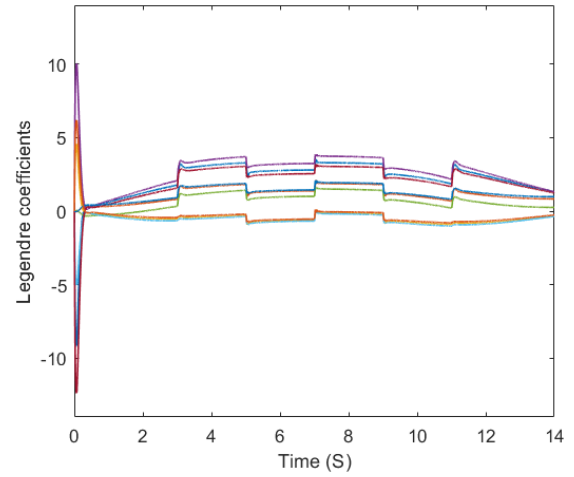


Fig. 6. Legendre parameters profiles

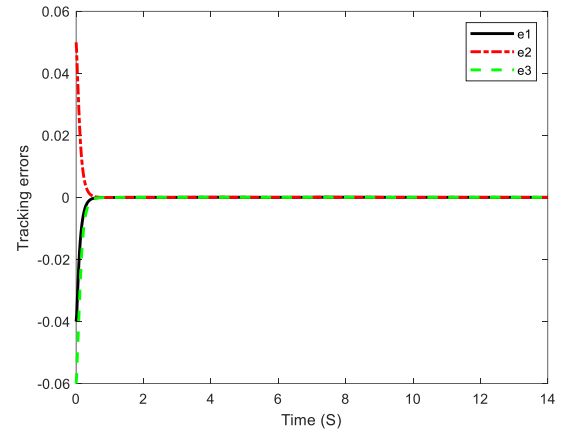


Fig. 7. Tracking errors under the conventional SMC method

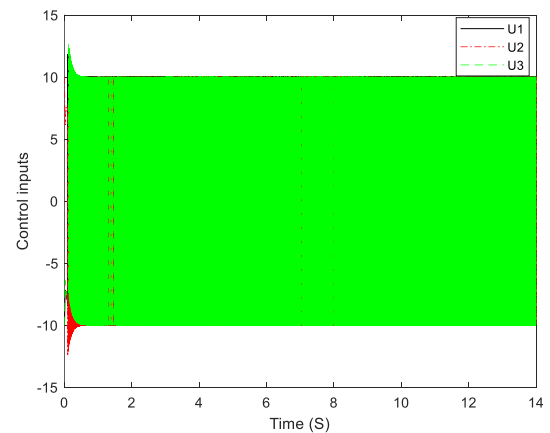


Fig. 8. Control signals under the conventional SMC method

VII. Future Research Directions

Future work may focus on the following directions: tuning the parameters of the proposed adaptive controller using novel optimization methods [43-50], incorporating observer-based

velocity estimation, extending the proposed method to flexible robotic manipulators, and exploring alternative uncertainty estimation techniques.

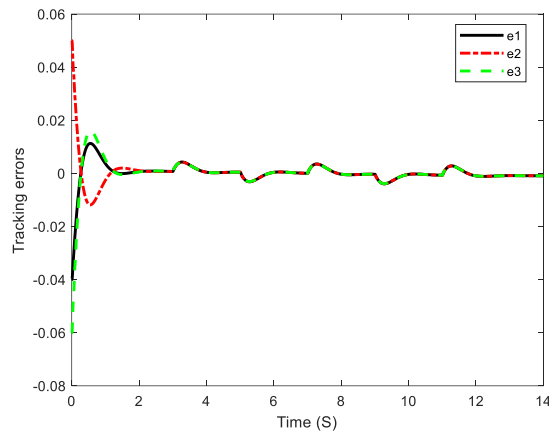


Fig. 9. Tracking errors under the Dynamic SMC method

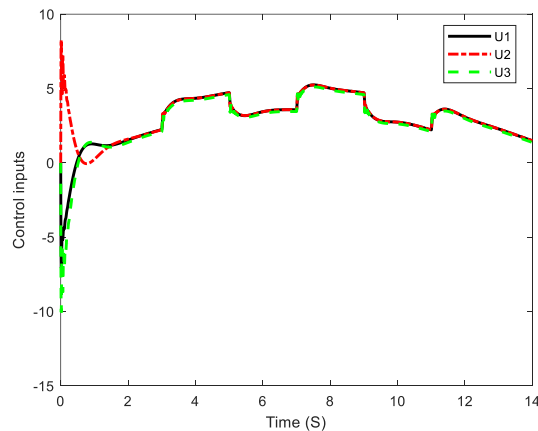


Fig. 10. Control signals under the Dynamic SMC method

VIII. Conclusions

An adaptive sliding mode control structure has been developed by integrating system uncertainty estimation via orthogonal functions and the upper bound of external disturbances through an adaptive law into the sliding mode control framework. Moreover, the proposed control scheme has eliminated the need for a priori knowledge of the upper bound of external disturbances, as it has employed an adaptive law to estimate this bound online. Legendre polynomials have been employed for estimating system uncertainties due to their simple structure. The control law has been formulated to compute the input voltages directly applied to the motors. The simulation results have demonstrated the effectiveness of the proposed method.

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Transformer Winding-to-Ground Connection Internal Fault Locating using the Estimation Error Standard Deviation Index

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Article Info	ABSTRACT		
Article type: Research Article Article history: Received: 12 January 2026 Received in revised form: 02 May 2026 Accepted: 13 May 2026 Published online: 22-Dec-2026 Keywords: Estimation, Extended Kalman filter (EKF), Fault location, Inrush current, Standard deviation.	Protective relay in the transformer may accept inrush current as fault current, thus, distinguishing the inrush current from the internal fault current is important in transformers. To estimate the transformer inrush current, the extended Kalman filter (EKF) is applied in this paper. Therefore, three phase transformer initial inrush currents are estimated using the extended Kalman filter formulation and a nonlinear model is used to simulate inrush currents. Then, type of current, whether it is a fault or an inrush current, is determined by comparing this current and the current calculated from the transformer model. It is considered as a fault current, if the inrush current estimation error surpasses a certain limit compared to the transformer actual inrush current. On the other hand, connection of the winding to the ground fault location has been also presented in this paper which has been less investigated in previous studies. Also, the relation of the fault location to the estimation error standard deviation is also studied and an index β is introduced. If the value of this index is greater than a specific value in any phase of the transformer, a fault condition is identified. Finally, the proposed technique is implemented for another transformer to calculate the estimation error standard deviation and index β in the faulty condition. The results show the index β values are in agreement with the index limits and validate the accuracy of the proposed method.		
NOMENCLATURE <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> f Frequency: Hz i_{ψ} The magnetizing current: A i_{inr} The inrush current: A l The transformer winding self-inductance: H m The transformer winding mutual inductance: H </td><td style="width: 50%; vertical-align: top;"> r The transformer winding resistance: Ω r_{mag} The magnetizing branch resistance: Ω v_s Source voltage: V α Switching angle: $^{\circ}$ ψ Core flux-turn: Wb.t </td></tr> </table>		f Frequency: Hz i_{ψ} The magnetizing current: A i_{inr} The inrush current: A l The transformer winding self-inductance: H m The transformer winding mutual inductance: H	r The transformer winding resistance: Ω r_{mag} The magnetizing branch resistance: Ω v_s Source voltage: V α Switching angle: $^{\circ}$ ψ Core flux-turn: Wb.t
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I. Introduction

One of the most important protections which is used for power transformers is differential protection [1]. In transient conditions, while the transformer is initially connected to the voltage source, an inrush current is generated, in which the relay should not work. In these conditions, harmonic inhibition methods usually are used to stabilize the relay against the transformer inrush current. But in transformers with better quality cores, the amount of second and fifth harmonics are low, so their detection may be done with error. To solve this problem, studies have been carried out to detect the transformer inrush current in the past years. The frequency domain reflectometry (FDR) technique for detecting and locating transformer winding defects has been proposed in

[2]. FDR measures the wave impedance and its change along the measured windings. In some methods, considering that the inrush current has only an active power close to zero, but in the fault mode active power is consumed, the transformer input power is checked to discriminate the inrush current from the internal fault current. The important disadvantage of this method is its slow response time, which is more than one cycle. The current variation characteristics of inter-turn short-circuit faults in grounding transformers under various operating conditions has been presented in [3] through theoretical analysis, simulations, and case studies. An online identification method is proposed in [3] to detect inter-turn short-circuit faults in grounding transformer windings and to distinguish affected phases based on current variation characteristics.

Some researchers have proposed artificial intelligence methods, including fuzzy methods and neural networks for high speed distinguishing the transformer inrush current from the internal fault current. Although these methods increase the speed and bring it below one cycle, but those are among learning-based methods and require large datasets [4]. In [5], a fault diagnosis method is presented for identifying multi-point grounding faults in transformer cores based on the finite element method (FEM). On the other hand, the internal temperature distribution within the transformer and the condition of the transformer oil are comprehensively analyzed and validated by using an electromagnetic-fluid-solid coupled finite element simulation. It is important to mention that the fault occurrence in a point of the transformer winding causes the hot spot creation and ultimately leads to the insulation destruction. A new transformer fault diagnosis method has been investigated in [6] based on the parallel AdaBoost-Naive Bayes algorithm. This method allows for resampling and reweighting, making the model pay more attention to samples that are difficult to classify and thereby improving performance on imbalanced datasets. Frequency response analysis (FRA) is one of the winding fault detection methods and the FRA is offline which it is a negative feature [7].

In [8] finite element method has been used for fault location in the transformer. In this paper force measuring sensors have been also placed in parts of the transformer windings to measure the forces applied to the windings during short circuit as online. The most disadvantages of the method in [8] are due to placing the sensor and additional winding in the transformer. Therefore, the flux-based technique is introduced in [9] to determine the location of the fault in the winding and is used to solve the problems in [8]. Among the other methods in fault location are artificial intelligence based methods. But these methods, in spite of many advantages, require a lot of dataset and system initial training [10]. The wavelet transform method and artificial intelligence are used in [11] to show the location of the fault in the transformer winding-to-ground connection. A time-domain analytical-numerical method has been proposed in [12] for inrush current modeling in three-phase core-type transformers, incorporating the nonlinear magnetic behavior of the core. A novel algorithm for transformer differential protection is proposed in [13] which distinguishes internal faults from inrush currents in the presence of CT saturation. To reduce the effect of CT saturation, the currents are compensated by a CT saturation compensation method. Then, wavelet transform is used to discriminate transformer internal fault from inrush currents. In [14] a differential protection method is suggested, which decomposes the differential current signal to multiple energy levels through the multi-resolution analysis (MRA) and selects the most useful feature to feed to the bidirectional gated recurrent unit (BIGRU) to classify the events.

To differentiate the transformer's internal fault from the inrush current, convolutional neural network (CNN) is used in [15]. Therefore, a combination of the higher order spectral estimations that perform a deep learning classification with high accuracy, are proposed. In this paper, the extended Kalman filter has been used to distinguish between inrush current and fault current in the transformer winding as well as to show the location of the winding fault. The EKF does not have the disadvantages of the above mentioned methods. This filter has the online nature, low computational rate, and the elimination (or maximum reduction) of process and measurement noises [16]. The nonlinear model of a single-phase transformer is derived in [17] which includes the hysteresis and magnetic saturation, and then an EKF is used for estimation of the primary winding current to distinguish the transformer inrush current from an internal fault current.

A statistical algorithm for discrimination of fault and inrush current signals under CT saturation scenarios is proposed in [18] and twofold discrimination indices centered on the concept of fourth-order statistic moment are presented. A method is suggested in [19] to distinguish the internal fault of transformers and other normal transient conditions, and operational characteristic is defined for classification of the instantaneous current behavior based on the modal transform. The Hermite transform method is considered in [20] to identify transient events in power transformers caused by energizations and current transformer (CT) saturation. In this approach, the positive half-cycle is analyzed firstly and then negative half-cycle is studied. A sensitive method based on wavelet transform for detecting inter-turn faults during energization of single-distribution transformers under healthy and faulty conditions is suggested in [21]. The waveform features are obtained by the mathematical morphology gradient (MMG) operator in [22] and then the morphological pattern spectrum (MPS) is applied to discriminate between inrush current and fault current. A transformer differential protection algorithm is presented in [23] so that between inter-turn, low-level internal faults, and inrush current is differentiated. Feature is extracted using difference function and maximum overlapped discrete wavelet transform energy in this method. In [24] digital image processing method on a new FRA signature is used, so that the frequency, magnitude, and phase angle of the conventional FRA signature are integrated into one 3D plot to classify various faults. The measurement process overview of fault detection and location in power transformers using frequency response analysis is proposed in [25]. The use of the EKF method to estimate the single-phase transformer inrush current have been investigated in various references, but based on the authors' best knowledge, the extended Kalman filter has not been developed for fault location in three-phase transformer. Therefore, in the present paper, to discriminate inrush current from fault current without complex calculations, an algorithm

has been developed. On the other hand, a method has been proposed and formulated to show location of the winding fault using variance of the Kalman filter estimation error. The paper is arranged as follows: Section II describes three phase transformer inrush current modeling. Extended Kalman filter and inrush current simulation results are proposed in section III. Fault current simulation is investigated in section IV. Discriminating the fault from the inrush current, the winding fault location and verification of the accuracy of the proposed technique are studied in section V. Lastly, conclusion is brought in section VI.

II. Three Phase Transformer Inrush Current Modeling

A suitable model of the transformer is necessary to use the Kalman filter. Therefore, transformer nonlinear model is considered in this paper. On the other hand, for this purpose, nonlinear Kalman filter such as the extended Kalman filter should be developed. The nonlinearities of the transformer core have the greatest impact in the transformer inrush current modeling. Due to the low frequency of the inrush current phenomenon, there is no need to enter the transformer parameters frequency dependence into the modeling [26]. Figure 1 shows a three-phase transformer electrical circuit.

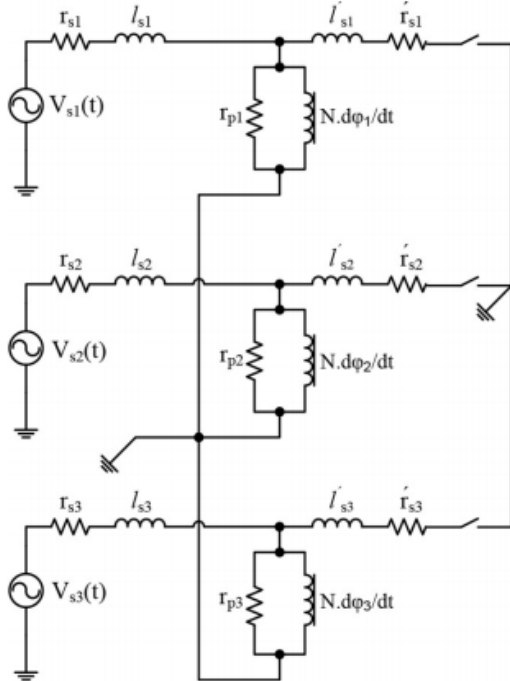


Fig.1. Three phase transformer electrical circuit [27]

Matrices of the winding resistances and inductances are defined as:

$$[R] = \begin{bmatrix} r_{s1} & 0 & 0 \\ 0 & r_{s2} & 0 \\ 0 & 0 & r_{s3} \end{bmatrix} \quad (1)$$

$$[L] = \begin{bmatrix} l_{s1} & m_{12} & m_{13} \\ m_{21} & l_{s2} & m_{23} \\ m_{31} & m_{32} & l_{s3} \end{bmatrix} \quad (2)$$

where l_{s1}, l_{s2} and l_{s3} are leakage inductance (H), and m_{12}, m_{23} and m_{31} are mutual inductance (H). The current $i_\psi(t)$ must be expressed in a nonlinear way in Figure 1. Therefore, the magnetizing current is considered as [28]:

$$i_\psi(t) = 0.7576 \psi(t) + 1.03 \times 10^7 \psi^{19}(t) \quad (3)$$

Equations (4) to (7) describe the three phase circuit of the Figure 1 as:

$$\begin{aligned} v_{s1}(t) &= r_{s1} i_{inr1}(t) + l_{s1} \frac{di_{inr1}(t)}{dt} + m_{12} \frac{di_{inr2}(t)}{dt} \\ &+ m_{13} \frac{di_{inr3}(t)}{dt} + v_1(t) = r_{s1} i_{inr1}(t) + l_{s1} \frac{di_{inr1}(t)}{dt} \\ &+ m_{12} \frac{di_{inr2}(t)}{dt} + m_{13} \frac{di_{inr3}(t)}{dt} + \frac{d\psi_1}{dt} \end{aligned} \quad (4)$$

$$\begin{aligned} v_{s2}(t) &= r_{s2} i_{inr2}(t) + l_{s2} \frac{di_{inr2}(t)}{dt} + m_{21} \frac{di_{inr1}(t)}{dt} \\ &+ m_{23} \frac{di_{inr3}(t)}{dt} + v_2(t) = r_{s2} i_{inr2}(t) + l_{s2} \frac{di_{inr2}(t)}{dt} \\ &+ m_{21} \frac{di_{inr1}(t)}{dt} + m_{23} \frac{di_{inr3}(t)}{dt} + \frac{d\psi_2}{dt} \end{aligned} \quad (5)$$

$$\begin{aligned} v_{s3}(t) &= r_{s3} i_{inr3}(t) + l_{s3} \frac{di_{inr3}(t)}{dt} + m_{31} \frac{di_{inr1}(t)}{dt} \\ &+ m_{32} \frac{di_{inr2}(t)}{dt} + v_3(t) = r_{s3} i_{inr3}(t) + l_{s3} \frac{di_{inr3}(t)}{dt} \\ &+ m_{31} \frac{di_{inr1}(t)}{dt} + m_{32} \frac{di_{inr2}(t)}{dt} + \frac{d\psi_3}{dt} \end{aligned} \quad (6)$$

$$i_{inrj}(t) = \frac{v_j(t)}{r_{mag}} + i_{\psi j}(t) = \frac{1}{r_{mag}} \frac{d\psi_j}{dt} + i_{\psi j}(t), \quad j=1,2,3 \quad (7)$$

Therefore, transformer state space model is given as:

$$\begin{bmatrix} l_{s1} & m_{12} & m_{13} & 1 & 0 & 0 \\ m_{21} & l_{s2} & m_{23} & 0 & 1 & 0 \\ m_{31} & m_{32} & l_{s3} & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} \end{bmatrix} \times \begin{bmatrix} \frac{di_{inr1}(t)}{dt} \\ \frac{di_{inr2}(t)}{dt} \\ \frac{di_{inr3}(t)}{dt} \\ \frac{d\psi_1(t)}{dt} \\ \frac{d\psi_2(t)}{dt} \\ \frac{d\psi_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} -r_{s1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_{s2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_{s3} & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} i_{inr1}(t) \\ i_{inr2}(t) \\ i_{inr3}(t) \\ \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} v_{s1}(t) \\ v_{s2}(t) \\ v_{s3}(t) \\ i_{\psi 1}(t) \\ i_{\psi 2}(t) \\ i_{\psi 3}(t) \end{bmatrix} \quad (8)$$

where $v_s(t)$ is the voltage applied to the transformer primary and $\psi(t)$ is the core flux-turn (Wb.t). we can define:

The equivalent circuit of the transformer for the delta-connected windings is shown in Figure 2. where Ψ_k is flux linkage associated with the transformer's primary coil, in the k -th column of the transformer, $i_k^{(g)}$ is transformer primary

$$A = \begin{bmatrix} l_{s1} & m_{12} & m_{13} & 1 & 0 & 0 \\ m_{21} & l_{s2} & m_{23} & 0 & 1 & 0 \\ m_{31} & m_{32} & l_{s3} & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} \end{bmatrix}^{-1} \times \begin{bmatrix} -r_{s1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_{s2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_{s3} & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$= \begin{bmatrix} L & I_{3 \times 3} \\ 0 & \frac{-1}{r_{mag}} I_{3 \times 3} \end{bmatrix}^{-1} \times \begin{bmatrix} -R & 0 \\ -I & 0 \end{bmatrix} = \begin{bmatrix} L^{-1} & r_{mag} L^{-1} \\ 0 & -r_{mag} I \end{bmatrix} \times \begin{bmatrix} -R & 0 \\ -I & 0 \end{bmatrix} = \begin{bmatrix} -(L^{-1}R + r_{mag}L^{-1}) & 0 \\ r_{mag}I & 0 \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} l_{s1} & m_{12} & m_{13} & 1 & 0 & 0 \\ m_{21} & l_{s2} & m_{23} & 0 & 1 & 0 \\ m_{31} & m_{32} & l_{s3} & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{mag}} \end{bmatrix}^{-1} \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} L^{-1} & r_{mag}L^{-1} \\ 0 & -r_{mag}I \end{bmatrix} \quad (11)$$

$$C = I_{6 \times 6} \quad (12)$$

Therefore, we have:

$$\dot{x} = Ax + Bu \quad (13)$$

$$y = Cx \quad (14)$$

The state space model in equation (1) is continuous in time. But, to implement the extended Kalman filter, we need a discrete time model. By discretizing the derivatives, we have:

$$\frac{di_{inrj}(t)}{dt} \approx \frac{i_{inrj}(t+\Delta t) - i_{inrj}(t)}{\Delta t} \quad (15)$$

$$\frac{d\psi_{inrj}(t)}{dt} = \frac{\psi_{inrj}(t+\Delta t) - \psi_{inrj}(t)}{\Delta t} \quad (16)$$

Now, we substitute these discrete derivatives into the original equations and rearranging the equations. Equation (3) shows the discrete state space model as follows:

current, $R_{Fe,k}$ is equivalent resistance representing the iron core losses, $i_{Fe,k}$ is active component of the transformer's idle current, R_s is equivalent resistance of the power grid (power source), L_s is equivalent inductance (reactance) of the power grid (power source), R_g is primary winding resistance, L_g is leakage inductance of the primary winding, Φ is main flux leakage (effective value of the flux), N_g is number of turns in the primary windings. Equations for three independent circulations appeared as a result of applying Kirchhoff's

second law to the modeled equivalent circuits of the primary side of the transformer are:

$$\begin{bmatrix} r_{s1}\Delta t + l_{s1} & m_{12} & m_{13} & 1 & 0 & 0 \\ m_{21} & r_{s2}\Delta t + l_{s2} & m_{23} & 0 & 1 & 0 \\ m_{31} & m_{32} & r_{s3}\Delta t + l_{s3} & 0 & 0 & 1 \\ r_{mag}\Delta t & 0 & 0 & -1 & 0 & 0 \\ 0 & r_{mag}\Delta t & 0 & 0 & -1 & 0 \\ 0 & 0 & r_{mag}\Delta t & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} i_{inr1}(t + \Delta t) \\ i_{inr2}(t + \Delta t) \\ i_{inr3}(t + \Delta t) \\ \psi_1(t + \Delta t) \\ \psi_2(t + \Delta t) \\ \psi_3(t + \Delta t) \end{bmatrix} = \quad (17)$$

$$\begin{bmatrix} v_{s1}(t)\Delta t + l_{s1}i_{inr1}(t) + m_{12}i_{inr2}(t) + m_{13}i_{inr3}(t) + \psi_1(t) \\ v_{s2}(t)\Delta t + m_{21}i_{inr1}(t) + l_{s2}i_{inr2}(t) + m_{23}i_{inr3}(t) + \psi_2(t) \\ v_{s3}(t)\Delta t + m_{31}i_{inr1}(t) + m_{32}i_{inr2}(t) + l_{s3}i_{inr3}(t) + \psi_3(t) \\ i_{\psi 1}(t)(r_{mag}\Delta t) - \psi_1(t) \\ i_{\psi 2}(t)(r_{mag}\Delta t) - \psi_2(t) \\ i_{\psi 3}(t)(r_{mag}\Delta t) - \psi_3(t) \\ -R_si_1^s(t) - L_s \frac{di_1^s(t)}{dt} + R_si_2^s(t) + L_s \frac{di_2^s(t)}{dt} \\ + R_g i_2^{(g)}(t) + L_g \frac{di_2^{(g)}(t)}{dt} + \frac{d\psi_2(t)}{dt} = e_{12}^{(s)}(t) \end{bmatrix} \quad (18)$$

$$-R_si_2^s(t) - L_s \frac{di_2^s(t)}{dt} + R_si_3^s(t) + L_s \frac{di_3^s(t)}{dt} \quad (19)$$

$$+ R_g i_3^{(g)}(t) + L_g \frac{di_3^{(g)}(t)}{dt} + \frac{d\psi_3(t)}{dt} = e_{23}^{(s)}(t) \quad (20)$$

$$R_g i_1^{(g)}(t) + L_g \frac{di_1^{(g)}(t)}{dt} + R_g i_2^{(g)}(t) + L_g \frac{di_2^{(g)}(t)}{dt} + R_g i_3^{(g)}(t) + L_g \frac{di_3^{(g)}(t)}{dt} + \frac{d\psi_1(t)}{dt} + \frac{d\psi_2(t)}{dt} + \frac{d\psi_3(t)}{dt} = 0 \quad (21)$$

where

$$i_1^s(t) = i_1^{(g)}(t) - i_2^{(g)}(t) \quad (22)$$

$$i_2^s(t) = i_2^{(g)}(t) - i_3^{(g)}(t) \quad (23)$$

$$i_3^s(t) = i_3^{(g)}(t) - i_1^{(g)}(t) \quad (24)$$

The resulting system of equations is ordered in matrix form:

$$\begin{bmatrix} -L_s & 2L_s + L_g & -L_s \\ -L_s & -L_s & 2L_s + L_g \\ L_g & L_g & L_g \end{bmatrix} \times \begin{bmatrix} \frac{di_1^{(g)}(t)}{dt} \\ \frac{di_2^{(g)}(t)}{dt} \\ \frac{di_3^{(g)}(t)}{dt} \end{bmatrix} = \begin{bmatrix} R_s & -2R_s - R_g & -R_s \\ R_s & R_s & -2R_s - R_g \\ -R_g & -R_g & -R_g \end{bmatrix} \begin{bmatrix} i_1^{(g)}(t) \\ i_2^{(g)}(t) \\ i_3^{(g)}(t) \end{bmatrix} \quad (25)$$

$$- \begin{bmatrix} \frac{d\psi_2(t)}{dt} \\ \frac{d\psi_3(t)}{dt} \\ 0 \end{bmatrix} + \begin{bmatrix} e_{12}^{(s)}(t) \\ e_{23}^{(s)}(t) \\ 0 \end{bmatrix}$$

Using the circuit description in modeling the magnetic circuit of the transformer, we can state that:

$$\frac{d\psi_k(t)}{dt} = R_{Fe,k} i_{Fe,k}(t) \quad (26)$$

Taking into account Kirchhoff's first and second laws for the magnetic circuit of the transformer, we have:

$$\left(\frac{1}{R_{Fe1}} + \frac{1}{R_{Fe2}}\right) \frac{d\psi_2(t)}{dt} + \frac{1}{R_{Fe1}} \frac{d\psi_3(t)}{dt} = i_2^{(g)}(t) - i_1^{(g)}(t) - \frac{U_{m1}}{N_g}(\psi_2(t) + \psi_3(t)) - \frac{U_{m2}}{N_g} \psi_2(t) \quad (27)$$

$$\frac{1}{R_{Fe2}} \frac{d\psi_2(t)}{dt} - \frac{1}{R_{Fe3}} \frac{d\psi_3(t)}{dt} = i_3^{(g)}(t) - i_2^{(g)}(t) - \frac{U_{m,2}}{N_g} \psi_2(t) + \frac{U_{m,3}}{N_g} \psi_3(t) \quad (23)$$

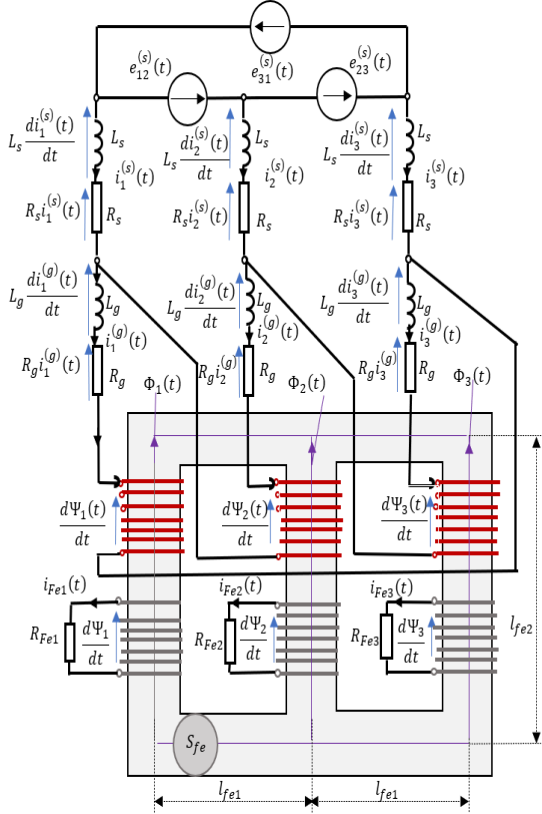


Fig.2. Equivalent circuit of the D-y three-phase transformer

III. Extended Kalman Filter and Inrush Current Simulation Results

The Kalman filter was introduced in 1961 by Rudolph Kalman (Hungarian-American electrical engineer and mathematician) and Bossi. This filter is a mathematical technique that is widely used in digital signal processing, navigation and control systems [29]. In fault location or satellite position detection, detection based methods use fewer sensors and have lower cost and more reliability than the other methods. In some situations, the estimation of the state variables can be more efficient than the direct measurement of them due to the effect of the noise phenomenon in the measurement. Kalman filter is a powerful tool for extracting information in the presence of uncertainties and provides an optimal solution for tracking and forecasting problems.

Kalman filter (KF) is applied for linear systems and provides optimal estimates for linear systems, while the extended Kalman filter (EKF) is an extension that can be applied to nonlinear systems and provides approximate estimates for nonlinear systems. The EKF works by transforming the nonlinear models at each time step into linearized systems of Equations. In a single-variable model, you would do this using the current model value and its

derivative; the generalization for multiple variables and Equations is the Jacobian matrix. The linearized Equations are then used in a similar manner to the standard Kalman filter. If you have a bad initial guess of the underlying system's state, in many cases where you approximate a nonlinear system with a linear model, the EKF will not perform well. The EKF is merely an extension of the linear-system technique to a wider class of problems. The nonlinear EKF model of the system can be represented as follows:

$$\begin{aligned} \dot{x} &= f(x, V(t), \omega, t) \\ y &= h(x, v, t) \\ \omega &= (0, Q) \\ v &= (0, R) \end{aligned} \quad (24)$$

In Equation (24), v and ω are uncorrelated white noises with zero mean and x is the state vector. Moreover, ω_0 and v_0 are zero. y and v are the transformer measured inrush current and input voltage. Moreover, Q and R matrices are chosen as follows:

$$Q = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \quad (25)$$

$$R = I_{6 \times 6} \quad (26)$$

Therefore,

$$f(x, V(t), \omega, t) = Ax + Bu + \omega$$

$$= \begin{bmatrix} -(L^{-1}R + r_{mag}L^{-1}) & 0 \\ r_{mag}I & 0 \end{bmatrix} x \quad (27)$$

$$+ \begin{bmatrix} L^{-1} & r_{mag}L^{-1} \\ 0 & -r_{mag}I \end{bmatrix} u + \omega$$

$$h(x, v, t) = Cx + v \quad (28)$$

$$F = \frac{\partial f}{\partial x}$$

$$= \begin{bmatrix} -(L^{-1}R + r_{mag}L^{-1}) & r_{mag}L^{-1} \times \begin{bmatrix} \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \\ \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \\ \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \end{bmatrix} \\ r_{mag}I & -r_{mag} \begin{bmatrix} \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \\ \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \\ \frac{\partial i_{\psi 1}}{\partial \psi_1} & \frac{\partial i_{\psi 2}}{\partial \psi_2} & \frac{\partial i_{\psi 3}}{\partial \psi_3} \end{bmatrix} \end{bmatrix} \quad (29)$$

$$L = \frac{\partial f}{\partial \omega} = I_{6 \times 6} \quad (30)$$

$$H = \frac{\partial h}{\partial x} = C \quad (31)$$

$$M = \frac{\partial h}{\partial v} = I_{6 \times 6} \quad (32)$$

To realize the EKF filter, the state variable and the estimation error covariance matrix are initialized as:

$$\hat{x}_0 = E(x_0) \quad (33)$$

$$P = P_0 = E\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\} \quad (34)$$

where E is mathematical expectation of variable and P is the estimation error covariance matrix. The estimation equations are stated as:

$$K = P \times H^T \times (M \times R \times M^T)^{-1} \quad (35)$$

$$\hat{x} = f(\hat{x}, V(t), \omega_0, t) + K \times (y - h(\hat{x}, v_0, t)) \quad (36)$$

where K is the estimator gain. Updating equation is:

$$\dot{P} = F \times P + P \times F^T + L \times Q \times L^T - K \times C \times P \quad (37)$$

To calculate the inrush current, equation (15) must be solved using MATLAB software. It should be noted that the three phase currents of the primary side and the value of the windings flux are as 6 state variables in equation (15). The specifications of the studied transformer have been brought in Table 1.

Table 1. Specifications of the first studied transformer

Name	Value
Voltage of phase a	$V_a(t) = 110 \cos(\omega t - \alpha)$
Voltage of phase b	$V_b(t) = 110 \cos(\omega t - (120^\circ + \alpha))$
Voltage of phase c	$V_c(t) = 110 \cos(\omega t - (240^\circ + \alpha))$
Switching angle	$\alpha = 42.97^\circ$
Power	4.5 kVA
Frequency	50 Hz

After the inrush current calculations, we have to implement the EKF filter. The filter can be useful if its estimation of the current, with an error close to zero, is consistent with the current of the previous step. This filter is implemented on the transformer state equations. The inrush currents are shown in figure 3.

IV. Fault Current Simulation

External and internal faults are two general classes of electrical faults in the transformer. External faults are not related to the transformer itself, but they directly affect it. Internal faults also occur inside the transformer and are related to the equipment of the transformer. Among the internal faults, we can mention phase-to-phase connection of the windings, connection of the phase to the ground, turn to turn fault in the winding [30]. In this section, connection of the winding to the ground is considered. In the following, in the presence of this fault, transformer inrush current is simulated, and then the results of the inrush current estimation are proposed. Finally, by comparing the calculated/measured current and the estimated current, the estimation error signal or the residual signal is obtained, and at that time, a decision is made about the fault condition or the normal condition of the transformer. Then, if the fault current is detected, using the β index, location of the fault in the winding is determined. If a ground connection fault occurs in one of the windings, it divides the winding into two parts and causes the resistance and inductance matrices to be 7×7 matrices:

$$[R] = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_6 \end{bmatrix} \quad (38)$$

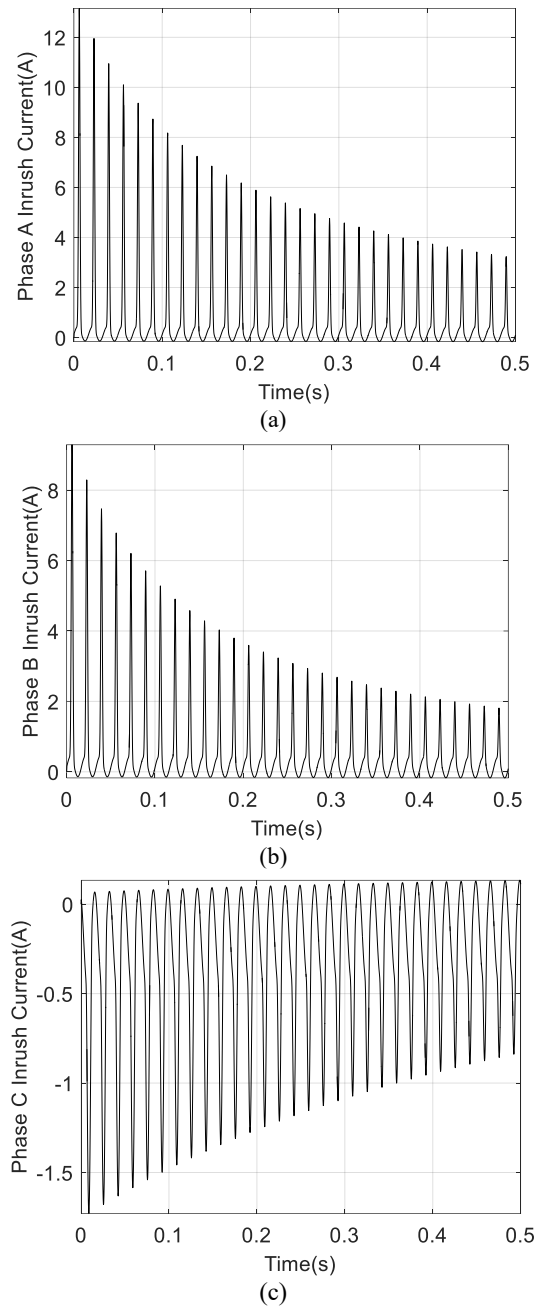


Fig. 3. Inrush currents of the transformer; (a) Phase A, (b) Phase B, (c) Phase C

$$[L] = \begin{bmatrix} L_1 & M_{12} & M_{1a} & M_{1b} & M_{14} & M_{15} & M_{16} \\ M_{21} & L_2 & M_{2a} & M_{2b} & M_{24} & M_{25} & M_{26} \\ M_{a1} & M_{a2} & L_a & M_{ab} & M_{a4} & M_{a5} & M_{a6} \\ M_{b1} & M_{b2} & M_{ba} & L_b & M_{b4} & M_{b5} & M_{b6} \\ M_{41} & M_{42} & M_{4a} & M_{4b} & L_4 & M_{45} & M_{46} \\ M_{51} & M_{52} & M_{5a} & M_{5b} & M_{54} & L_5 & M_{56} \\ M_{61} & M_{62} & M_{6a} & M_{6b} & M_{64} & M_{65} & L_6 \end{bmatrix} \quad (39)$$

The fault in the winding has caused to have the number of N_a turns of the winding in the upper part and N_b turns of the winding in the lower part of the winding. The unknown resistances R_a and R_b are given as:

$$R_a = \frac{N_a}{N_3} R_3, R_b = \frac{N_b}{N_3} R_3 \quad (40)$$

The other unknown values in the equation (39) are brought as follows:

$$a = \frac{N_a}{N_b} \quad (41)$$

$$L_a = \frac{a^2 L_3}{a^2 + 2a + 1} \quad (42)$$

$$L_b = \frac{L_3}{a^2 + 2a + 1} \quad (43)$$

$$M_{ab} = \frac{a L_3}{a^2 + 2a + 1} \quad (44)$$

$$M_{3i} = M_{ai} + M_{bi} \quad (45)$$

$$\frac{M_{ai}}{M_{bi}} = a \quad (46)$$

$$M_{ai} = \frac{a}{1+a} \times M_{3i} \quad (47)$$

$$M_{bi} = \frac{1}{1+a} \times M_{3i} \quad (48)$$

Therefore, the resistance and inductance matrices of the transformer in the section 2 considering connection of winding to the ground are given as:

$$[R] = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 \\ 0 & 0 & R_b & 0 \\ 0 & 0 & 0 & R_3 \end{bmatrix} \quad (49)$$

$$[L] = \begin{bmatrix} L_1 & M_{1a} & M_{1b} & M_{13} \\ M_{a1} & L_a & M_{ab} & M_{a3} \\ M_{b1} & M_{ba} & L_b & M_{b3} \\ M_{31} & M_{3a} & M_{3b} & L_3 \end{bmatrix} \quad (50)$$

Because the resistance and inductance values change depending on the location of the fault, unknown values in (49) and (50) are calculated for 9 positions of the winding connection to the ground. These values have been considered in the upper 10% of the winding to the lower 10% of the winding respectively. It should be noted that the fault is considered in the middle phase and the transformer windings connection is star-shaped. According to the new resistance and inductance values in the fault conditions, it is possible to calculate the fault current during the startup of the transformer in every 10% of the winding. The inrush current estimation in the fault mode is also done using the EKF formulation. It should be noted that the simulation has been performed at switching angle 42.97° . Figure 4 shows the difference between measured and estimated current in faulty transformer which the fault occurs in the upper 10% of the winding. The difference between the actual and estimated inrush current for the faulty transformer with the location of the fault in the half of the winding have been shown in figure 5. As it can be seen from the simulation results, depending on the location of the fault, there is always a percentage of estimation error between the measured and the estimated signal. Figure 6 shows the difference between measured and estimated current in faulty transformer which the fault occurs in the lower 10% of the winding. As it can be seen in figure 6, in this the location of the fault, the fault current is very close to the normal inrush current.

V. Discriminating the Fault from the Inrush Current and the Winding Fault Location

A) The estimation error standard deviation

In this section, the covariance matrix of the estimation error is introduced in order to determine when the error becomes close to zero. In order to check the estimation error changes

relative to time, the trace of estimation error covariance matrix is used. The trace of a matrix, which is defined only for square matrices, is equal to the sum of the eigenvalues of the matrix.

$$\text{trace}(A) = \sum \lambda_i \quad (51)$$

It can be shown that the estimation error starts from 0.01 at the beginning and increases, but after a period of time (about 10 milliseconds), it reaches its final value. Estimation error covariance matrix is used to monitor the steady state estimation error value. On the other hand, to obtain the estimation error in the steady state, the standard deviation index is also used so that in addition to the covariance matrix, which provides an intuitive view of the estimation error, a comparable numerical index is also available.

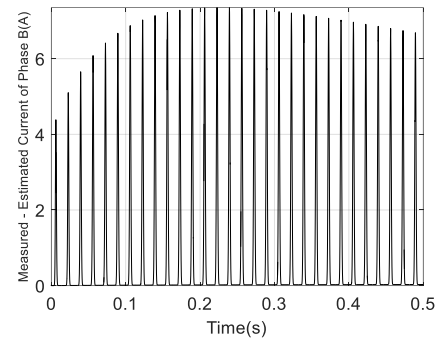


Fig. 4. Difference between measured and estimated current with the location of the fault in the upper 10% of the winding in phase B of the transformer

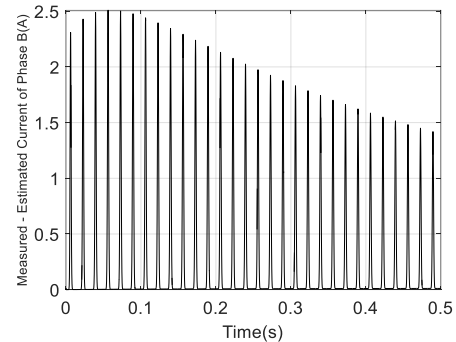


Fig. 5. Difference between measured and estimated current with the location of the fault in the half of the winding in phase B of the transformer

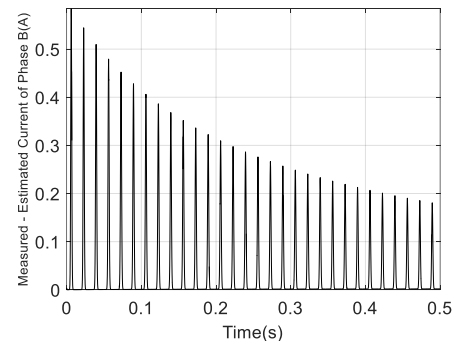


Fig. 6. Difference between measured and estimated current with the location of the fault in the lower 10% of the winding in phase B of the transformer

The standard deviation is the root of the variance. Variance is defined as the average value of the square of the differences of the values from the mean. Standard deviation is one of the dispersal indicators so that shows the data how far from the average value. The estimation error standard deviation of the transformer fault current in phases a, b and c are equal to 0.004, 0.01 and 0.02 respectively. In the previous section, the simulation results showed that the estimated current of the transformer is almost the same as the measured current in the healthy transformer, and the estimation error eventually tended to zero. On the other hand, it was shown that in a faulty transformer, there is always a significant difference between the estimated and the measured inrush current relative to the location of the fault. But in order to obtain the numerical value of this difference and to discriminate between the normal inrush current and the fault current, a numerical index is required. Table 2 shows the estimation error standard deviation of the healthy three phase transformer at different switching angles.

Table 2. The estimation error standard deviation of the healthy transformer

Switching angle(°)	Phase A	Phase B	Phase C
5	0.004	0.01	0.02
42.97	0.01	0.008	0.01
80	0.05	0.07	0.06

Figure 7 shows the estimation error standard deviation of the faulty three phase transformer at different switching angles with 9 separate locations of the fault in the winding of the phase B. The standard deviation index at different switching angles does not show a constant value to identify the location of the fault. To solve this problem, an index β is defined so that it is equal to the second norm of the inrush current vector divided by the standard deviation of the estimation error. The estimation error is equal to the difference between the measured and the estimated inrush current:

$$\hat{i}_{inr} = i_{inr, meas} - i_{inr, est} \quad (52)$$

Therefore, the standard deviation of the estimation error is calculated as:

$$i_{inr, sd} = \sqrt{\frac{\sum_{j=1}^N (i_{inr}(j) - i_{inr, mean})^2}{N}} \quad (53)$$

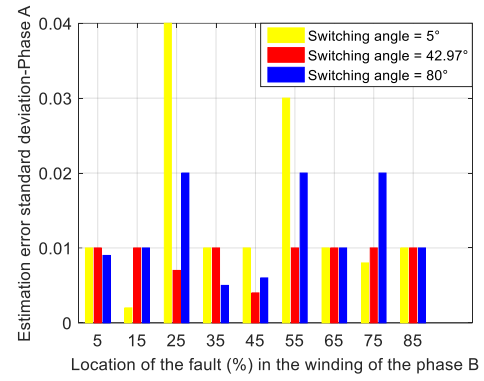
where:

$$\hat{i}_{inr, mean} = \frac{\hat{i}_{inr1} + \hat{i}_{inr2} + \dots + \hat{i}_{inrN}}{N} \quad (54)$$

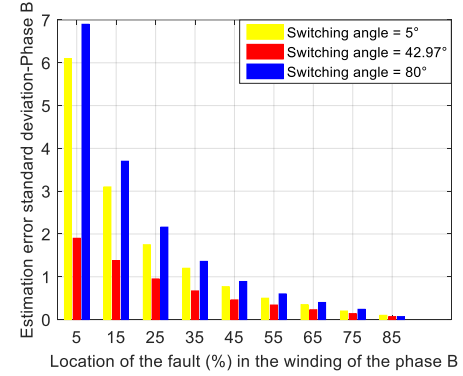
Index β is given as:

$$\beta = \frac{\sqrt{\sum_{j=1}^N i_{inr}^2(j)}}{i_{inr, sd}} \times 100 = \frac{\sqrt{\sum_{j=1}^N i_{inr}^2(j)}}{\sqrt{\frac{\sum_{j=1}^N (i_{inr}(j) - i_{inr, mean})^2}{N}}} \times 100 \quad (55)$$

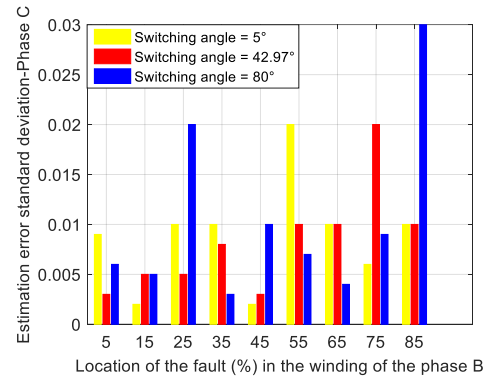
This index has been calculated in Table 3 in the healthy transformer at three different switching angles. On the other hand, this index has been calculated in Table 4 in the faulty transformer, with 9 separate locations of the fault in the winding, and with three different switching angles.



(a)



(b)



(c)

Fig. 7. The estimation error standard deviation of the faulty transformer at different switching angles; a) Phase A, b) Phase B, c) Phase C

The estimation error in table 2 shows that in the healthy transformer (phase B), the maximum value of the estimation error is 0.01, on the other hand in the faulty transformer with different percentages of error, the minimum value is 0.07. Therefore, it is possible to use the estimation error standard deviation index to determine whether the transformer is faulty or is not. Its adjustment value is set at 0.05 in this paper. In this way, if the value of this index is greater than 0.05 in any phase, it can be shown that there is a fault condition. The index β in table 3 shows that for a healthy transformer (phase B), the minimum value of this index is 6.2%. The maximum value of this index is 5.9% for the faulty phase with the location of the fault in 90% of the winding. Therefore, it is possible to classify the location of the fault in the table 5.

Table.3.The index β of the healthy transformer

Switching angle(°)	Phase A	Phase B	Phase C
5	34.2	85.1	5.5
42.97	27.6	6.2	29.4
80	24.3	78	7.4

Table.4.The index β of the faulty transformer

Location of the fault (%) in the winding of phase B	Switching angle(°)	Phase A	Phase B	Phase C
	5	13.8	0.4	12
Upper 10%	42.97	27.9	0.49	39.3
	80	7.4	0.41	13
	5	68.5	0.51	53.5
10 to 20	42.97	28.5	0.53	12
	80	6.7	0.51	14.8
	5	6.75	0.55	10.7
20 to 30	42.97	40.2	0.59	23.2
	80	3.35	0.53	3.7
	5	13.8	0.63	10.6
30 to 40	42.97	28.5	0.69	14.75
	80	13.4	0.64	24.6
	5	13.6	0.79	5.5
40 to 50	42.97	70	0.86	39.4
	80	11.1	0.79	7.4
	5	4.5	1	5.35
50 to 60	42.97	28.7	1.1	118
	80	3.35	1	10.5
	5	13.8	1.3	10.6
60 to 70	42.97	28.8	1.46	11.7
	80	6.7	1.32	18.5
	5	17.3	2	17.6
70 to 80	42.97	27.8	2.16	59
	80	3.35	1.97	8.2
	5	13.6	3.2	10.6
80 to 90	42.97	28.4	4	12
	80	6.7	5.9	2.46

Table 5. The location of the fault classification in faulty phase of the transformer

The location of the fault in the winding	Index β (%)
Upper 10%	0.4 < β < 0.49
10 to 20	0.5 < β < 0.54
20 to 30	0.55 < β < 0.6
30 to 40	0.6 < β < 0.69
40 to 50	0.7 < β < 0.89
50 to 60	0.9 < β < 1.1
60 to 70	1.2 < β < 1.6
70 to 80	1.7 < β < 2.8
80 to 90	2.9 < β < 6.0

It is worth noting that fault localization has not been performed in reference [17]. To identify nonlinear model, Equation (56) has been used for modelling the magnetic core behavior in [17]:

$$i_{\psi}(t) = 16\psi^5 - 47\psi^4 + 43\psi^3 - 11.5\psi^2 - 0.5\psi + 0.02 \quad (56)$$

B) Validation of the results

Table 6 shows comparison of the other references in fault detection and location as follows:

Table 6. Comparison of the other references in fault detection and location

Ref.	Method	Transformer	Fault type	Location capability	Measurement
[7]	FRA	A winding	Turn-to-turn	Yes	Yes
[11]	Wavelet Transform	Three phase	Winding to the ground	Yes	No
[15]	Neural Network	Single phase	Winding to the ground	No	No
[16]	EKF	Single Phase	Winding to the ground	No	No
[17]	EKF	Single Phase	Winding to the ground	No	Yes
[20]	Hermite transform	Three phase	Internal faults	No	No
[24]	FRA	Three phase	Internal faults	No	Yes
[25]	FRA	Three phase	Insulation faults	Yes	Yes

The presented method has been validated using the single phase transformer in [17]. The specifications of this transformer have been brought in Table 7.

Table.7. Parameters of the single phase transformer in [17]

Name	Value
Power	15 kVA
Ratio	2300/230
Frequency	50 Hz
Primary winding	3.225 Ω
Primary winding	4.85 Ω
Core resistance	105 k Ω

Equivalent circuit of the transformer is shown in Figure 8.

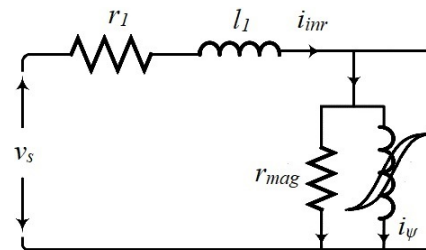


Fig.8. Equivalent circuit of the single phase transformer

State space equations for this single phase transformer is given as follows:

$$\begin{bmatrix} l \\ 0 \end{bmatrix} - \frac{1}{r_{mag}} \times \begin{bmatrix} \frac{di_{inr}(t)}{dt} \\ \frac{d\psi(t)}{dt} \end{bmatrix} = \begin{bmatrix} -r & 0 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} v_s(t) \\ i_\psi(t) \end{bmatrix} \quad (57)$$

Or:

$$\begin{aligned} \underbrace{\begin{bmatrix} \frac{di_{inr}(t)}{dt} \\ \frac{d\psi(t)}{dt} \end{bmatrix}}_{\dot{x}} &= \underbrace{\begin{bmatrix} -\frac{r+r_{mag}}{l} & 0 \\ r_{mag} & 0 \end{bmatrix}}_A \times \underbrace{\begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix}}_x \\ &+ \underbrace{\begin{bmatrix} \frac{1}{l} & \frac{r_{mag}}{l} \\ 0 & -r_{mag} \end{bmatrix}}_B \times \underbrace{\begin{bmatrix} v_s(t) \\ i_\psi(t) \end{bmatrix}}_u \\ y &= \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_C \begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix} \end{aligned} \quad (58)$$

The nonlinear EKF model of the system can be represented as follows:

$$\begin{aligned} \dot{x} &= f(x, V_s(t), \omega, t) \\ y &= h(x, v, t) \\ \omega &= (0, Q) \\ v &= (0, R) \end{aligned} \quad (59)$$

Matrices Q and R are given as follows [17]:

$$Q = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix} \quad (60)$$

$$R = 1 \quad (61)$$

We have:

$$\begin{aligned} f(x, V_s(t), \omega, t) &= \begin{bmatrix} -\frac{r+r_{mag}}{l} & 0 \\ r_{mag} & 0 \end{bmatrix} \begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{l} & \frac{r_{mag}}{l} \\ 0 & -r_{mag} \end{bmatrix} \times \begin{bmatrix} v_s(t) \\ i_\psi(t) \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \end{aligned} \quad (62)$$

$$h(x, v, t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (63)$$

$$F = \frac{\partial f}{\partial x} = \begin{bmatrix} -\frac{r+r_{mag}}{l} & \frac{r_{mag}}{l} \cdot \frac{\partial i_\psi}{\partial \psi} \\ r_{mag} & -r_{mag} \cdot \frac{\partial i_\psi}{\partial \psi} \end{bmatrix} \quad (64)$$

$$L = \frac{\partial f}{\partial \omega} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (65)$$

$$H = \frac{\partial h}{\partial x} = C \quad (66)$$

$$M = \frac{\partial h}{\partial v} = I_{2 \times 2} \quad (67)$$

The residual flux is initially assumed to be zero. Figure 9 shows the inrush current of the transformer with zero residual flux at switching angles 0° (zero voltage). Now the residual flux is assumed to be 20%. Figure 10 shows the inrush current of the transformer with 20% residual flux at switching angles 0°. On the other hand, the proposed method has 9ms mean detection time in distinguishing inrush current from fault current and fault location. Figure 11 shows the inrush current of the transformer with zero residual flux and CT saturation at switching angles 0°. Besides, Figure 12 shows the inrush current of the transformer with 20% residual flux and CT saturation at switching angles 0°.

Table 8 shows the estimation error standard deviation of the second studied transformer in the faulty condition with 9 separate locations of the fault in the winding. To examine the sensitivity and robustness of the proposed method for fault location, we first make 5% change in the transformer resistance and leakage inductance values. Secondly, we apply a 10% change to the noise matrices values in the EKF method. Table 8 shows the percentage of changes in the standard deviation of the estimation error from its previous values. As it can be seen from Table 8, the estimation error standard deviation has a very small deviation from the mentioned changes.

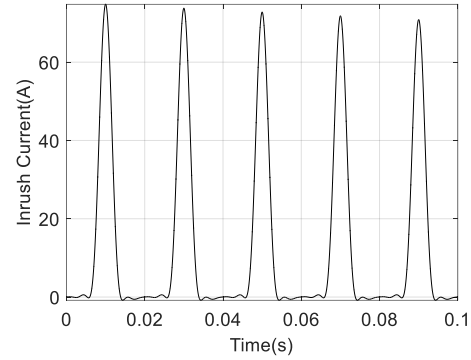


Fig. 9. Transformer Inrush current with zero residual flux at switching angle 0°

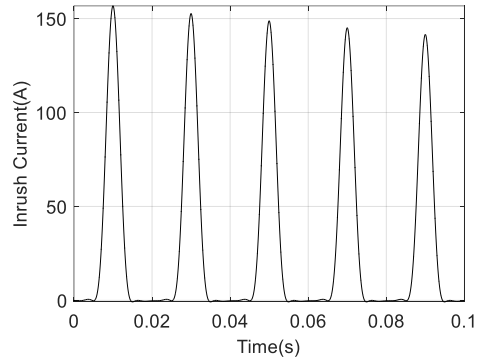


Fig. 10. Transformer Inrush current with 20% residual flux at switching angle 0°

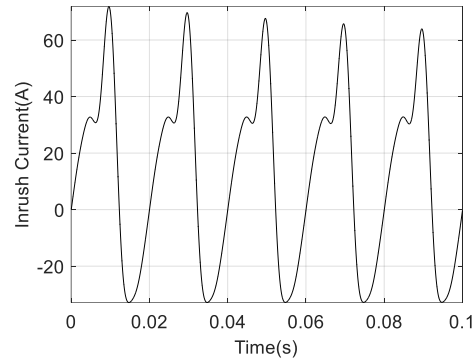


Fig. 11. Transformer Inrush current with zero residual flux and CT saturation at switching angle 0°

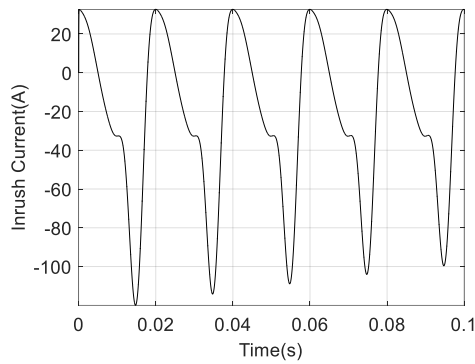


Fig. 12. Transformer Inrush current with 20% residual flux and CT saturation at switching angle 0°

In Table 9, in the faulty condition, index β has been calculated for the second studied transformer. The index β in table 9 shows that the maximum value of this index is 3.8% for the faulty phase with the location of the fault in 90% of the winding. Therefore, the index β values in table 9 are in agreement with the index limits. On the other hand, Table 9 shows the percentage of changes in the index β from its previous values. As it can be seen from Table 9, index β has a very small deviation from the noise and parameter mismatch changes.

The recognition time of inrush current from fault current, and fault location in the transformer has been shown in Table

Table 8. The estimation error standard deviation (EESD) of the second studied transformer in the faulty condition, the percentage of changes in EESD with 5% change in the transformer resistance and leakage inductance values, and 10% change to the noise matrices values

Location of the fault (%) in the winding	EESD	5% change in the resistance and leakage	The percentage of changes (%) in EESD	10% change to the noise matrices values	The percentage of changes (%) in EESD
Upper 10%	1.85	1.87	1.08	1.85	0
10 to 20	1.34	1.35	0.75	1.34	0
20 to 30	0.91	0.92	1.09	0.90	1.09
30 to 40	0.64	0.65	1.56	0.64	0
40 to 50	0.42	0.43	2.3	0.43	2.38
50 to 60	0.32	0.32	0	0.32	0
60 to 70	0.20	0.21	5.0	0.21	0
70 to 80	0.12	0.12	0	0.12	0
80 to 90	0.07	0.07	0	0.07	0

10. Intelligent algorithms studied in the other works and customary fault location methods have been compared with the proposed detection time. On the other hand, references

Table 9. The index β of the second studied transformer in the faulty condition, the percentage of changes in index β with 5% change in the transformer resistance and leakage inductance vales, and 10% change to the noise matrices values

Location of the fault (%) in the winding	Index β (%)	5% change the resistance and leakage inductance values	The percentage of changes (%) in Index β	10% change to the noise matrices values	The percentage of changes (%) in Index β
Upper 10%	0.47	0.47	0	0.47	0
10 to 20	0.51	0.51	0	0.51	0
20 to 30	0.58	0.58	0	0.58	0
30 to 40	0.68	0.68	0	.68	0
40 to 50	0.86	0.87	1.16	0.86	0
50 to 60	1.08	1.10	1.85	1.09	0.92
60 to 70	1.44	1.47	2.04	1.46	1.37
70 to 80	1.95	1.98	1.53	1.96	0.51
80 to 90	3.8	3.88	2.10	3.83	0.79

[16] and [17] have not given the fault location in the transformer. Table 10 shows that the proposed algorithm has a sensibly well detection time compared to the other methods in fault location and differentiating inrush current from fault current. Besides, the residual signal which is the difference between the estimated and measured currents in case of energizing a faulty transformer has been considered in Table 10. Therefore the absolute residual signal (ARS) using the proposed technique has been compared with the other algorithms in Table 10. Therefore, Table 10 shows that the

proposed algorithm has a reasonably well residual signal compared to the other methods in discriminating inrush current from fault current.

Besides, the other algorithms such as wavelet transform require higher sampling rate and are extremely sensitive to noise and unpredicted disturbances. The EKF-based methods have very good robustness since they deal with the measurement and process noises suitably, unlike the wavelet transform that is highly affected by noise and disturbances.

Table 10 Localization speed, detection time, and ARS of inrush current from fault current using different algorithms

Algorithm	Mean detection time (ms)	ARS (A)
Proposed	9	5
Wavelet Transform [11]	72	12
Convolutional Neural Network[15]	34	---
[16]	115	14
EKF [17]	5	7
Hermite transform [20]	26	10
Decision Tree [31]	3	---
Artificial Neural Network [31]	487.5	---

C) The practical implementation and limitations of the proposed method

The electrical fault detection and real-time monitoring system integrates multi microcontroller architecture, real-time sensor monitoring (voltage and current), wireless communication , and cloud-based data logging to create a comprehensive power fault management solution. The system must simulate a miniaturized power distribution network with capabilities to detect Line-to-Ground (L–G) faults, measure real-time electrical parameters, automatically isolate faulty sections using relay-based switching, and provide multi-platform monitoring through OLED displays and web-based dashboards. When a fault is detected, the system sends an SMS to the maintenance personnel notifying them of the type of fault and where it is. This is a real-time alert system that reduces downtimes and allows the services to be restored faster.

The convergence of the extended Kalman filter is locally for nonlinear systems. If the initial estimation error is not too large then as time goes to infinity, the error goes to zero exponentially. In this paper, the convergence of this filter is strongly dependent on the nonlinear characteristic of the core and operating point for fault recognition. Divergence probability of the filter increases if the amplitude of the inrush current is large. Therefore, the probability of divergence is high in power transformers with large changes in core nonlinear characteristic and with large magnitude in inrush current. Besides, the EKF convergence is dependent to the amount of the residual flux. Even by reduction of the time step, convergence is not obtained in these cases. As a result, it is unimaginable that the extended Kalman filter can converge for all values of the switching angle or residual fluxes.

VI. Conclusion

To differentiate between the fault current and the inrush current in a transformer, EKF can be used. The EKF has

resistance against measurement noises, the high estimation speed and as well as appropriate estimation accuracy. In this paper, the nonlinear Kalman filter or EKF is used to more accurately estimate the transformer inrush current. The simulated results are presented in different percentages of the winding to the ground fault and is determined the location of the fault. On the other hand, to obtain the estimation error in the steady state, the standard deviation index is also introduced so that to provide an intuitive view of the estimation error. Then again, it is shown that in a faulty transformer, there is always a significant difference between the estimated and the measured inrush current relative to the location of the fault. In addition, an index β is defined for any phase of the transformer so that if the value of this index is in a specific range of value, the location of the fault at the winding can be shown. Thus, it is possible to classify the location of this type of fault in the transformer. Finally, the proposed technique is implemented for another transformer to calculate the estimation error standard deviation and index β in the faulty condition.

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A New Quadratic Voltage Lift Cascaded Boost Topology for Hydrogen Extraction

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Article Info	ABSTRACT
Article type: Research Article	This paper presents a novel non-isolated DC-DC converter topology with several significant advantages. First, the input current remains continuous, thereby reducing the current stress on the input filter capacitor. Second, the converter utilizes the same number of inductors as conventional topologies such as SEPIC, Zeta, and Ćuk converters. Third, it achieves high voltage gain at relatively low duty cycles. Fourth, the maximum voltage stress on the semiconductors remains well below the output voltage, ensuring improved device reliability. Fifth, the design incorporates only a single switch, simplifying the drive circuitry. Sixth, the voltage stress on the switch is significantly lower than the output voltage. Seventh, a quadruple voltage-lift is realized using an enhanced diode-capacitor voltage multiplier integrated at both stages of the converter. Finally, although the power circuit employs 15 diodes and 14 capacitors, the topology maintains a high voltage gain density, justifying the component count. Experimental results are provided to validate the theoretical analysis. The implemented prototype successfully boosts an input voltage of 20 V to an output of 1200 V at a 50% duty cycle, delivering an output power of less than 200 W.
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Keywords: CUK-Based, Step-up dc-dc converter, Quadratic.	

I. Introduction

Hydrogen is widely recognized as a clean and sustainable energy source for the future [1]. It can be efficiently extracted from water, a resource that covers the majority of Earth's surface [2], [3]. Although water electrolysis can occur at low voltages, applying high-voltage pulses with a low average value has been shown to significantly enhance hydrogen production efficiency due to kinetic and ohmic losses [4], [5]. High voltage enables rapid gas output in a given cell, which is crucial for applications demanding high throughput or compact system size. High voltage can forcibly drive a substantial current even in systems with high internal resistance (e.g., cells with large electrode gaps, low-conductivity electrolytes, or resistive solid electrolytes). Low-voltage DC sources such as batteries, fuel cells, and photovoltaic (PV) panels are readily available [6]-[8].

However, their output voltages are typically insufficient to drive the input terminals of pulse power converters used in high-efficiency water electrolysis systems [9], [10]. Therefore, a high-gain DC-DC conversion stage is essential. Transformer-based DC-DC converters are one possible solution [11], [12]. However, high-frequency transformers increase the overall system cost, size, weight, electromagnetic interference (EMI), and voltage stress on semiconductors [13], [14]. As electrical isolation is not always required in these applications, non-isolated high-gain DC-DC converters offer a more compact and cost-effective alternative [15].

Among non-isolated converter topologies, the conventional boost converter (Fig. 1(a)) remains the most widely adopted due to its simplicity, compactness, and reliability. However, its voltage gain is inherently limited, making it unsuitable for high-voltage applications. One common approach to enhance

its performance is by integrating diode–capacitor voltage multiplier cells, as illustrated in Figs. 1(b) and 1(c). These configurations utilize the voltage-lift technique to boost the output voltage. Nevertheless, the resulting voltage gain is typically limited to only twice that of the conventional boost converter. Another method to achieve higher voltage gain is through the cascaded boost converter topology (Fig. 1(d)), which provides a quadratic voltage gain relative to the original boost converter. Despite this improvement, the topology requires duty cycles greater than 50% to surpass the gain achieved by modified boost structures. Additionally, it suffers from a major drawback: the voltage stress on the switch equals the full output voltage, which can reduce reliability and increase cost. The topology shown in Fig. 1(e) addresses the issue of switch voltage stress; however, it still falls short in achieving sufficiently high voltage gain for demanding applications.

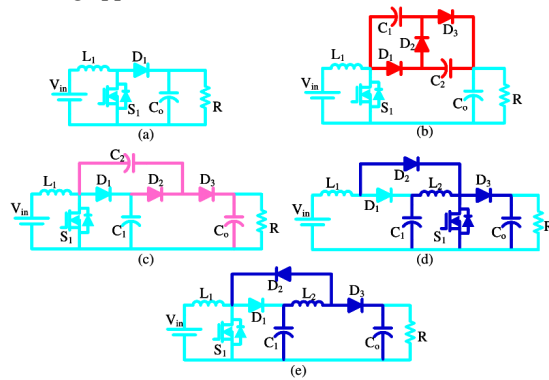


Fig. 1. The classic boost topology. (a) Classic boost converter, (b) and (c) Boost converter using diode–capacitor voltage multiplier cells, (d) Classic quadratic converter, and (e) Boost converter with reduced voltage stress.

The topologies proposed in [16]–[48] represent various improved non-isolated DC-DC converters. Among these, the structures presented in [16]–[20] offer multi-mode operation, functioning as step-up, step-down, and pass-through converters. However, they require relatively high duty cycles to achieve voltage gains exceeding that of the conventional boost converter. Additionally, the voltage stress on the semiconductors in these designs often exceeds the output voltage, which is generally undesirable for practical applications.

The converters introduced in [21]–[22] exhibit improved voltage gain compared to those in [16]–[20], yet they suffer from the same drawbacks—namely, high voltage stress and dependency on large duty cycles. Similarly, the topologies described in [23]–[25] offer marginally better voltage gain but continue to exhibit the aforementioned limitations. Furthermore, they introduce an additional issue: reversed output polarity, which complicates integration with standard loads.

The quadratic boost topologies in [26]–[27] achieve better voltage gains than the aforementioned designs. However,

their performance still falls short of the high-gain requirements necessary for applications such as hydrogen generation. The structure proposed in [28] improves upon the classic boost converter by incorporating voltage-lift techniques, yet the resulting gain does not sufficiently justify the increased number of components.

The converters presented in [29]–[32] achieve approximately double the voltage gain of the conventional boost topology; however, this increase does not constitute a significant performance enhancement. In contrast, the topology in [33] integrates two voltage multiplier cells within the classic boost converter structure. Although this design successfully isolates the common ground between the input source and the load, it is unable to generate sufficiently high output voltages from low input levels, limiting its applicability in hydrogen extraction systems. Reference [34] proposes an improved version of the classic buck-boost converter, achieving higher voltage gain than the base topology. Nonetheless, this design fails to address the inherent topological limitations of the original converter, and while its voltage gain surpasses that of traditional converters, it remains modest compared to other advanced topologies. In addition, in [49] and [50], two new quadratic step-up DC-DC converters with low input current ripple using switched capacitor techniques are suggested. However, these structures cannot offer ultra-high voltage gain.

This study proposes an improved cascaded boost topology featuring ultra-high voltage gain. The design incorporates an enhanced diode–capacitor voltage multiplier cell to achieve significant voltage lifting. Unlike conventional boost and cascaded boost converters, the proposed topology overcomes their main limitations, addressing issues related to voltage gain and component stress. The substantial voltage gain delivered by this converter makes it particularly well-suited for high-voltage applications, including dielectric test equipment and water electrolysis systems.

II. The Proposed Topology Function

The proposed topology is illustrated in Fig. 2(a). It is based on the integration of two diode–capacitor voltage multiplier cells—similar to those used in the improved topologies shown in Figs. 1(b) and 1(c)—combined with an enhanced cascaded boost topology, as depicted in Fig. 1(e). Notably, the design utilizes only two inductors, matching the number found in classic converter topologies such as SEPIC, Čuk, and Zeta. The input current in the proposed converter is continuous, with the input current ripple matching that of the inductor L_1 . The switching device is positioned in the low-voltage section of the circuit, resulting in reduced voltage stress on the switch. The equivalent circuits corresponding to the first and second operating modes are illustrated in Figs. 1(b) and 1(c).

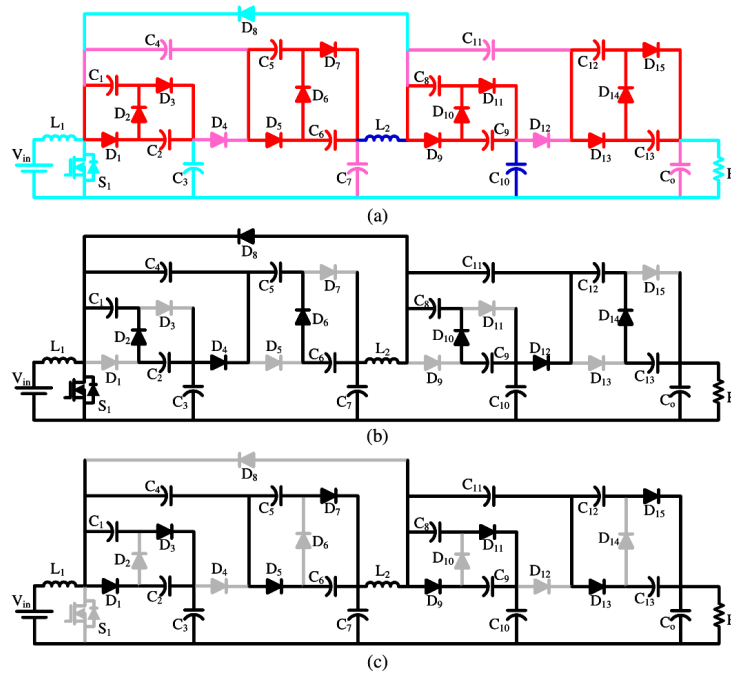


Fig.2. (a) Proposed converter, (b) The equivalent circuit of the first mode, (c) The equivalent circuit of the second mode.

During the first operating mode, the voltage applied to the inductors is positive, causing them to magnetize. In this mode, capacitors C_3 and C_4 , C_{10} and C_{11} , the series combination of C_1 , C_2 , and C_3 , the series combination of C_8 , C_9 , and C_{10} , the series connection of C_4 through C_6 and C_7 , and the series connection of C_{11} through C_{13} and C_o are all connected in parallel. In the second operating mode, the applied voltage to the inductors reverses polarity, causing them to demagnetize. During this interval, capacitors C_1 and C_2 , C_5 and C_6 , C_8 and C_9 , C_{12} and C_{13} , the series combination of C_2 through C_5 , and the series combination of C_9 through C_{12} with C_o are connected in parallel. According to the expressed concepts and the equivalent circuits, the inductors' voltage and capacitors' current in the steady state are as (1)-(16):

$$v_{L1} = DV_{in} + (1-D)(V_{in} + V_{C2} - V_{C3}) \quad (1)$$

$$v_{L2} = DV_{C7} + (1-D)(V_{C7} + V_{C9} - V_{C10}) \quad (2)$$

$$i_{C1} = Di_3 + (1-D)(-i_1) \quad (3)$$

$$i_{C2} = Di_3 + (1-D)(i_1 + i_6 - i_{L1}) \quad (4)$$

$$i_{C3} = D(-i_3 - i_4) + (1-D)(i_{L1} - i_6) \quad (5)$$

$$i_{C4} = D(i_4 + i_7) + (1-D)(-i_6) \quad (6)$$

$$i_{C5} = D(i_7) + (1-D)(-i_5) \quad (7)$$

$$i_{C6} = D(i_7) + (1-D)(i_5 - i_6) \quad (8)$$

$$i_{C7} = D(i_{10}) + (1-D)(-i_8) \quad (9)$$

$$i_{C8} = D(i_{10}) + (1-D)(-i_1) \quad (10)$$

$$i_{C9} = D(i_{10}) + (1-D)(i_8 + i_{14} - i_{12}) \quad (11)$$

$$i_{C10} = D(-i_{10} - i_{11}) + (1-D)(i_{L2} - i_{13}) \quad (12)$$

$$i_{C11} = D(i_{11} + i_{14}) + (1-D)(-i_{13}) \quad (13)$$

$$i_{C12} = D(i_{14}) + (1-D)(-i_{12}) \quad (14)$$

$$i_{C13} = D(i_{14}) + (1-D)(i_{12} - i_{13}) \quad (15)$$

$$i_{C_o} = D(-i_{14} - I_o) + (1-D)(i_{13} - I_o) \quad (16)$$

In this context, currents i_1 , i_3 through i_8 , and i_{10} through i_{14}

represent the inrush currents generated during the parallel connection of the capacitors. Based on the voltage-second balance principle, the average voltage across the inductors is zero. Similarly, according to the current-second balance, the average current flowing through the capacitors is zero. By applying these principles to equations (1)–(16), the average inductor currents, average capacitor voltages, and capacitor inrush currents are derived as shown in equations (17) and (28).

$$V_{C1} = V_{C2} = V_{C5} = V_{C6} = \frac{V_{in}}{1-D} \quad (17)$$

$$V_{C3} = V_{C4} = V_{C7} = \frac{4V_{in}}{1-D} \quad (18)$$

$$V_{C8} = V_{C9} = V_{C12} = V_{C13} = \frac{4V_{in}}{(1-D)^2} \quad (19)$$

$$V_{C10} = V_{C11} = \frac{8V_{in}}{(1-D)^2} \quad (20)$$

$$V_{C_o} = \frac{8V_{in}}{(1-D)^2} \quad (21)$$

$$i_{L1} = \frac{16}{(1-D)^2} I_o \quad (22)$$

$$i_{L2} = \frac{4}{1-D} I_o \quad (23)$$

$$i_1 = i_5 = \frac{4}{(1-D)^2} I_o \quad (24)$$

$$i_3 = i_4 = i_7 = \frac{4}{D(1-D)} I_o \quad (25)$$

$$i_6 = \frac{8}{(1-D)^2} I_o \quad (26)$$

$$i_8 = i_{12} = i_{13} = \frac{I_o}{1-D} \quad (27)$$

$$i_{10} = i_{11} = i_{14} = \frac{I_o}{D} \quad (28)$$

The provided voltage gain in the cascaded boost converter is $\frac{1}{(1-D)^2}$. Comparing this voltage gain with the one provided in the proposed topology shows that the voltage gain in the proposed topology is 16 times that of the cascaded boost converter. Based on the derived voltage gain, Fig. 3 compares the performance of the proposed converter with the topologies presented in Fig. 1.

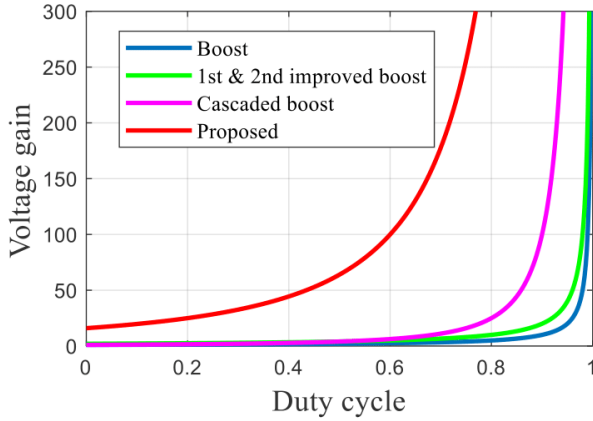


Fig. 3. Voltage gain comparison.

It is evident that at low duty cycles, the proposed topology delivers a higher voltage gain than the boost, improved boost, and cascaded boost converters operating at higher duty cycles. This comparison clearly demonstrates the superior performance of the proposed topology. The semiconductors' voltage/current stresses are as follows ((28)-(34)):

$$V_S = V_{D1-D7} = \frac{V_{in}}{1-D} \quad (28)$$

$$V_{D8} = \frac{(3+D)}{(1-D)^2} V_{in} \quad (29)$$

$$V_{D9} = V_{D10-D15} = \frac{4}{(1-D)^2} V_{in} \quad (30)$$

$$I_S = \frac{15+2D-D^2}{(1-D)^2} I_o \quad (31)$$

$$I_{D1-D7} = \frac{4}{1-D} I_o \quad (32)$$

$$I_{D8} = \frac{3+D}{1-D} I_o \quad (33)$$

$$I_{D9-D15} = I_o \quad (34)$$

To evaluate the operating conditions of the semiconductors, the voltage stresses are compared against the output voltage, while the current stresses are compared with the input current. The normalized values are illustrated in Fig. 4. As shown in Fig. 4(a), the voltage stresses on the semiconductors are significantly lower than the output voltage. Except for the switch, whose current stress approaches the input current level, the current stresses on all other components remain considerably below the input current. Therefore, all

semiconductor devices operate within safe limits. The relatively low voltage stress on the semiconductors makes this converter especially suitable for low-voltage applications requiring high voltage gain, such as water electrolysis.

The simplified current ripple of the inductors and capacitors' voltage ripple are as (35) to (41).

$$\Delta i_{L1} = \frac{DV_{in}}{L_1 f_s}, \quad \Delta i_{L2} = \frac{4DV_{in}}{(1-D)L_2 f_s} \quad (35)$$

$$\Delta V_{C1} = \frac{4I_o}{(1-D)C_1 f_s}, \quad \Delta V_{C2} = \frac{4I_o}{(1-D)C_2 f_s} \quad (36)$$

$$\Delta V_{C3} = \frac{8I_o}{(1-D)C_3 f_s}, \quad \Delta V_{C4} = \frac{4I_o}{(1-D)C_4 f_s} \quad (37)$$

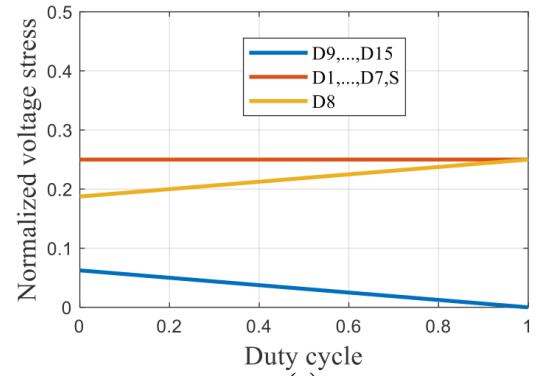
$$\Delta V_{C5} = \frac{4I_o}{(1-D)C_5 f_s}, \quad \Delta V_{C6} = \frac{4I_o}{(1-D)C_6 f_s} \quad (38)$$

$$\Delta V_{C7} = \frac{4(1+D)I_o}{(1-D)C_7 f_s}, \quad \Delta V_{C8} = \frac{I_o}{C_8 f_s} \quad (39)$$

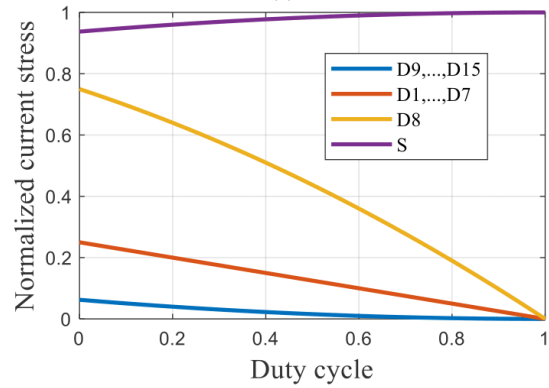
$$\Delta V_{C9} = \frac{I_o}{C_9 f_s}, \quad \Delta V_{C10} = \frac{2I_o}{C_{10} f_s}, \quad \Delta V_{C11} = \frac{2I_o}{C_{11} f_s} \quad (40)$$

$$\Delta V_{C12} = \frac{I_o}{C_{12} f_s}, \quad \Delta V_{C13} = \frac{I_o}{C_{13} f_s}, \quad \Delta V_{Co} = \frac{(1+D)I_o}{C_o f_s} \quad (41)$$

Here, ΔV_{Ci} and f_s are the allowable voltage ripple of the capacitors and switching frequency, respectively.



(a)



(b)

Fig. 4. Normalized voltage/current stresses of the semiconductors. (a) Normalized voltage stress, (b) Normalized current stress.

The operation of the converter in continuous conduction mode (CCM) depends on the inductors' values and their average currents. Specifically, the inductors' values influence the magnitude of the current ripple. To maintain CCM, the peak-to-peak current ripple of each inductor must be less than twice its average current. Based on this criterion, the

minimum inductance required to sustain CCM is given by the following expression:

$$L_1 > \frac{D(1-D)^4 R}{512 f_s}, L_2 > \frac{D(1-D)^2 R}{32 f_s} \quad (42)$$

Here, R denotes the output resistance. The average current through each inductor must exceed half of its corresponding current ripple to ensure continuous conduction mode (CCM). This average inductor current depends on both the duty cycle and the average output current. Fig. 5 illustrates the converter's operating regions, distinguishing between CCM and discontinuous conduction mode (DCM) based on output current and duty cycle. The equations used to generate this plot are provided in (43).

$$I_{OB} = \frac{V_{in}}{2L_2 f_s} D = \frac{V_o}{32L_2 f_s} D(1-D)^2 \quad (43)$$

Which, I_{OB} is the boundary value of the output current between CCM and DCM. Notably, any operating point above the curves shown in Fig. 5 corresponds to continuous conduction mode (CCM), points below the curves indicate discontinuous conduction mode (DCM), and points lying on the curves represent the boundary between these two modes. Notably, the effect of the parasitic components of the circuit elements shows their effect in the higher percentages of the duty cycle. In other words, in the majority of the duty cycle percentage, the converter's function and parameters have the same behavior in both the model and non-ideal modes.

III. Non-Ideal Voltage Gain and Efficiency

The voltage gain derived in the previous section does not account for variations in component quality or changes in output power. By incorporating the resistances of the inductors (r_L), switch (r_s), and diodes (r_D), along with the load resistance (R), the non-ideal voltage gain is expressed as shown in equation (44).

$$\frac{V_o}{V_{in}} = \frac{16}{(1-D)^2} - \frac{r_L}{R} \frac{16^2}{(1-D)^6} - \frac{r_{sc}}{R} \frac{16^2 D}{(1-D)^6} - \frac{r_D}{R} \frac{16^2}{(1-D)^5} \quad (44)$$

Fig. 6 compares the ideal and non-ideal voltage gains. As shown, both curves closely align when the duty cycle is below 80%. Fig. 7 illustrates the voltage gain behavior under different output power levels and component variations. The results indicate that output power has the most significant impact on voltage gain. Additionally, the choice of inductor cores influences the equivalent series resistance, while different switches with varying dynamic resistances and diodes with distinct forward voltage drops are considered in this sensitivity analysis. Considering the conduction loss of inductors (P_L), switch (P_{sc}), and diodes (P_D) and ignoring the frequency loss of the components, the efficiency of the converter can be modelled as (45)-(48). Fig. 8 shows the efficiency sensitivity analysis considering different output

power levels and circuit components. The results indicate that variations in output power have the greatest impact on efficiency. Changes in the type of inductor and switch exhibit approximately equal effects on efficiency.

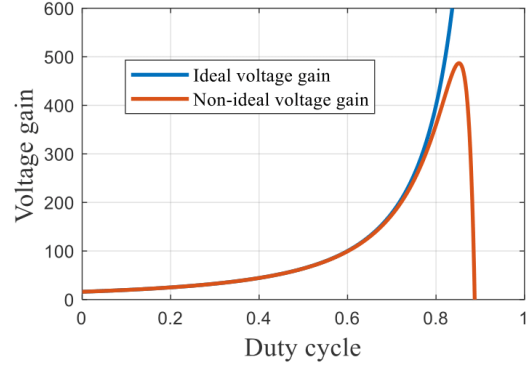


Fig. 6. Comparison of the ideal and non-ideal voltage gains.

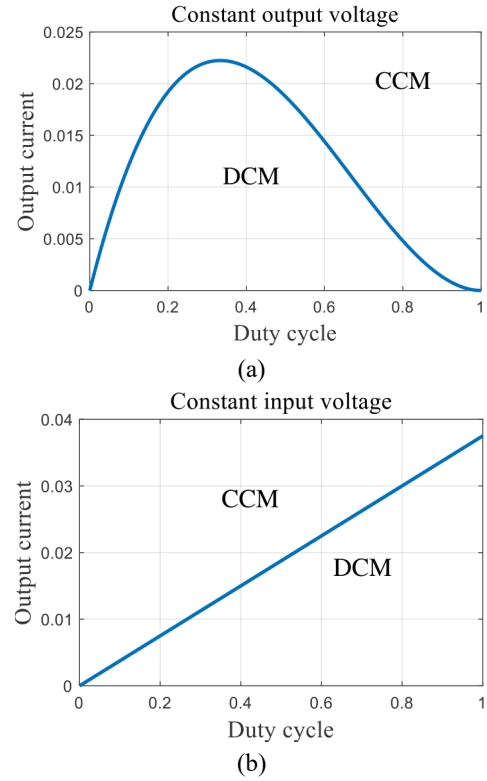


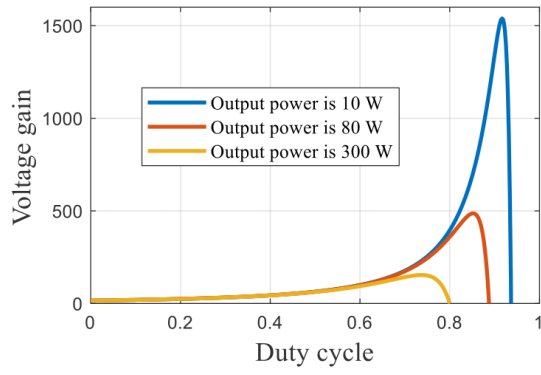
Fig. 5 The operational region of the converter in CCM and DCM while: (a) constant output voltage, (b) constant input voltage.

$$P_L = \frac{16(17 - 2D + D^2) r_L}{(1-D)^4} \frac{P_o}{R} \quad (45)$$

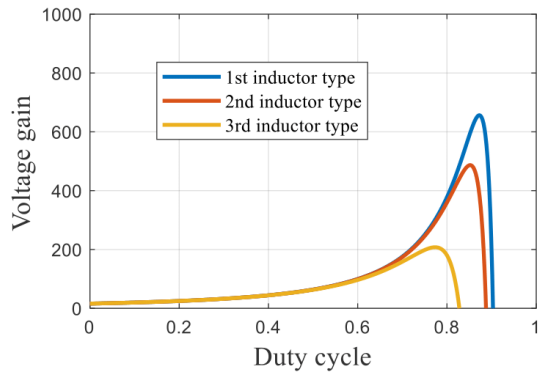
$$P_{sc} = \frac{(6 + 11D - D^2) r_s}{D(1-D)^4} \frac{P_o}{R} \quad (46)$$

$$P_D = \frac{(38 - 6D)}{1-D} V_{DF} I_o \quad (47)$$

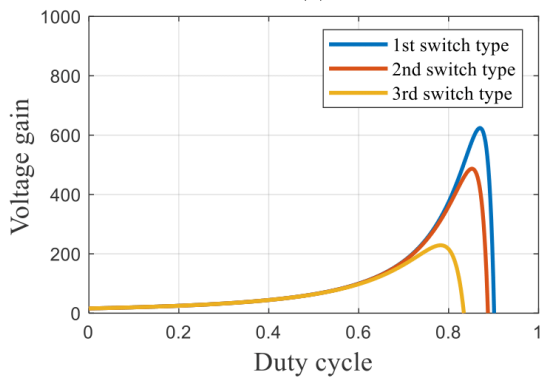
$$\eta = \frac{P_o}{P_o + P_L + P_{sc} + P_D} \quad (48)$$



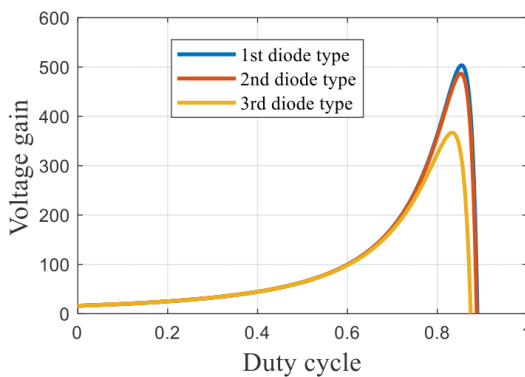
(a)



(b)

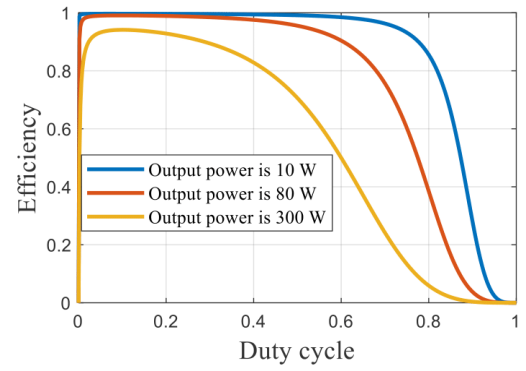


(c)

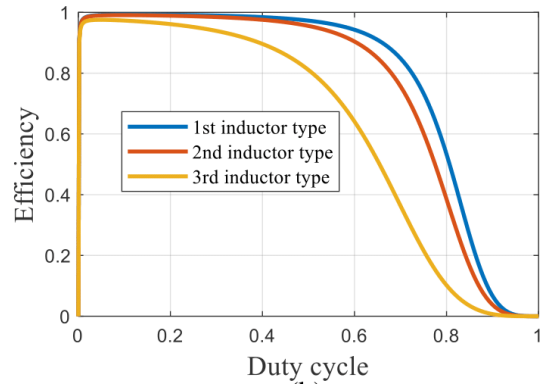


(d)

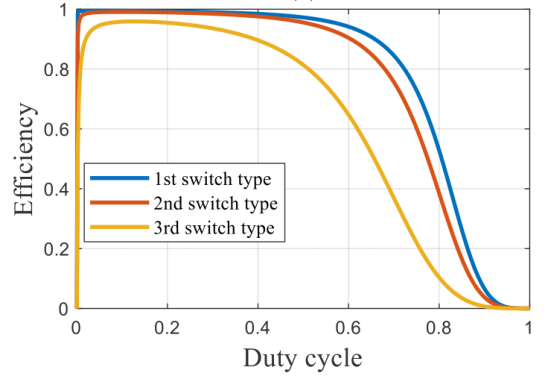
Fig. 7. Non-ideal voltage gain behavior while: (a) the output power is changing, (b) the inductor type is changing, (c) the switch type is changing, (d) the diode type is changing.



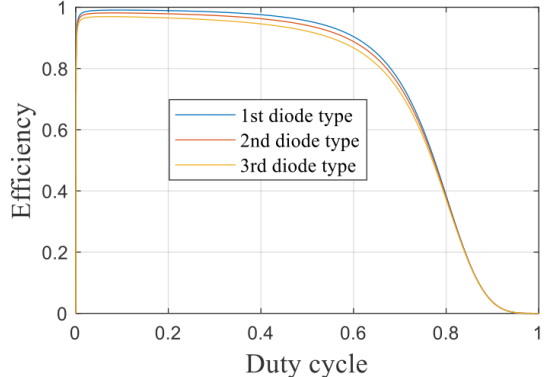
(a)



(b)



(c)



(d)

Fig. 8. Efficiency behavior while: (a) the output power is changing, (b) the inductor type is changing, (c) the switch type is changing, (d) the diode type is changing.

IV. Application of the Proposed Topology

The electrolysis of water is an electrochemical process that uses direct electric current (DC) to drive the non-spontaneous decomposition of water into its constituent elements, hydrogen (H_2) and oxygen (O_2). The Materials and Apparatus for this process are: Electrolytic Cell, Electrodes, Electrolyte, Power Supply, and Measurement Devices. Hydrogen is widely regarded as a clean energy source for the future. One common method of hydrogen production is water electrolysis, which can be performed at relatively low voltages. However, at low voltages, bubble formation around the electrodes tends to accumulate, reducing the efficiency of the electrolysis

process. To mitigate this issue, applying a high pulsed voltage with an average value of approximately 10 V has been proposed, as it helps decrease bubble congestion around the electrodes. A high-gain DC-DC converter can be employed to supply the elevated voltage required for the DC link of the pulse power converter. Such a converter enables the generation of high voltages from low-voltage sources like batteries, power supplies, or photovoltaic panels. Fig. 9-10 provide the schematic illustration of the application of a high-gain DC-DC converter for water electrolysis in hydrogen extraction.

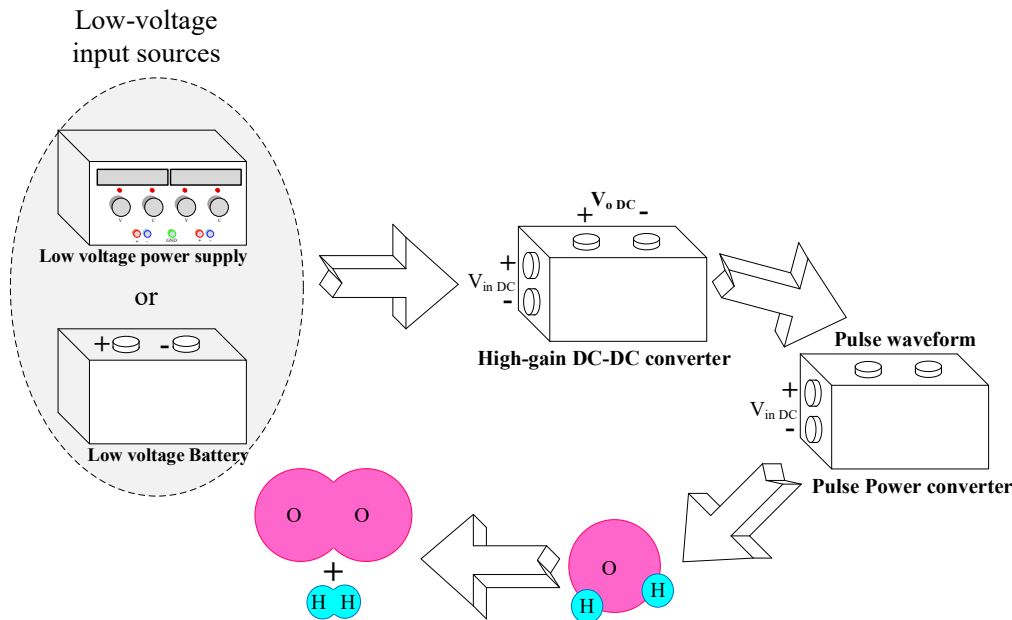


Fig.9. Symbolic presentation of using a high-gain DC-DC converter to electrolyze the water for hydrogen extraction.

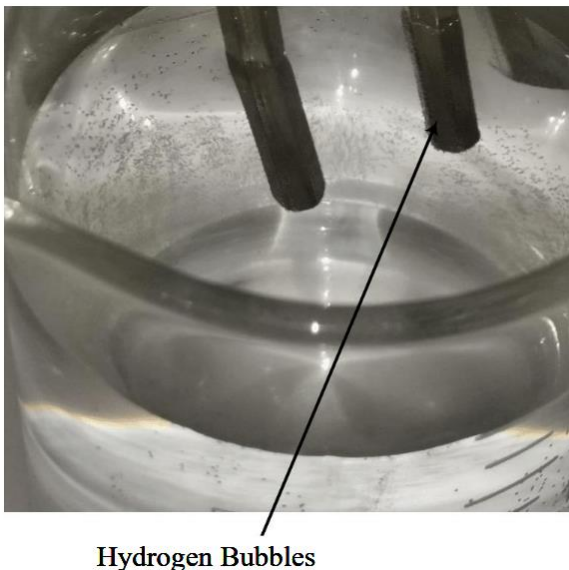


Fig. 10. The actual performance of the proposed converter in a water electrolysis setup.

V. Comparison Study

Fig. 11 compares the voltage gain and voltage gain density of various topologies. As shown, the proposed topology achieves a significantly higher voltage gain than those presented in [16]–[20]. Specifically, the voltage gain produced at low duty cycle percentages in the proposed converter exceeds that of the referenced topologies even at higher duty cycles. While the increased number of components in the proposed topology might be perceived as the reason for this improved gain, Fig. 11(b) addresses this by comparing voltage gain density as “voltage gain per total number of active and passive components (inductors, capacitors, diodes, and switches)”. In this analysis, the voltage gain of each topology is normalized by its respective component count to provide a fair comparison. As illustrated in the figure, the proposed topology exhibits a higher voltage gain density compared to the others, demonstrating its superior design for achieving elevated voltage gain.

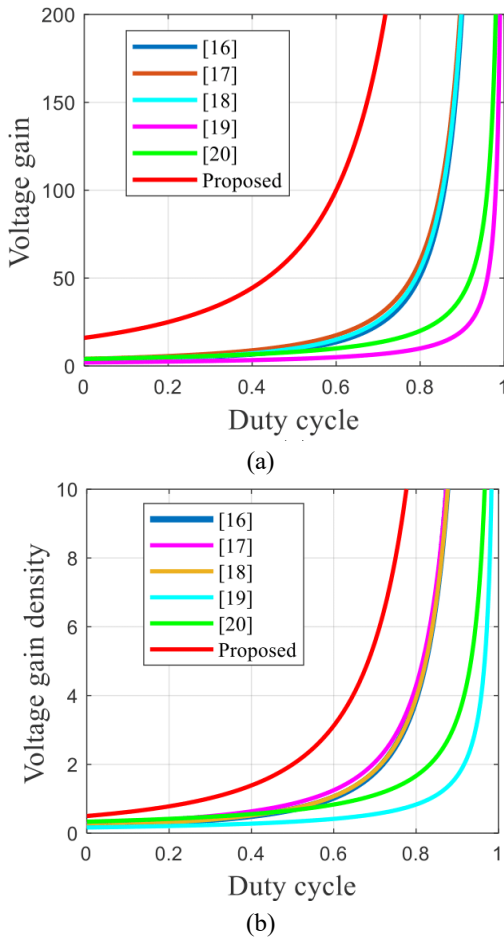


Fig. 11. Comparison results of the proposed topology and recently suggested topologies. (a) Voltage gain (b) voltage gain density

VI. Experimental Results

The prototype converter is designed by specifying the voltage and current characteristics of the circuit components and selecting appropriate values for the capacitors and inductors. Table I summarizes key parameters, including input voltage, switching frequency, inductor current ripple, capacitor voltage ripple, output current, and duty cycle. Based on these values and the equations presented in Section 2, Table II details the average capacitor voltages, average inductor currents, and voltage/current stresses on the semiconductor devices. Additionally, Table II presents the minimum required inductance and capacitance values derived from the current and voltage ripple specifications discussed in Section 2. Based on the components' expressed voltage/current characteristics, the component types used in the prototype are listed in Table III. Fig. 12 depicts the constructed prototype of the proposed converter based on the outlined design concepts. The first step to reduce the EMI is to provide the continuity of the input current. This factor was provided in the proposed topology. The second effective factor in EMI is the hot loops. Hot loops are current paths,

which their current value changes from zero to a non-zero value. The most important hot-loop in DC-DC converters is the current path of the input current to switch. In the proposed topology, the switch is used at the first part of the cascaded parts, and the switch position is as close as possible to the input source terminal. In order to reduce the EMC, the continuity of the input current is provided. Additionally, the inductors were shielded to reduce the DM noises. Experimental results are presented in Fig. 13-19, illustrating the current waveforms of the inductors, voltage waveforms of the capacitors, and voltage and current waveforms of the semiconductor devices.

A comparison between these measured waveforms and the corresponding values reported in Table II demonstrates strong agreement. The minor discrepancies observed are attributed to losses and voltage drops caused by parasitic elements.

Fig. 20 shows the efficiency characteristics under different conditions: Fig. 20(a) displays efficiency versus output current at a constant output voltage, while Fig. 20(b) shows efficiency versus output voltage at a constant output power. The results confirm that the converter maintains acceptable efficiency levels.

According to the presented curves in Fig. 20, the extracted efficiency at $P_o=80$ W, and $V_o=1200$ V, is about 94%. Notably, this high efficiency is due to the high value of the voltages and low value of the currents, which keeps the losses in a low value and negligible range. In other words, changing the operating point can change the extracted efficiency. The reported efficiency in the recently suggested converters is shown in Table IV. It is worth noting that for output voltages exceeding 1200 V (as demonstrated in the experiments), two such converters can be connected in series, as illustrated in Fig. 21. In this configuration, the input voltages must be isolated from one another, and the switch of the second converter requires a high-side drive.

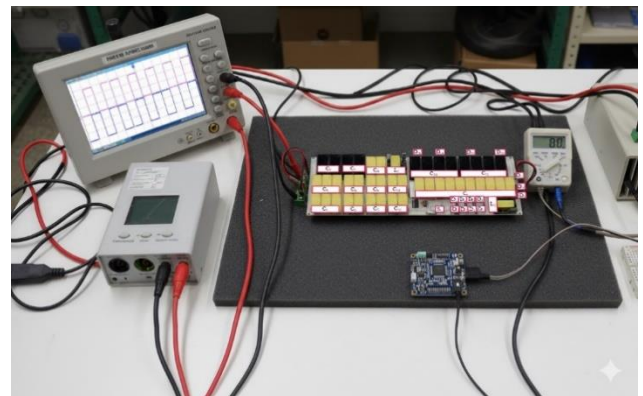


Fig. 12. A photo of the prototype of the proposed converter.

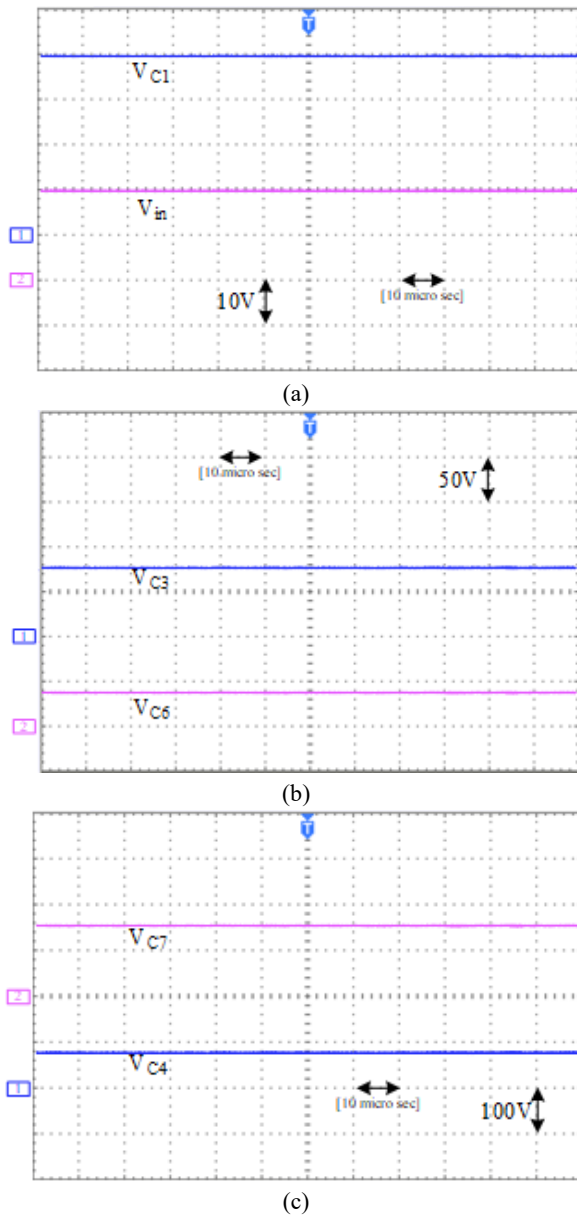
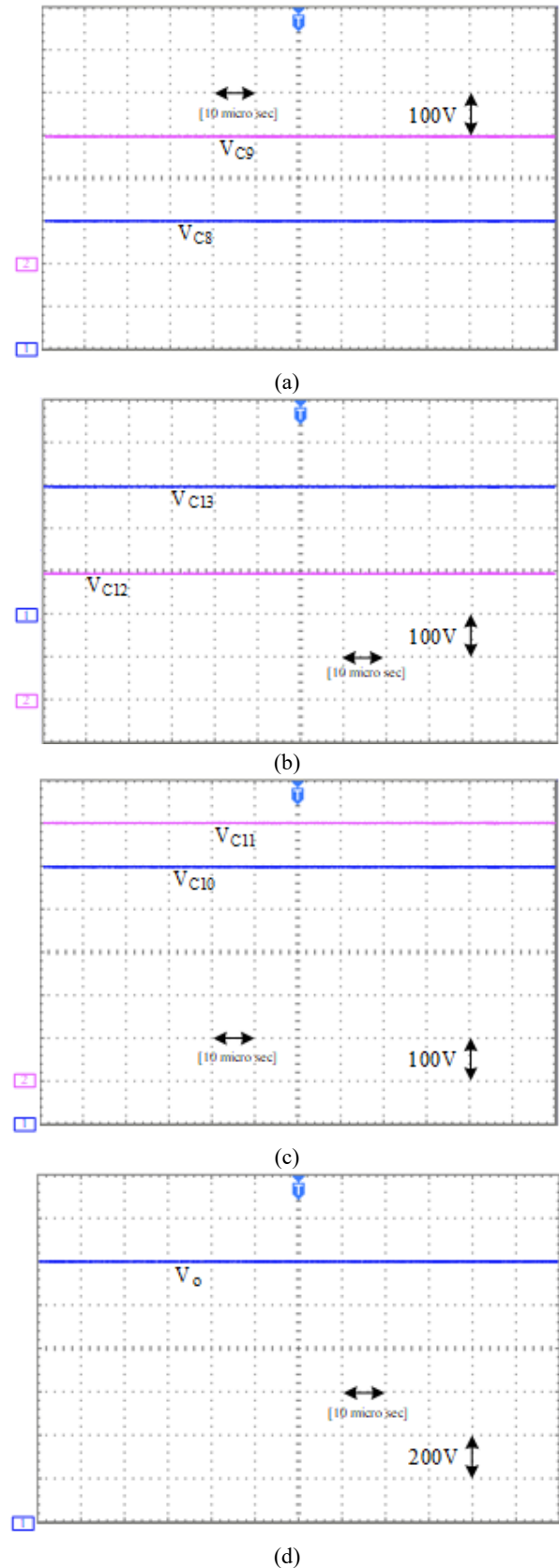
V_{in}	I_o	f_s	D	Δi_L	Δv_C
20V	62.5mA	50kHz	0.5	5%	30%

TABLE II. Predicted values by the theoretical relations.

parameter	value	parameter	value	parameter	value
$V_{C1,2,5,6}$	40 V	$V_{C3,4}$	80 V	V_{C7}	160 V
$V_{C8,9,12,13}$	320 V	$V_{C10,C11}$	640 V	V_o	1280 V
I_{L1}	4 A	I_{L2}	0.5 A	$V_{S,D1,...,D7}$	40 V
$V_{D9,...,D1}$	320 V	V_{D8}	280 V	$I_{D1,...,D7}$	0.5 A
$I_{D9,...,D1}$	62.5 mA	I_{S1}	3.93 A	I_{D8}	0.43 A
$L1$	166 μ H	$L2$	10 mH	$C1,...,6$	5 μ F
$C8,...,13$	78 nF	C_o	30 nF		

TABLE III. Components type.

Inductor	Capacitor	diode	Switch
EC type	MKT	DTV56F	IRF540

Fig. 13. Experimental results. (a) V_{C1} and V_{in} (b) V_{C3} , V_{C6} , and (c) V_{C4} , V_{C7} .Fig. 14. Experimental results. (a) V_{C9} , V_{C8} , (b) V_{C12} , V_{C13} , (c) V_{C11} , V_{C10} , and (d) V_o .

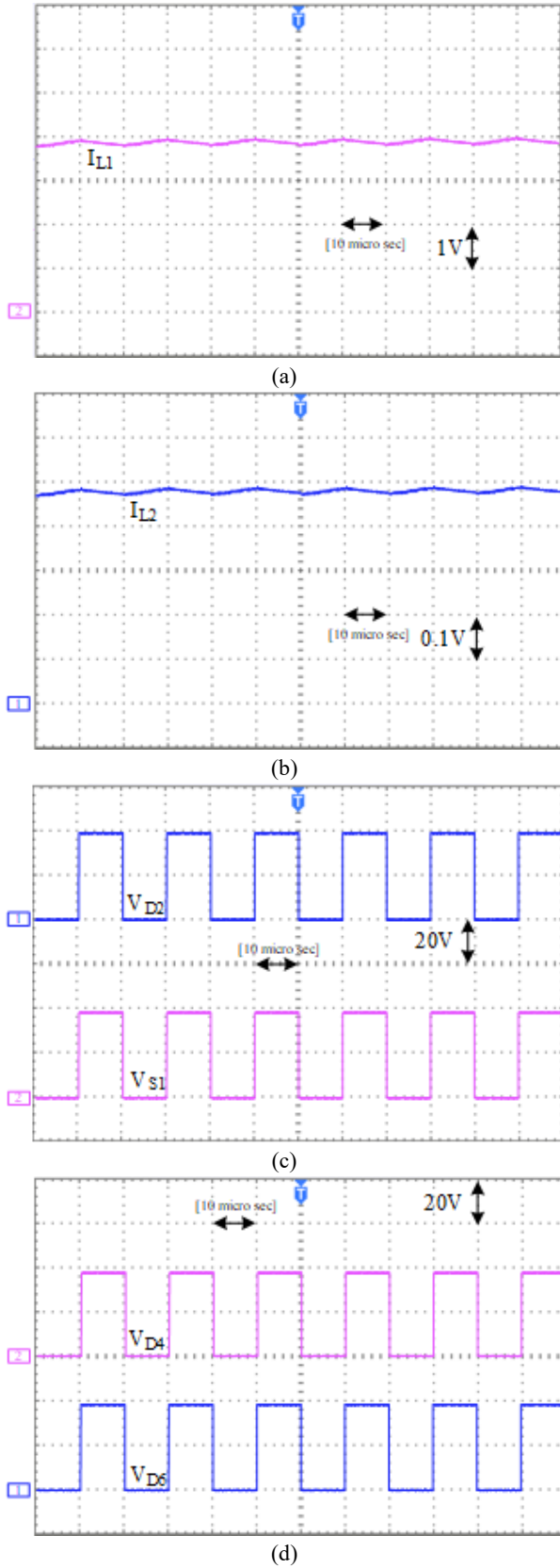


Fig. 15. Experimental results of the proposed converter. (a) I_{L1} , (b) I_{L2} , (c) V_{S1} , V_{D2} , and (d) V_{D4} , V_{D6} .

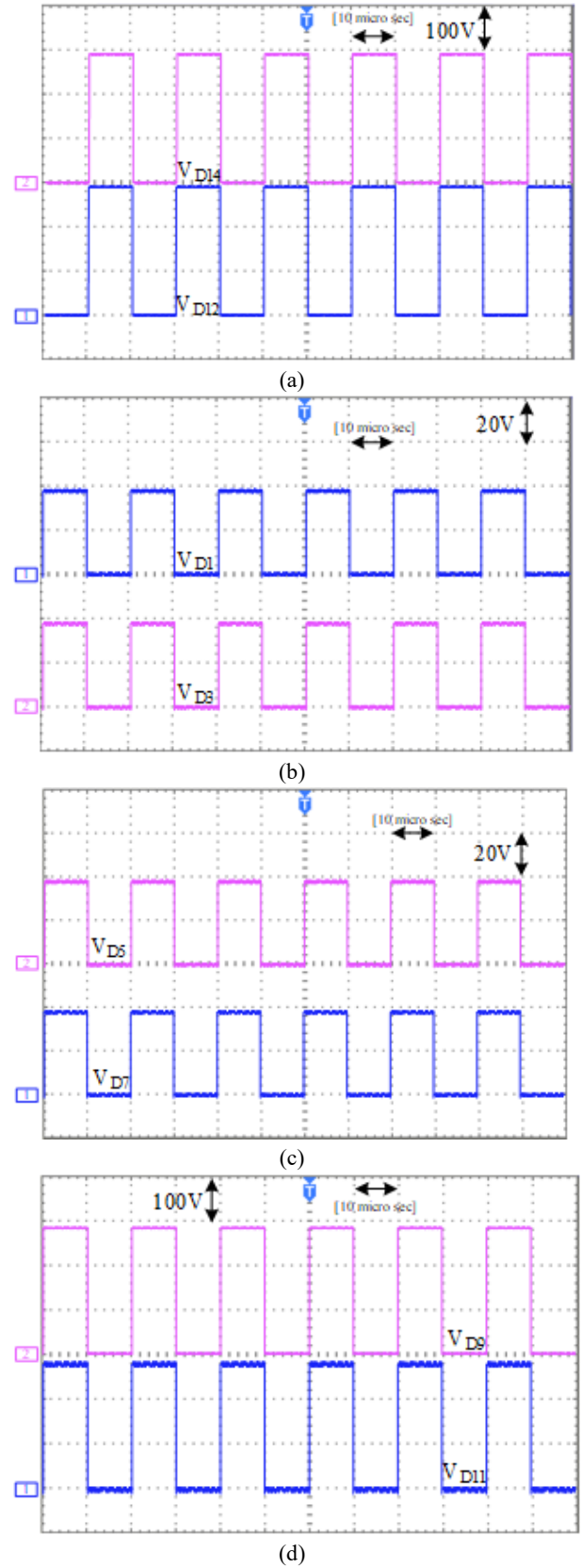
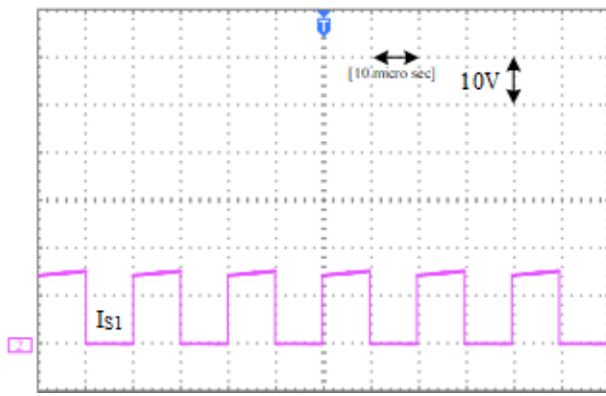
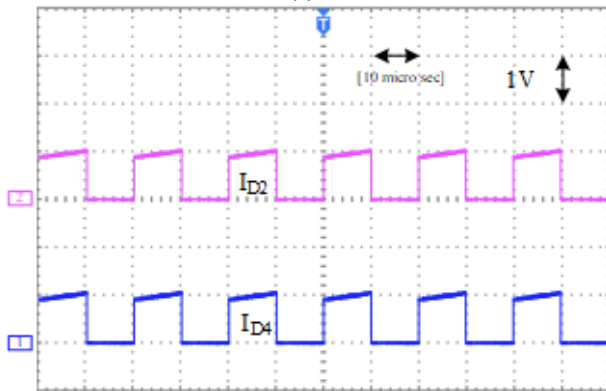


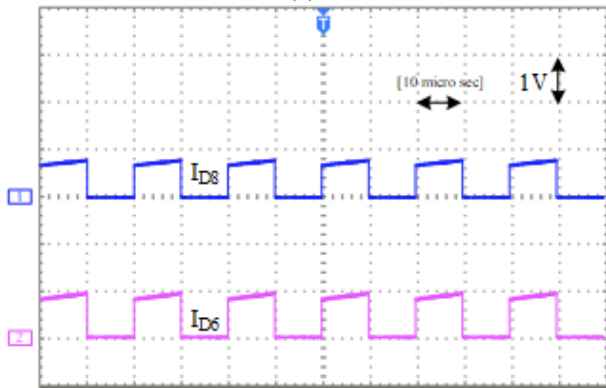
Fig. 16. Experimental results. (a) V_{D12} , V_{D14} , (b) V_{D1} , V_{D3} , (c) V_{D5} , V_{D7} , and (d) V_{D9} , V_{D11} .



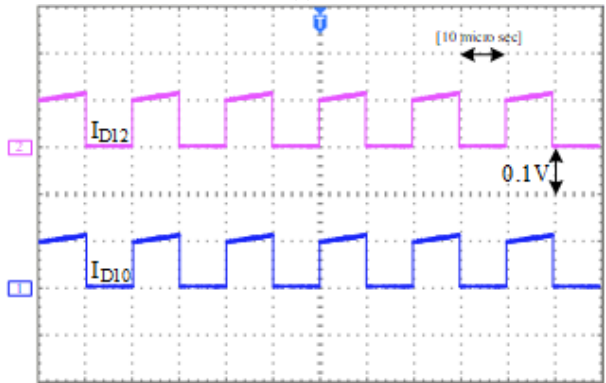
(a)



(b)

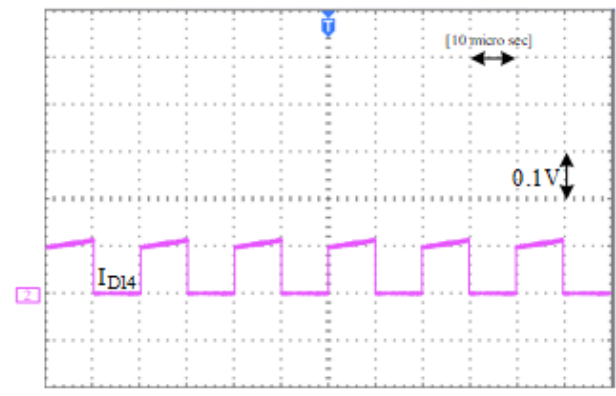


(c)

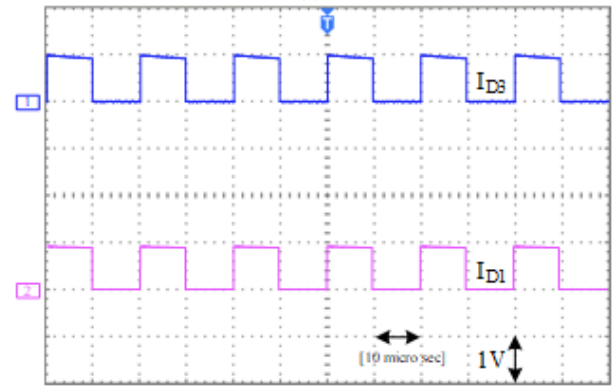


(d)

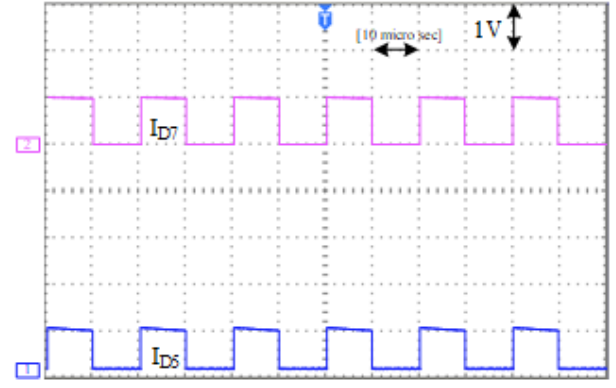
Fig. 17. Experimental results of the proposed converter. (a) I_{S1} , (b) I_{D2} , I_{D4} , (c) I_{D6} , I_{D8} and (d) I_{D12} , I_{D10} .



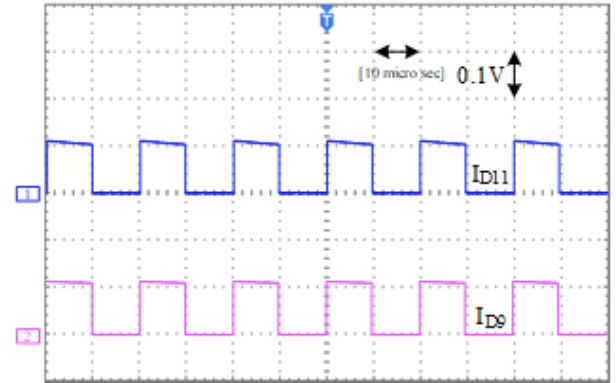
(a)



(b)



(c)



(d)

Fig. 18. Experimental results of the proposed converter. (a) I_{D14} , (b) I_{D1} , I_{D3} , (c) I_{D5} , I_{D7} , and (d) I_{D9} , I_{D11} .

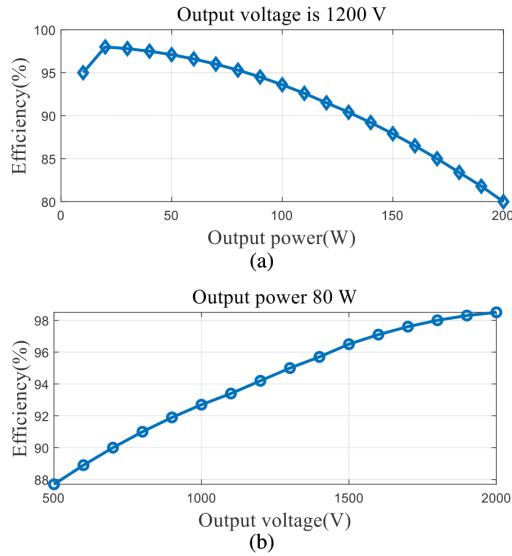


Fig. 20. The measured efficiency of the proposed converter.

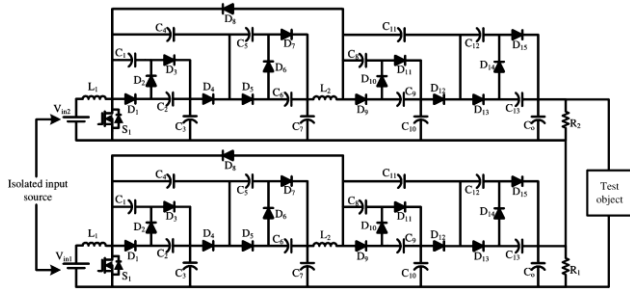


Fig.21. Two series connection of such converters.

TABLE IV. Components of the measured efficiency of the converters.

Converter	Measured efficiency	Rated Power
[16]	93%	60W
[17]	90%	100 W
[18]	92.96%	150 W
[19]	93%	70 W
[20]	94.6%	250 W
[21]	90%	100 W
[22]	93.8%	100 W
[23]	93.1%	50 W
[24]	94.7%	100 W
[25]	97%	100 W
[27]	90%	100 W
[29]	94%	100 W
[30]	95.5%	100 W
[35]	93%	100 W
[36]	95.8%	100 W
[37]	93%	100 W
[38]	91%	100 W
[42]	97%	100 W
[43]	94.2%	100 W
[44]	96.5%	100 W
[45]	90%	100 W
[56]	91%	100 W
[47]	90.5%	100 W
[48]	95.1%	100 W

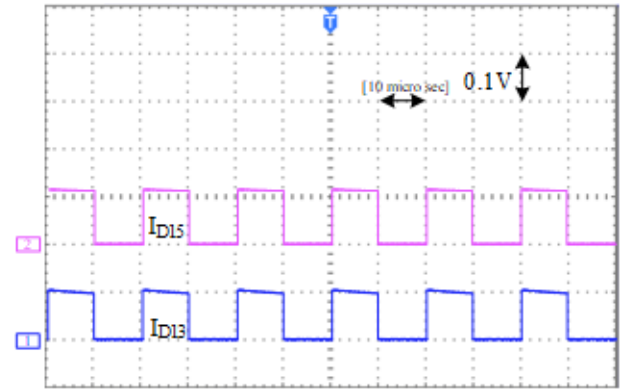


Fig. 19. Experimental results of Id13, Id15.

VII. Conclusion

This study presented an improved cascaded boost topology featuring a novel quadratic voltage-lift effect, achieving significantly higher voltage gains at lower duty cycles. In addition to the high voltage gain, the voltage stress on the semiconductors remains well below the output voltage, and their current stress is considerably less than the input current, enhancing device reliability. Experimental results demonstrated the converter's ability to boost a 20 V input to 1200 V output at a 50% duty cycle, validating the theoretical analysis. Furthermore, the compact prototype design enables series connection of multiple converters to reach output voltages up to approximately 6 kV, making it suitable for high-voltage applications such as dielectric testing.

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A Min-Max Approach for Energy Management of Renewable-Based Electricity-Hydrogen Microgrids with Demand Response



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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 28 December 2025 Received in revised form: 24 April 2026 Accepted: 30 April 2026 Published online: 22-Dec-2026 Keywords: Renewable energy, Demand response, Microgrid, Hydrogen-based microgrid, Long short-term memory.	This paper presents a multi-objective energy management strategy for a hybrid electricity-hydrogen microgrid (MG) aimed at improving economic and operational performance. The proposed framework simultaneously minimizes total operating cost and reduces peak load, which are conflicting objectives in MG operation. To address this challenge, the problem is formulated as a multi-objective optimization model using a normalized weighted sum approach, enabling an effective trade-off between objectives while preventing the creation of new peaks in the load profile. The studied hydrogen-based MGs includes renewable energy sources such as solar and wind power, a battery energy storage system (BESS), a diesel generator, and demand response programs. Hydrogen is also utilized as an energy carrier to enhance system flexibility and support improved integration of renewable generation. Coordinated scheduling of generation units, storage systems, and flexible loads improves system efficiency. To enhance model accuracy, a long short-term memory (LSTM) forecasting method is applied to predict solar irradiance and wind speed using historical data, supporting more reliable scheduling decisions. The proposed model is validated through a general case study. Simulation results show a peak load reduction of 442 kW and an improvement in the load factor of 16.75%, confirming the effectiveness of the proposed approach.

NOMENCLATURE			
DR	Demand response	h	Index of hydrogen system
MG	Microgrids	i	Index of non-renewable resources
RES	Renewable energy sources	l	Index of load
SoC	State-of-charge	s	Index of scenarios
b	Index of battery storage system	t	Index of times
c	Index of capacitors		

I. Introduction

Renewable energy sources (RES) such as photovoltaic, wind energy are becoming important as the world seeks sustainable alternatives to fossil fuels [1]. Their environmental benefits, including reduced carbon footprint and lower ecological impact, make them essential components of modern energy strategies. However, the intermittent and variable nature of many RES creates different challenges for stability, reliability, and efficiency of the energy systems [2, 3]. To address these challenges, the concept of microgrids (MGs) has been created as a flexible and resilient solution [4, 5]. A MG is a local energy system

that integrates distributed energy resources, energy storage, and controllable loads. The MGs are able to operate as islanded mode or grid-connected mode. By incorporating RES with smart control technologies, MGs enhance energy reliability, support grid stability, and enable efficient energy management at the community, industrial, or campus scale [6, 7]. As a result, MGs are known as key enablers of the transition toward a low carbon energy system [8].

In recent years, different studies are focused on the energy scheduling of MGs. Reference [9] presents a hybrid operation scheduling and peer-to-peer energy trading for MGs to maximize its profit and reduce the electricity unbalance. The

proposed model is formulated as mixed integer linear programs, where the deep learning approach has been developed to forecast the loads and RES generation. The authors in [10] proposed a multi-objective optimization for economic and sustainable operation of MGs. The proposed structure integrates the RES, fuel cell, microturbine, energy storage systems, electric vehicles, and demand response programs (DRP) to investigate the impact of different weather conditions on the MG operation. An advanced dynamic programming approach has been developed in [11] to manage the uncertainty of RES in the energy management of MGs. In this study, the probability distribution functions have been utilized to model the uncertainty of RES. The authors in [12] studied the supply-demand balance in RES-based MG.

Reference [13] studies the optimal sizing of hybrid MG that integrates the RES, AC loads, DC loads, and energy storage systems. The presented model decides to minimize the leveled cost of energy and carbon emission in the scheduling horizon. A two-stage robust scheduling mechanism has been developed in [14] to manage the operation of multi-carrier MG. In the first stage, the optimal sizing of photovoltaic panel, wind turbine, combined heat and power has been defined. The second stage is responsible for determining the optimal operation of electrical and heat systems. Reference [15] proposed a techno-economic model for optimal sizing and optimal operation of AC MG using genetic algorithm. The proposed model investigates the voltage stability and resiliency of MGs in both islanded and grid-connected modes. The particle swarm optimization algorithm has been developed for the AC/DC MG under dynamic tariffs. The proposed model considered the DRP and battery energy storage systems to minimize the total cost of MGs. A Honey Badger Algorithm has been developed in [16] to improve the energy flow in MGs. However

The authors in [17] propose a stochastic approach for peer-to-peer energy trading in the integrated electricity and gas networks. The power-to-gas systems are considered to increase the flexibility of the MGs. However, the peak load reduction is not the main focus of the proposed study. A multi-objective scheduling algorithm is presented in [18] that simultaneously optimizes the operating cost and flexibility of MGs. In this study, the energy storage systems are considered as the main flexible resources in the MGs. Although the proposed model considers the uncertainty of RES, the Long Short-Term Memory (LSTM) is not utilized to predict the RES generation from historical data. Reference [19] considers the economic, environmental, and reliability indices to present a comprehensive scheduling for the MG. To this end, the artificial bee colony and a sine-cosine algorithm are utilized to find the optimal solution for MGs. However, the peak load reduction by the min-max algorithm is not studied.

In this paper, we propose a multi-objective optimization model to simultaneously minimize the total cost and peak

load in a hydrogen-based MG. The proposed MG is operated in the grid-connected mode and is integrated with RES, diesel generator, battery storage system, and controllable loads. Also, the MG system utilizes the power-to-hydrogen, hydrogen to power, and hydrogen storage tank to increase the flexibility of MGs and manage the uncertainty of RES. To estimate the production of the RES, the long short-term memory approach has been developed in the proposed model. This approach considers the weather conditions for 365 days to provide more reliable data for RES generation. The main contribution of the work is formulated as follows:

- Proposing a multi-objective model that simultaneously minimizes the total cost and peak load of MGs.
- The proposed model provides a link between electricity and hydrogen in the local energy system. Also, the MG is able to sell the hydrogen to industrial to get more profit.
- Applying the long short-term memory approach to estimate the production of RES and create more reliable results.

The rest of this paper is organized as follows:

Section 2 presents the mathematical formulation of the proposed model. Section 3 describes the numerical results and simulation analysis. Finally, section 4 presents the conclusion and future works.

II. Mathematical formulation of hydrogen-based MG

This section models the objective functions and the equal and unequal constraints of the hydrogen-based MG.

A. Objective functions

The hydrogen-based MG considers both operating cost and peak reduction as objective functions and solves a multi-objective optimization to define the best scheduling. Equations (1) and (2) model the operation cost and peak load reduction, respectively.

$$\min \sum_t \delta_t^E P_t^{Grid} - \sum_t \delta_t^E P_t^{sell} + \sum_i \sum_t (\alpha_i + \beta_i \zeta_i) P_{i,t} + \gamma_i u_{i,t} \quad (1)$$

$$- \sum_h \sum_t \delta_t^h H_{h,t}^{sell} \quad (2)$$

The first term of Eq. (1) refers to the cost of imported electricity from the grid. The second term shows the selling electricity to the grid. The third term in Eq. (1) considers the production cost of local resources. The fourth term of Eq. (1) shows the selling hydrogen to industries. According to Eq. (2), a min-max approach is presented to decrease the peak load. The Equation (2) can be replaced by Eq. (3).

$$\min \lambda \quad (3)$$

$$\lambda \geq P_{l,t}^{DR}$$

λ is a free variable that finds the peak load and Eq. (3) tries to reduce λ . Actually, λ is an auxiliary variable that finds the highest demand over the time horizon. By minimizing this variable, the optimization problem effectively reduces the peak load. This approach simplifies the original optimization problem while keeping the reasoning clear and easy to follow.

To solve this multi-objective optimization problem, a normalized approach is used according to Eq. (4) to convert the problem into a single-objective.

$$\min \left\{ \frac{(\sum_t \delta_t^E P_t^{Grid} + \sum_i \sum_t (\alpha_i + \beta_i \zeta_i) P_{i,t} + \gamma_i u_{i,t} - \sum_h \sum_t \delta_t^h H_{h,t}^{sell})}{Cost^*} + \frac{\lambda}{\lambda^*} \right\} \quad (4)$$

S.t:

$$\lambda \geq P_{l,t}^{DR}$$

where $Cost^*$ and λ^* are the ideal solutions for cost function and λ , respectively. The normalized weighted sum approach provides different advantages for MGs. This method is able to consider the objective functions with different types and scales. Also, the simplicity of this method is another of its advantages. This method presents a unique solution, and there is no need for solution ranking methods. Due to these advantages, the normalized weighted sum approach is used to handle the proposed multi-objective optimization. However, the main limitation of the proposed model is its sensitivity to the weights of objective functions.

B. Distributed energy resources

Equations (5)-(8) presents formulation of distributed energy resources.

$$u_{i,t} P_i^{min_{i,t}, max_{i,t}} \quad (5)$$

$$P_{i,t} - P_{i,t-1} \leq R_i^{max, up} \quad (6)$$

$$P_{i,t-1} - P_{i,t} \leq R_i^{max, down} \quad (7)$$

$$cost^{DG} = \sum_i \sum_t (\alpha_i + \beta_i \zeta_i) P_{i,t} + \gamma_i u_{i,t} \quad (8)$$

Equation (5) shows the acceptable generation of resources. The ramp-up and ramp-down limits are represented in Eqs. (6) and (7), respectively. Finally, Eq. (8) calculates the production cost of resources.

C. Energy trading with main grid

Equation (9) limits the imported electricity from the main grid. Also, Eq. (10) shows the selling energy with main grid.

$$pg_{min_t}^{Grid^{g,max}} \quad (9)$$

$$psell_{min_t}^{sell^{sell,max}} \quad (10)$$

D. Demand response programs

Equations (11)-(13) present the formulation of DR programs. Equation (11) shows the acceptable load shifting

level or DR participation level. Equation (12) calculates the shifted load to each time slot. Finally, Eq. (13) presents the new load profile after DR programs.

$$DR^{min_{l,t}, max_{l,t}} \quad (11)$$

$$\sum_{t=1}^T ldr_{l,t} = \sum_{t=1}^T P_{l,t}^{load} DR_{l,t} \quad (12)$$

$$P_{l,t}^{DR} = P_{l,t}^{load} (1 - DR_{l,t}) + ldr_{l,t} \quad (13)$$

E. Battery storage devices

Equations (14)-(19) model the operation constraints of energy storage devices.

$$SoC_{b,t+1} = SoC_{b,t} + \Delta T (\eta_b^{ch} P_{b,t}^{ch} - \frac{P_{b,t}^{disch}}{\eta_b^{disch}}) \quad (14)$$

$$SoC_b^{min_{b,t}, max_{b,t}} \quad (15)$$

$$0 \leq P_{b,t}^{ch} \leq X_{b,t}^{ch} P_b^{ch} \quad (16)$$

$$0 \leq P_{b,t}^{disch} \leq X_{b,t}^{disch} P_b^{disch} \quad (17)$$

$$X_{b,t}^{ch} + X_{b,t}^{disch} \leq 1 \quad (18)$$

$$SoC_{b,t1} = SoC_{b,t4} \quad (19)$$

The stored electricity in the battery is given by Eq. (14). The minimum and maximum stored electricity is determined by Eq. (15). Equations (16) and (17) limit the charging and discharging rates, respectively. Equation (18) prevents from both charging and discharging at each time slot. Finally, Eq. (19) ensures that the initial and final energy in storage system should be equal.

F. Photovoltaic energy

Equation (20) presents the production of photovoltaic panels.

$$P_{i,t}^{PV} = \eta^{PV} N_i^{PV} S_i^{PV} I_t (1 - 0.005(T_t^{out} - 25)) \quad (20)$$

G. Wind energy

Equation (21) models the production of wind turbines.

$$P_{i,t}^{WT} = \begin{cases} 0 & 0 \leq v_t \leq v_{ci} \text{ or } v_{co} \leq v_{t,s} \\ P_{i,r}^{WT} \frac{v_{t,s}^2 - v_{ci}^2}{v_{i,r}^2 - v_{ci}^2} & v_{ci} \leq v_t \leq v_{i,r} \\ P_{i,r}^{WT} & v_{i,r} \leq v_t \leq v_{co} \end{cases} \quad (21)$$

H. Hydrogen systems

Equations (22)-(29) developed the mathematical formulation of hydrogen storage system, hydrogen-to-electricity, and electricity-to-hydrogen.

$$P_{h,t}^{E \rightarrow H_2} \chi_{h,t} = H_{h,t}^{in} \quad (22)$$

$$HL_{h,t} = \begin{cases} HL_{h,t}^0 & t = 1 \\ HL_{h,t-1} + \left(\phi_{h,t} H_{h,t}^{in} - \frac{H_{h,t}^{out}}{\phi_{h,t}} \right) & t \geq 2 \end{cases} \quad (23)$$

$$H_{h,t}^{out} = H_{h,t}^E + H_{h,t}^{sell} \quad (24)$$

$$H_{h,t}^E \psi_h = P_{h,t}^{H_2 \rightarrow E} \quad (25)$$

$$HL_h^{min_{h,t}, max_{h,t}} \quad (26)$$

$$HL_{h,t}^E \leq \chi_{h,t} HL_h^{E,max} \quad (27)$$

$$HL_{h,t}^{sell} \leq \sigma_{h,t} HL_{h,t}^{sell,max} \quad (28)$$

$$\sigma_{h,t} + \chi_{h,t} \leq 1 \quad (29)$$

Equation (22) shows the amount of generated hydrogen by electrolyzer unit as electricity to hydrogen unit. Equation (23) describes the hourly stored hydrogen in the storage tank. According to Eq. (24), the output hydrogen of hydrogen storage tank can be sold to industries or consumed by fuel cell unit to generate the electricity. Equation (25) calculates the amount of generated electricity by hydrogen-to-electricity unit. Equation (26) limits the minimum and maximum capacity of hydrogen storage tank. Equations (27) and (28) show the maximum bounds of input hydrogen to fuel cell and selling hydrogen to the industries, respectively. Finally, Eq. (29) shows that the hydrogen system sells the generated hydrogen or consumes it by fuel cell units at time t .

I. Active and reactive power balances

The active and reactive balance in the hydrogen-based MG is presented in Eqs. (30) and (31), respectively.

$$P_t^{Grid} + \sum_i P_{i,t} + \sum_i (P_{i,t}^{PV} + P_{i,t}^{WT}) + \sum_b (P_{b,t}^{disch} - P_{b,t}^{ch}) + \sum_h P_{h,t}^{H_2 \rightarrow E} = \quad (30)$$

$$P_t^{sell} + \sum_h P_{h,t}^{E \rightarrow H_2} + \sum_i P_{i,t}^{flex} + Q_t^{Grid} + \sum_i (Q_{i,t} + Q_{i,t}^{PV} + Q_{i,t}^{WT}) + \sum_b Q_{b,t} + \sum_c Q_{c,t} = \sum_i Q_{i,t} \quad (31)$$

III. Simulation Results

A. Long Short-Term Memory (LSTM) Model for Renewable Generation Forecasting

To enhance the reliability of the proposed energy management system, a Long Short-Term Memory (LSTM) neural network is employed to forecast the intermittent generation of renewable energy sources (RES), specifically photovoltaic (PV) and wind turbine (WT) outputs. LSTM is a type of recurrent neural network (RNN) well-suited for time-series forecasting due to its ability to capture long-term dependencies and temporal patterns in sequential data. The LSTM model consists of memory cells that regulate the flow of information through three gates: the input gate, forget gate, and output gate. These gates enable the model to retain or discard information over long sequences, mitigating the vanishing gradient problem commonly encountered in traditional RNNs.

Historical data of weather parameters-including solar radiation, ambient temperature, and wind speed-are used as inputs to the LSTM model. The dataset comprises 365 days of hourly records, ensuring seasonal and diurnal variations are captured. The model is trained to predict the next 24-hour generation profile for PV and WT systems. The LSTM model

is implemented using the TensorFlow/Keras framework. The dataset is split into training (70%), validation (15%), and testing (15%) sets. The model is trained using the Adam optimizer with mean squared error (MSE) as the loss function. Early stopping and dropout regularization are applied to prevent overfitting.

The performance of the LSTM forecaster is evaluated using the Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE), according to Eqs. (32) and (33), respectively.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2} \quad (32)$$

$$MAPE = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad (33)$$

where y_i is the actual generation and $\hat{y}_i$ is the forecasted value. The internal structure of the Long Short-Term Memory (LSTM) unit used for the renewable energy forecasting model is shown in Fig. 1. The diagram illustrates the information flow through the cell state (C_t) and hidden state (h_t), controlled by the three distinct gating mechanisms: the Forget Gate (discarding irrelevant historical weather data), the Input Gate (updating the cell with new solar/wind features), and the Output Gate.

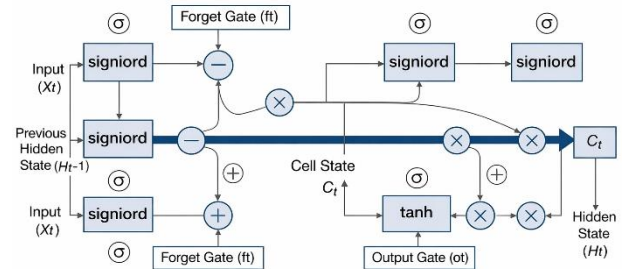


Fig. 1 LSTM model

B. Results

The multi-objective optimization model is tested on 33-bus IEEE test system. Three wind turbines and photovoltaic panels are located in the test system. Table 1 presents the characteristics of wind turbines. Also, three diesel generators are located in bus number 6, 19, and 32 whose characteristics are provided in Table 2. Besides, Fig. 2 and Fig. 3 show the wind speed and solar radiation, respectively.

The maximum DR participation is considered 20%. The maximum power purchased from the electricity grid is 1200 kW for each time slot. Two hydrogen storage tank is considered with capacities 100 and 150 m^3 . The two following case studies are assumed:

Table 1 characteristic of wind turbine

	Rated power (kW)	Rated speed (m/s)	Cut-in speed (m/s)	Cut-out speed (m/s)
WT1	225	13.5	3.5	25
WT2	100	13	4.5	25
WT3	90	16.8	3.3	28

Table 2 characteristic of diesel generators

	Capacity (kW)	Min generation (kW)	Ramp-up (kW)	Ramp-down (kW)
DG1	100	10	50	50
DG2	500	100	125	125
DG3	500	100	125	125

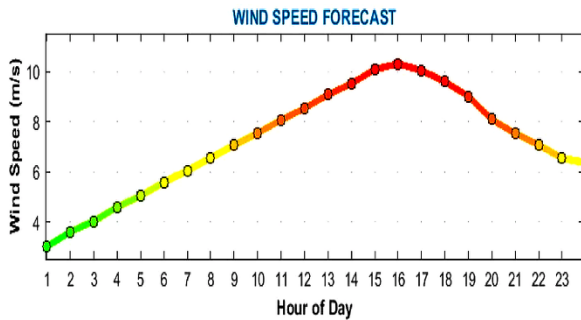


Fig. 2 generated wind speed by LSTM

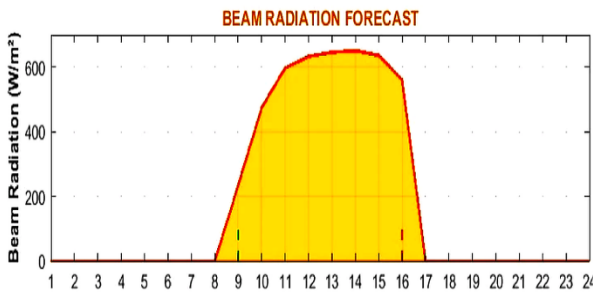


Fig. 3 generated solar radiation by LSTM

- Case study 1: The hydrogen-based MGs only consider the operation cost as an objective function and peak reduction is not considered.
- Case study 2: In this case study, the hydrogen-based MGs considers both operation cost and peak reduction simultaneously.

Table 3 compares the performance of case studies 1 and 2.

Table 3 comparison of case studies 1 and 2

Case study	Operating cost (\$)	Peak load (kW)	L.F (%)
Single-objective	6299.18	2637.89	68.03
Multi-objective	6678.65	2195.92	81.72

Table 3 shows that the proposed multi-objective model reduces the peak load by 16.75% because it considers peak reduction as one of its objective functions. Therefore, the MGs perform a coordinated DR program to reduce the peak load. Also, this coordinated DR program increases the load factor and uniformizes the load profile. According to the results, the proposed multi-objective model improves the load factor by 20.12 %. This improvement is provided because the coordinated DR programs decrease the peak load and fill the valley load. Finally, Fig. 4 shows the load profile of the hydrogen-based MGs in both case studies.

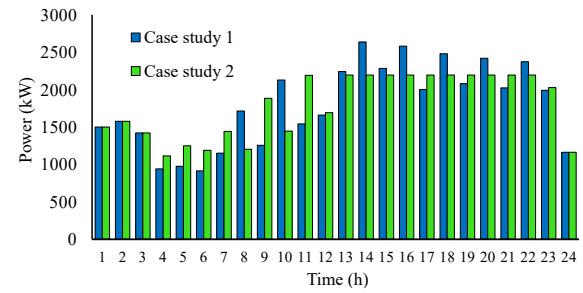


Fig. 4 load profile in case studies 1 and 2

The results of Fig. 4 indicate that the proposed model significantly decreases the peak load of the hydrogen-based MGs compared to case study 1. Actually, MGs reduce their energy consumption during peak hours and shift it to the off-peak hours. This performance reduces the operating cost of hydrogen-based MGs because electricity tariffs during off-peak hours are lower than those during peak hours. Fig. 5 presents the DR participation level for different loads.

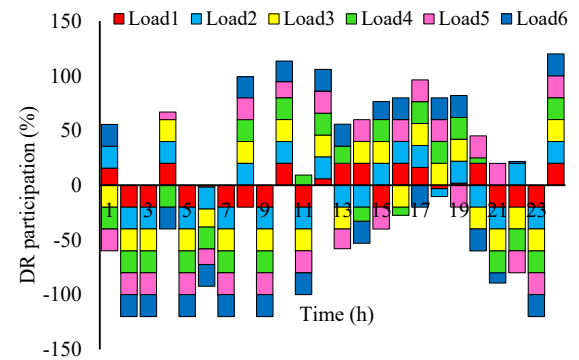


Fig. 5 DR participation level in the proposed model

The positive and negative values show the downward and upward DR, respectively. The results show that during peak hours, the hydrogen-based MGs reduce their energy consumption. Two reasons exist for this performance. First, the difference between off-peak and peak hours encourages the hydrogen-based MGs to shift their loads. Second, the proposed model attempts to reduce the peak load using a min-max approach. Fig. 6 presents the purchased electricity from the main grid.

The results of Fig. 6 show that at the beginning of the day, the hydrogen-based MGs import less energy from the main grid because its loads are lower and its distributed energy sources can supply the needed energy. However, during peak hours, the hydrogen-based MGs increase the imported energy from the electricity network to meet the required loads.

Figures 7 and 8 show a complete comparison of the three forecasting methods. In Figure 7, the solar energy production forecasts are displayed alongside the actual measured values. In this section, the base model, or Persistent, has the weakest results, as it is associated with a delay and cannot accurately represent the correct time and peak of daily production. This

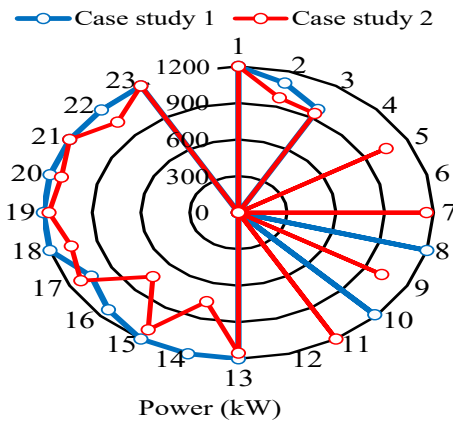


Fig. 6 purchased electricity from the main grid

result is not unexpected, since the model assumes that the next value is exactly equal to the last value and therefore cannot predict the daily cycle of the sun. The ARIMA model performs better than the base model and detects the overall daily trend. However, it still has significant errors during peak production hours and times when production decreases or increases. The reason for this limitation is that the ARIMA model is based on linear relationships and cannot model well the nonlinear variations in solar radiation that occur due to factors such as clouds and atmospheric conditions. However, the LSTM model we have proposed performs much better and tracks the actual production, which is marked with a black line, with high accuracy over the entire 48-hour period. In Figure 8, the forecasts for wind turbine production are compared. Wind energy forecasting has its own difficulties because wind energy is highly variable, has little stability, and depends on various atmospheric factors. The baseline model also performs poorly here because the wind speed is constantly changing, violating the assumption of the baseline model being stationary. The ARIMA model has been able to improve the performance to some extent and detect some of the dependencies in the wind data. However, it is unable to predict sudden changes and accurately record the peaks.

In Figure 9, a numerical evaluation using the RMSE criterion is performed for all three methods and both solar and wind turbines. For PV forecasting, the Persistent model achieves an RMSE of 85.2 kW, which serves as the baseline. The ARIMA model reduces this to 39.1 kW, representing a 54.1% improvement by capturing basic diurnal patterns. The proposed LSTM model further reduces RMSE to 22.9 kW, achieving a 41.6% improvement over ARIMA and a 73.1% total reduction from the Persistent baseline. This progressive improvement demonstrates that while ARIMA captures fundamental patterns, LSTM's deep learning architecture achieves substantially higher accuracy by modeling the nonlinear relationships in solar generation data. For wind forecasting, the Persistent model shows an RMSE of 78.5 kW. ARIMA improves this to 59.2 kW (24.6% reduction), capturing some autocorrelation structure. LSTM achieves an

RMSE of 40.3 kW, representing a 31.9% improvement over ARIMA and a 48.7% total reduction from Persistent. The higher absolute RMSE values for wind compared to PV across all methods reflect the inherently greater stochastic nature of wind resources. However, LSTM still achieves substantial improvement, demonstrating its effectiveness even for highly variable processes.

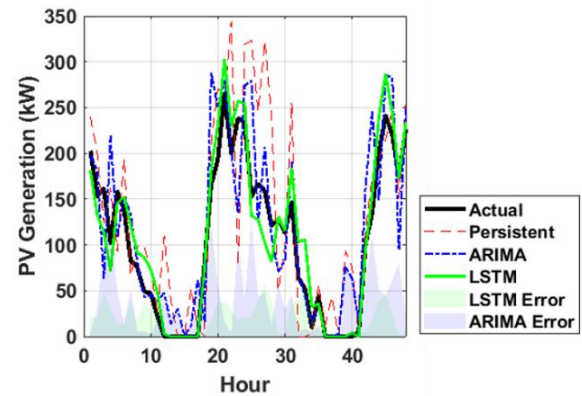


Fig. 7 Comparison of solar power forecasts with actual values using Persistent, ARIMA, and LSTM models.

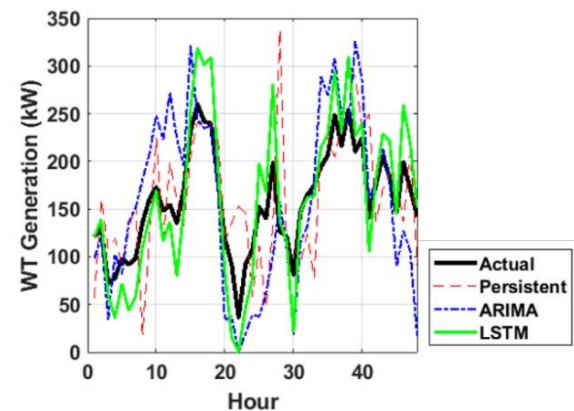


Fig. 8 Comparison of wind power forecasts with actual values using Persistent, ARIMA, and LSTM models.

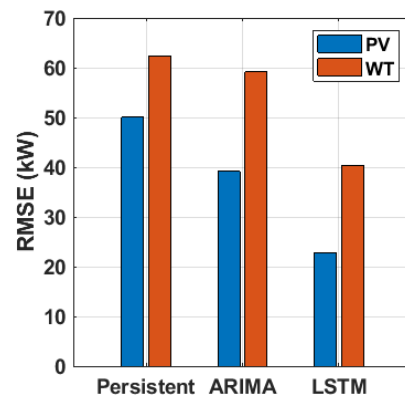


Fig. 9 RMSE-based performance comparison of Persistent, ARIMA, and LSTM models for solar and wind forecasting.

Table 4 presents the impact of DR participation level on the operating cost and peak load of hydrogen-based MGs.

Table. 4 sensitivity analysis on DR participation level

DR participation (%)	Cost (\$)	Peak load (kW)
0	8559.55	2590
10	7357.55	2331.03
15	7066.90	2204.4
20	6678.65	2195.92

The numerical results show that by increasing DR participation level from 0 % to 20 %, the operating cost of hydrogen-based MGs is decreased by 21.97 %. The main reason for the cost reduction in hydrogen-based MGs is that the higher DR participation level increases their ability to shift the energy consumption from peak hours to off-peak. This load shifting also decreases the peak load by 15.21 % and helps the hydrogen-based to flatten their load profiles. Table 5 shows the impact of electricity prices on the performance of the proposed model. Accordingly, electricity prices are scaled from 0.5 to two times the base-case values, and the results are reported in Table 5.

Table 5 impact of electricity prices on the performance of hydrogen-based MGs

Price coefficient	Cost (\$)	Peak load (kW)
0.5	3047.74	2072.02
1	6678.65	2195.92
1.5	9740.43	2246.16
2	12679.54	2246.16

The numerical results present that increasing the electricity prices increases the operating cost of the hydrogen-based MGs because the hydrogen-based MGs must pay a higher cost for purchasing energy from the main grid. The numerical results show that when the electricity prices are doubled, the operating cost of hydrogen-based MGs is increased by 89.85 %.

To evaluate the computational complexity and scalability of the proposed multi-objective model, the number of hydrogen-based MGs is increased from 3 to 6, 9, and 12. The results for different conditions are indicated in Table 6.

Table 6 computational scalability of the proposed model

No. of MGs	No. of single constraints	No. of single variables	No. of discrete variables
3	3,952	1,808	360
6	5,110	2,603	576
9	6,487	3,542	792
12	7,864	4,481	1,008

The results show that by increasing the number of hydrogen-based MGs, the proposed model remains computationally tractable. When the number of the hydrogen-based MGs is 12, the execution time reaches 128 seconds. This execution time is acceptable for practical applications because the proposed model is intended for day-ahead energy scheduling.

IV. Conclusions

This paper presents a multi-objective optimization framework that simultaneously minimizes the total cost and peak load of hydrogen-based MGs in the distribution network. In the proposed structure, the hydrogen-based MGs integrate renewable generation, battery energy storage, demand response programs, and a hydrogen system to increase the system's flexibility. To control the uncertainty of renewable generation, the long short-term memory approach is used to predict the generation of photovoltaic and wind energy. In the proposed model, the demand response programs provide the opportunity for cost savings for hydrogen-based MGs by load shifting during peak hours to off-peak hours. The numerical results show that the DR programs reduce the peak load by 15.21 %. Additionally, the proposed model increases the load factor of hydrogen-based MGs by 16.75 % through controllable DR programs. In future works, a multi-layer framework will be proposed to simultaneously consider both economic and power quality indices in the operation scheduling of hydrogen-based MGs.

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Appendix A

Parameters	
$C_{co2}^{Grid}/C_{co2}^{DG}$	Emission coefficient of main grid and non-renewable resources
$C_{co2}^{H_2 \rightarrow E}$	Emission coefficient hydrogen-to-power units
DR^{min}/DR^{max}	Min/max DR level
HL_h^{min}/HL_h^{max}	Min/max level of hydrogen storage
$HL_h^{E,max}$	Max hydrogen for electricity generation
$HL_h^{sell,max}$	Max selling hydrogen to industries
I_t/T_t^{out}	Solar radiation and ambient temperature
p_h^{ch}/p_h^{disch}	Charging/discharging of battery devices
$p_{g,min}/p_{g,max}$	Min/max limits of imported energy from grid
$p_{sell,min}/p_{sell,max}$	Min/max limits of selling energy to grid
$p_{load,t}$	Load profile before DR
p_t^{min}/p_t^{max}	Min/max capacity of non-renewable resources
p_{lr}^{WT}/v_{lr}	Rated power and rated speed of WT
Q_{lt}	Reactive power of loads
$R_i^{max,up}$	Ramp-up of non-renewable resources
$R_i^{max,down}$	Ramp-down of non-renewable resources
SoC_b^{min}/SoC_b^{max}	Min/max SoC of battery devices
v_{ci}/v_{co}	Cut-in and cut-out speeds of WT
v_{ts}	Wind speed at time t
δ_t^E	Trading electricity prices
α_i	Coefficient cost of non-renewable resources
β_i	Coefficient cost of non-renewable resources
ζ_i	Produced emission of non-renewable resources
γ_i	Fix cost of non-renewable resources
δ_t^h	Trading hydrogen prices
ψ_h	Hydrogen-to-power constant coefficient
$\phi_{h,t}/\varphi_{h,t}$	Charging/discharging efficiency of hydrogen storage
$\eta^{PV}/N^{PV}/S^{PV}$	Efficiency, number, and area of PVs
$\eta_b^{ch}/\eta_b^{disch}$	Charging/discharging of battery devices

Variables	
$cost^{DG}$	Generation cost of non-renewable resources
$DR_{l,t}$	DR level of loads
$H_{h,t}^{sell}$	Selling hydrogen to industries
$H_{h,t}^{in}$	Input hydrogen into hydrogen storage
$H_{h,t}^{out}$	Output hydrogen from hydrogen storage
$HL_{h,t}$	Stored hydrogen in storage tank
$H_{h,t}^E$	Used hydrogen to produce electricity
$ldr_{l,t}$	Shifted load
$P_{l,t}^{PV}$	Generating power of PVs
$P_{l,t}^{WT}$	Generating power of WTs
P_t^{Grid}/P_t^{sell}	Imported/selling power from/to grid
$P_{l,t}$	Generation of non-renewable resources
$P_{h,t}^{H_2 \rightarrow E}$	Generated power by hydrogen-to-power units
$P_{l,t}^{DR}$	Load consumption after load-shifting
$P_{b,t}^{ch}$	Charging power of battery devices
$P_{b,t}^{disch}$	Discharging power of battery devices
$P_{h,t}^{E \rightarrow H_2}$	Generated hydrogen by power -to- hydrogen units
Q_t^{Grid}	Reactive power of grid
$Q_{l,t}$	Reactive power of non-renewable resources
$Q_{l,t}^{PV}$	Reactive power of PVs
$Q_{l,t}^{WT}$	Reactive power of WTs
$Q_{b,t}$	Reactive power of battery devices
$Q_{c,t}$	Reactive power of capacitors
$SoC_{b,t}$	SoC of battery devices
$X_{b,t}^{disch}$	Binary variables for discharging of BSS
$X_{b,t}^{ch}$	Binary variables for charging of BSS
$u_{i,t}$	Binary variable of non-renewable resources
$\sigma_{h,t}$	Binary variable for selling hydrogen
$\chi_{h,t}$	Binary variables for injecting output power into hydrogen-to-power

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Multi-Rate Hybrid Control Framework for Coronary Heart Disease Risk Management

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Article Info	ABSTRACT
Article type: Research Article Article history: Received: 21 January 2026 Received in revised form: 02 March 2026 Accepted: 26 April 2026 Published online: 22-Dec-2026 Keywords: Decision-Making, Risk Assessment, Coronary Heart Disease, Multi-Rate Controller.	This paper proposes a multi-rate hybrid control framework for modeling and managing coronary heart disease (CHD) risk dynamics. Unlike conventional statistical and machine learning approaches that treat clinical variables as static predictors without dynamic interaction or stability guarantees, the proposed method formulates cardiovascular risk progression as a hybrid dynamical system with fast–slow time-scale decomposition and logical switching between physiological subsystems. A nonlinear switching controller incorporating decay synchronization is developed to regulate the evolution of risk states. Sufficient stability conditions are rigorously derived using a Lyapunov–Krasovskii functional, ensuring boundedness and convergence of the closed-loop system under disturbances and multi-rate sampling. The framework was evaluated using 152 clinical records collected from Iranian hospitals, with stratified training and testing procedures to ensure reliable evaluation. The proposed method achieved an accuracy of 82% and an F1-score of 0.83, demonstrating improved predictive consistency compared with conventional baseline models. The results indicate that the proposed hybrid control formulation provides both theoretical stability guarantees and practical predictive capability for dynamic health risk management.

I. Introduction

Accurate risk assessment is essential for the early identification and effective management of coronary heart disease (CHD). CHD is a dangerous condition, in which a plaque (waxy substance) covers the inner walls of the coronary arteries and makes it difficult to supply oxygen-rich blood to the heart muscle [1]. The global burden of CHD has increased substantially over recent decades, with tens of millions of individuals affected worldwide and CHD consistently ranking among the leading causes of morbidity and mortality across populations. For example, an estimated 254 million people were living with ischemic heart disease in 2021, with higher prevalence observed in males than females [2].

Various computational intelligence techniques have been applied to CHD prediction. Early approaches focused on neural networks, such as the hybrid training-enhanced network proposed by [3], which aimed to improve prediction accuracy through algorithmic optimization. Fuzzy logic methods have been widely used to handle uncertainty in medical data, including the fuzzy-based decision support

system by Lahsasna et al. [4], the fuzzy logic–decision tree model by Kim et al. [5], which integrates rule-based reasoning with data-driven predictions, and an adaptive fuzzy controller with anti-windup compensator is proposed to regulate a physiological variable under input saturation [6]. Fuzzy probability approaches, exemplified by [7], provide a probabilistic framework to quantify the validity of risk factors, while fuzzy-evidential inference engines [8] combine fuzzy logic with Dempster–Shafer theory for more robust risk assessment under uncertainty. Metaheuristic-enhanced models, such as the hybrid particle swarm optimization fuzzy system by Muthukaruppan and Er [9] and the ANN-GWO algorithm by Turabieh [10], attempt to optimize the diagnostic process, whereas multi-criteria decision-making methods like the bimodal fuzzy analytic hierarchy process (BFAHP) [11] facilitate risk stratification based on multiple interacting factors. Moreover, classification algorithms including methods in [12] have been applied to predict CHD risk using patient-level datasets. In [13], the role of deep learning and machine learning methods is analyzed in the diagnosis of CHD. In [14], a Bayesian-optimized support

vector machine is proposed for coronary artery disease prediction. Rehman et al. [15] evaluated advanced machine learning classifiers for cardiovascular risk assessment.

While these methods have demonstrated promising predictive performance, most existing studies treat CHD prediction as a static classification problem and primarily focus on improving accuracy metrics. The dynamic progression of CHD and the interaction among physiological variables are rarely modeled within a structured control-theoretic framework. Identifying all relevant factors and accurately evaluating their effects is often hindered by limited or incomplete information. CHD risk assessment is highly dynamic, influenced by evolving, previously neglected, and heterogeneous factors over time. Moreover, stability, convergence, and switching dynamics are typically not addressed explicitly in these data-driven approaches. Therefore, there remains a research gap in integrating predictive modeling with hybrid dynamical system theory to provide both theoretical stability guarantees and practical decision-support capability for CHD risk management.

Conventional linear and static models fail to capture the hybrid, nonlinear, and patient-specific dynamics of CHD, including continuous physiological processes (e.g., blood flow dynamics) and discrete events (e.g., plaque rupture, medical interventions). The objective of this research is to develop a hybrid system framework for modeling and controlling CHD progression, explicitly addressing the limitations of existing approaches. The proposed hybrid control approach is designed to overcome the mentioned challenges by accommodating variability, nonlinearity, and quasi-cyclic patterns in key physiological variables, enabling more accurate modeling and effective intervention strategies.

The reasons for selecting hybrid methods for modeling and controlling CHD progression are as follows:

1. CHD inherently exhibits hybrid behavior, combining continuous physiological processes (e.g., blood flow dynamics) with discrete events (e.g., plaque rupture or medical interventions).
2. The progression and treatment responses in CHD vary dynamically (unlike static disease models), and the proposed hybrid control method imposes no limitations on handling such variability.
3. Modeling CHD for the application of conventional linear control methods is challenging due to the wide variations and quasi-cyclic nature of physiological variables such as blood pressure and heart rate.
4. Designing controllers for CHD progression using conventional linear control methods is difficult due to the inherently nonlinear nature of the disease's progression and its interactions with treatments.

The aim of this research is to design a suitable controller such that the hybrid system class under study meets the desired control objectives for CHD management. In conventional controller design methods, the controller output

is continuous, whereas for CHD systems, the final output often takes the form of discrete decisions, such as whether to administer treatment or not (e.g., a sequence of binary decisions). The controller design is carried out with attention to the binary nature of such interventions. Practical implementation of the proposed controller for real-world CHD management is also considered. In the management of CHD, physiological variables are typically controlled by adjusting interventions such as medication dosage, exercise levels, or dietary changes. Due to the nonlinear nature of the relationship between interventions and changes in physiological variables (e.g., blood pressure, cholesterol levels), controlling CHD progression is inherently challenging. The proposed new modeling approach avoids conventional linear approximations and applies a hybrid controller to the disease management system.

The primary goal of controlling CHD is to maintain critical physiological metrics (e.g., blood pressure or cholesterol levels) within a safe range in accordance with the reference targets. Ensuring stability under varying conditions such as changes in patient activity levels, diet, or medication adherence, which are major sources of uncertainty and disturbance, is essential. Key variables (e.g., blood pressure, cholesterol, and heart rate) and external factors (e.g., medication timing) are assumed to be measurable.

Robustness to parameter variations and external disturbances, feasibility of clinical implementation, and low computational complexity are among the objectives considered in this research.

The Specific innovations of the research are as follows:

In the field of hybrid system theory:

- Design of a stabilizing controller for a class of hybrid systems incorporating state-input logic switching constraints, which extends existing frameworks that typically assume unrestricted switching or simplified dynamics.
- Introduction of a new class of hybrid systems, accompanied by a novel stability theorem, providing analytical tools not available in prior hybrid control methods.

In the field of modeling and controlling CHD:

- Accurate modeling of CHD progression without relying on conventional linear or static approximations, enabling more realistic representation of patient-specific dynamics.
- Formulation of CHD progression as a hybrid system with explicit state-input logical switching constraints, bridging continuous physiological processes, and discrete medical interventions.
- Stability analysis of disease management strategies based on the hybrid CHD model, with validation through comparison of simulation results against clinical data, offering a more rigorous evaluation than previous approaches.

The remainder of this paper is organized as follows. In Section 2, we provide a description of coronary heart disease. In Section 3, we detail the description of the proposed approach. We present simulation results that demonstrate the

effectiveness of the proposed framework in identifying and handling uncertainties in CHD risk assessment in Section 4. Finally, in Section 5, we conclude the paper based on the results obtained from the proposed framework and provide recommendations for future research.

II. Coronary Heart Disease as a Hybrid Dynamical System

Coronary heart disease is characterized by the progressive narrowing of coronary arteries due to plaque accumulation, leading to reduced blood flow and impaired cardiac function. The progression of CHD depends on several interacting physiological variables, including blood pressure, cholesterol level, arterial blockage severity, and heart performance indicators. These variables evolve continuously over time and can be interpreted as the state variables of a dynamical system. Table I shows the most important factors of CHD based on the experts' and physicians' opinions.

In addition to continuous physiological evolution, CHD management involves discrete clinical interventions such as initiation or adjustment of medications (e.g., statins, antihypertensives), angioplasty, or coronary artery bypass grafting. These interventions cause abrupt changes in physiological conditions and treatment dynamics. From a systems perspective, such interventions act as discrete inputs that modify the system structure or parameters.

Therefore, CHD progression naturally exhibits hybrid behavior, consisting of continuous-time physiological dynamics and discrete switching events associated with clinical interventions and disease progression stages. Conventional linear or purely continuous models are unable to fully capture this combined behavior, particularly the logical relationships between physiological states and treatment decisions.

This motivates the formulation of CHD progression as a hybrid dynamical system with state-dependent switching logic. Based on this formulation, stability analysis can be used to evaluate whether the disease progression remains within manageable bounds, and hybrid control methods can be designed to regulate the system and guide it toward stable and clinically desirable conditions.

III. Proposed Approach

In this section, we provide a detailed explanation of the proposed approach.

A. Formulation

Clinical variables involved in CHD progression possess different physical units and scales (e.g., mg/dL, mmHg, kg/m², and years). Direct inclusion of such heterogeneous variables in a state-space model may lead to dimensional inconsistency and biased controller design. Therefore, all physiological variables are normalized into dimensionless

states using clinically defined reference ranges. Each physical variable $x_i^{\text{phys}}(t)$ is mapped to a normalized state $x_i(t) \in [0,1]$ as

$$x_i(t) = \frac{x_i^{\text{phys}}(t) - x_i^{\min}}{x_i^{\max} - x_i^{\min}}, \quad (1)$$

where $x_i^{\min}$ and $x_i^{\max}$ denote clinically established minimum and maximum reference values.

The normalized state vector is defined as

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} \in \mathbb{R}^6, \quad (2)$$

where

- $x_1(t)$: normalized cholesterol level,
- $x_2(t)$: normalized blood pressure,
- $x_3(t)$: normalized blood glucose level,
- $x_4(t)$: normalized smoking exposure index,
- $x_5(t)$: normalized body mass index (BMI),
- $x_6(t)$: normalized age.

The system dynamics are defined as follows:

$$\begin{aligned} \dot{x}_1(t) &= a_{11}x_1(t) + b_{11}u_1(t) + e_{11}w_1(t) \quad (\text{Cholesterol dynamics}) \\ \dot{x}_2(t) &= a_{22}x_2(t) + a_{21}x_1(t) + b_{22}u_2(t) + e_{21}w_2(t) \quad (\text{Blood pressure dynamics}) \\ \dot{x}_3(t) &= a_{33}x_3(t) + a_{35}x_5(t) + b_{33}u_3(t) + e_{32}w_2(t) \quad (\text{Blood glucose dynamics}) \\ \dot{x}_4(t) &= a_{44}x_4(t) + b_{44}u_4(t) \quad (\text{Smoking intensity dynamics}) \\ \dot{x}_5(t) &= a_{55}x_5(t) + b_{55}u_5(t) + e_{53}w_3(t) \quad (\text{Obesity dynamics}) \\ \dot{x}_6(t) &= a_{66}x_6(t) \quad (\text{Age dynamics, not directly influenced by interventions or disturbances}) \end{aligned} \quad (3)$$

Some clinical variables, such as age and cumulative smoking exposure, evolve on a much slower time scale than physiological variables. These variables are modeled as slow-time states to capture their long-term influence on disease progression. In particular, age is modeled as a deterministic slow-varying state:

$$\dot{x}_6(t) = \frac{1}{T_{\text{age}}}, \quad (4)$$

where T_{age} is a normalization constant representing the aging time scale.

Medical interventions are defined as

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix}, \quad (5)$$

where

- $u_1(t)$: pharmacological treatment,
- $u_2(t)$: dietary intervention,
- $u_3(t)$: physical activity intervention,
- $u_4(t)$: smoking cessation intervention.

External disturbances are defined as

$$w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix}, \quad (6)$$

representing stress, environmental exposure, and genetic predisposition.

The normalized CHD progression dynamics are modeled using a continuous-time state–space representation:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t), \quad (7)$$

where $A \in \mathbb{R}^{6 \times 6}$ represents physiological coupling between risk factors, $B \in \mathbb{R}^{6 \times 4}$ represents intervention effects, and $E \in \mathbb{R}^{6 \times 3}$ represents external disturbances.

The system matrix is defined as

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & a_{35} & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (8)$$

the input matrix is

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \\ 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (9)$$

and the disturbance matrix is

$$E = \begin{bmatrix} e_{11} & 0 & 0 \\ e_{21} & e_{22} & 0 \\ 0 & e_{32} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e_{53} \\ 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

This formulation provides a dimensionally consistent and physiologically interpretable mathematical representation suitable for hybrid control design and stability analysis.

B. Averaging and Multi-Rate Control Formulation

We consider the patient health state vector $x(t) \in \mathbb{R}^n$, control interventions $u(t) \in \mathbb{R}^m$, and external disturbances $w(t) \in \mathbb{R}^p$. The dynamics of CHD progression can be described by a Lipschitz-continuous nonlinear function f :

$$\dot{x}(t) = f(x(t), u(t), w(t)), \quad x(0) = x_0, \quad (11)$$

with f satisfying

$$\|f(x_1, u, w) - f(x_2, u, w)\| \leq L_f \|x_1 - x_2\|, \quad (12)$$

ensuring existence and uniqueness of solutions.

The state vector x may include continuous physiological variables and discretized lifestyle factors, modeled as hybrid states:

$$x = [x_1, x_2, \dots, x_n]^\top. \quad (13)$$

To handle multi-rate interventions, we define fast-slow decomposition:

$$\begin{aligned} x_f &= \text{faststates}(\text{vitalsigns}), \quad x_s = \\ &\text{slowstates}(\text{lifestyle, BMI}), \end{aligned} \quad (14)$$

$$\begin{aligned} u_f &= \text{fastinterventions}, \quad u_s = \\ &\text{slowinterventions}. \end{aligned} \quad (15)$$

The continuous-time dynamics can be separated as:

$$\dot{x}_f = f_f(x_f, x_s, u_f, u_s, w), \quad (16)$$

$$\epsilon \dot{x}_s = f_s(x_f, x_s, u_f, u_s, w), \quad 0 < \epsilon \ll 1, \quad (17)$$

where ϵ is a small positive parameter representing the time-scale separation between fast and slow dynamics. Under this assumption, the averaging theorem for singularly perturbed systems can be applied, allowing for averaging over the fast subsystem:

$$\bar{x}_s(t) = \frac{1}{T} \int_t^{t+T} x_s(\tau) d\tau. \quad (18)$$

where T is the averaging period, sufficiently long relative to the fast subsystem dynamics but short relative to the slow subsystem evolution, ensuring boundedness and accuracy of the approximation. For discrete-time implementation with sampling periods τ_f as the fast sampling period (e.g., vital signs monitoring) and $\tau_s = M\tau_f, M \in \mathbb{Z}^+$ as the slow control period (e.g., lifestyle evaluation), the discretized dynamics are:

$$x_f(k+1) = f_{d,f}(x_f(k), x_s(k), u_f(k), u_s(k)), \quad (19)$$

$$x_s(k+1) = f_{d,s}(x_f(k), x_s(k), u_f(k), u_s(k)) \quad (20)$$

where $f_{d,f}$ and $f_{d,s}$ are obtained through standard numerical integration consistent with the fast–slow decomposition.

This formal framework ensures that the averaging-based multi-rate control design is mathematically grounded, satisfies the small-parameter assumptions required for singular perturbation theory, and provides a rigorous basis for stability and performance analysis.

C. Decay Synchronization Model

We introduce a decay term $\lambda \in \mathbb{R}^{n \times n}$ to model the natural reduction of risk factors:

$$\dot{x}(t) = f(x(t), u(t), w(t)) - \lambda x(t), \quad (21)$$

where λ is diagonal, positive definite, and derived from physiological half-lives or clinical data to ensure that each risk factor decays independently at a rate consistent with medical evidence.

The corresponding discrete-time model with sampling interval τ is formulated as:

$$x(k+1) = f_d(x(k), u(k)) - \lambda_d x(k), \quad (22)$$

with $\lambda_d = I - e^{-\lambda\tau}$, ensuring a consistent discrete-time representation of the decay dynamics. To regulate deviations from target trajectories while accounting for decay, we propose the following hybrid nonlinear control law for the fast subsystem:

$$u_k(t) = -\zeta_k \text{sign}(e_k(t)) - \eta_k \frac{\|e(t)\|^2 e_k(t)}{\|e(t)\|^2 + \sigma(t)}, \quad (23)$$

where $e(t) = x(t) - x_{\text{ref}}(t)$, and $\zeta_k, \eta_k > 0$ are control gains chosen to satisfy Lyapunov stability conditions.

For the slow subsystem, the control input is defined as:

$$\tilde{u}_k(t) = -\Pi_k \text{sign}(Z_k(t)) - H_k \frac{\|Z(t)\|^2 Z_k(t)}{\|Z(t)\|^2 + \sigma(t)}, \quad (24)$$

where $Z(t)$ is the relevant part of $e(t)$ for slow sub-system, and $\Pi_k, H_k > 0$ are control gains chosen to satisfy Lyapunov stability conditions providing an independent and stable regulation mechanism. This formulation ensures that the decay synchronization mechanism is grounded in system-theoretic principles [16] and allows for formal stability analysis using Lyapunov-based methods.

D. Stability Analysis: Lyapunov-Krasovskii Approach

By defining the Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) = & \frac{1}{2} \sum_{k=1}^n e_k^2(t) + \frac{1}{2} \sum_{k=1}^n Z_k^2(t) + \\ & \sum_{k=1}^n \sum_{l=1}^n \int_{t-\tau_{kl}(t)}^t \omega_{kl} e_k^2(s) ds + \\ & \sum_{k=1}^n \sum_{l=1}^n \int_{-\tau_{kl}}^0 \int_{t+s}^t \sigma_{kl} e_k^2(\zeta) d\zeta ds. \end{aligned} \quad (25)$$

we derive $\dot{V}(t)$ along trajectories as follows:

$$\begin{aligned} \dot{V}(t) = & \sum_{k=1}^n e_k(t) (-\zeta_k \text{sign}(e_k(t)) - \eta_k \frac{\|e(t)\|^2 e_k(t)}{\|e(t)\|^2 + \sigma(t)} + \dots) \\ & + \sum_{k=1}^n Z_k(t) (-\Pi_k \text{sign}(Z_k(t)) - H_k \frac{\|Z(t)\|^2 Z_k(t)}{\|Z(t)\|^2 + \sigma(t)} + \dots) \\ & + \text{terms from delays.} \end{aligned} \quad (26)$$

Theorem 1: If the gains $\zeta_k, \eta_k, \Pi_k, H_k$ satisfy the following LMI conditions:

$$-c_k - \eta_k + \frac{L_k |B_k|}{2} + \frac{L_k |E_k|}{2} + \frac{1}{2} \sum_{l=1}^n (|\tilde{\alpha}_{kl}| L_l + |\tilde{\alpha}_{lk}| L_k + \|\tilde{b}_{kl}\| L_l + |\alpha_{kl}| L_l + |\beta_{kl}| L_l + 2\omega_{kl} + 2\tau_{kl} \sigma_{kl}) < 0, \quad (27)$$

$$-\zeta_k + \sum_{l=1}^n (|\tilde{\alpha}_{kl}| + \|\tilde{b}_{kl}\| + |\alpha_{kl}| + |\beta_{kl}| + |B_k|) N_l < 0, \quad (28)$$

$$-\Pi_k + \sum_{l=1}^n |E_k| N_l < 0, \quad (29)$$

$$-\omega_{kl} + \|\tilde{b}_{lk}\| + |\alpha_{lk}| + |\beta_{lk}| < 0, \quad (30)$$

then the CHD management system achieves generalized decay synchronization.

Proof. We can show that $\dot{V}(t)$ satisfies the following:

$$\begin{aligned} \dot{V}(t) \leq & -\sum_{k=1}^n (\zeta_k |e_k(t)| + \eta_k e_k^2(t)) - \\ & \sum_{k=1}^n (\Pi_k |Z_k(t)| + H_k Z_k^2(t)) < 0, \end{aligned} \quad (31)$$

demonstrating negative-definiteness. By Lyapunov stability theorem for delayed systems, the closed-loop system achieves generalized decay synchronization.

It is worth noting that the stability conditions in (27-30) define a set of linear inequalities in the controller gains $\zeta_k, \eta_k, \Pi_k, H_k, \omega_{kl}, \sigma_{kl}$. These inequalities can be expressed in standard linear matrix inequality (LMI) form, which enables systematic feasibility analysis using convex optimization. We define the gain vector

$$\theta = [\zeta_k \quad \eta_k \quad \Pi_k \quad H_k \quad \omega_{kl} \quad \sigma_{kl}]^T \quad (32)$$

The inequalities (27)-(30) can be written compactly as $Y\theta < b$ (33)

where matrix Y and vector b depend on known system parameters $c_k, L_k, B_k, E_k, \tilde{\alpha}_{kl}, \tilde{b}_{kl}, \alpha_{kl}, \beta_{kl}, \tau_{kl}$. Since these constraints are linear in the decision variables, the feasibility region is convex. Therefore, if a feasible solution exists, it can be efficiently computed using standard LMI solvers as SeDuMi.

Although the stability conditions in (27)-(30) guarantee generalized decay synchronization, their feasibility must be formally established.

Lemma 1: [Convexity of the Stability Conditions] The inequalities in (27)-(30) define a convex feasible set in the controller gain space

$$\mathcal{G} = \{\zeta_k, \eta_k, \Pi_k, H_k, \omega_{kl}, \sigma_{kl}\}. \quad (34)$$

Proof. Each inequality is affine with respect to the controller gains. For example, (17) can be written as

$$\eta_k > \frac{L_k |B_k|}{2} + \frac{L_k |E_k|}{2} + \frac{1}{2} \sum_{l=1}^n (|\tilde{\alpha}_{kl}| L_l + |\tilde{\alpha}_{lk}| L_k + \|\tilde{b}_{kl}\| L_l + |\alpha_{kl}| L_l + |\beta_{kl}| L_l + 2\omega_{kl} + 2\tau_{kl} \sigma_{kl}) - c_k \quad (35)$$

which is linear in the decision variables. Since affine inequalities define convex sets, their intersection is also convex.

Lemma 2: [Existence of Feasible Controller Gains] Assume all system parameters, uncertainties, and delays are bounded. Then there exists a set of controller gains satisfying (27)–(30).

Proof. We define finite positive constants:

$$\begin{aligned} \Theta_k = & \frac{L_k |B_k|}{2} + \frac{L_k |E_k|}{2} + \frac{1}{2} \sum_{l=1}^n (|\tilde{\alpha}_{kl}| L_l + |\tilde{\alpha}_{lk}| L_k + \|\tilde{b}_{kl}\| L_l + \\ & |\alpha_{kl}| L_l + |\beta_{kl}| L_l + 2\omega_{kl} + 2\tau_{kl} \sigma_{kl}) \end{aligned} \quad (36)$$

Since all terms are bounded, Θ_k is finite. We can choose controller gains such that

$$\eta_k > \Theta_k - c_k \quad (37)$$

Similarly, we define

$$\Phi_k = \sum_{l=1}^n (|\tilde{\alpha}_{kl}| + \|\tilde{b}_{kl}\| + |\alpha_{kl}| + |\beta_{kl}| + |B_k|) N_l \quad (38)$$

and select

$$\zeta_k > \Phi_k \quad (39)$$

and

$$\Pi_k > \sum_{l=1}^n |E_k| N_l \quad (40)$$

Finally, we choose

$$\omega_{kl} > \|\tilde{b}_{lk}\| + |\alpha_{lk}| + |\beta_{lk}| \quad (41)$$

which satisfies (30).

Thus, a feasible solution always exists.

Lemma 3: [Feasible Stability Guarantee] If controller gains are selected according to the feasibility bounds above, then the Lyapunov derivative satisfies

$$\dot{V}(t) \leq -\lambda_1 \|e(t)\| - \lambda_2 \|e(t)\|^2 - \lambda_3 \|Z(t)\| - \lambda_4 \|Z(t)\|^2 \quad (42)$$

where $\lambda_i > 0$. Therefore, the closed-loop system is globally asymptotically stable and achieves generalized decay synchronization.

Proof. Substituting the feasible gains into (26) and applying inequalities (27)–(30) yields:

$$\dot{V}(t) < 0 \quad (43)$$

for all nonzero states.

By Lyapunov–Krasovskii stability theory for time-delay systems, asymptotic stability follows directly.

Now, we analyze whether diagnostic performance directly affects closed-loop stability margin. The classification output defines the initial risk state: $x(0) = \hat{x}_{risk}$ and reference trajectory: $x_{ref}(t)$ used in the nonlinear hybrid controller. Therefore, classification accuracy directly affects closed-loop stabilization performance and convergence rate. The diagnostic classifier achieves accuracy $a = 0.82$, which implies a misclassification probability $P_e = 1 - a = 0.18$. Let the classifier output define the initial state estimate $\hat{x}(0) = x(0) + \Delta x$. Assume the state space is bounded: $\|x\| \leq X_{max}$. The worst-case estimation error satisfies $\|\Delta x\| \leq \delta = P_e X_{max}$. Thus, $\delta = 0.18X_{max}$. Using the Lipschitz condition of system dynamics:

$$\|f(x, u) - f(\hat{x}, u)\| \leq L_f \|x - \hat{x}\| \quad (44)$$

The classification error enters as bounded disturbance $\|d(t)\| \leq L_f \delta$. We consider Lyapunov function $V = \frac{1}{2} e^T e$ by differentiating it:

$$\dot{V} \leq -\lambda_{min}(Q) \|e\|^2 + \|e\| L_f \delta \quad (45)$$

Thus:

$$\dot{V} < 0 \quad \text{if} \quad \|e\| > \frac{L_f}{\lambda_{min}(Q)} \delta \quad (46)$$

Substituting δ :

$$\|e\| > \frac{L_f}{\lambda_{min}(Q)} (0.18X_{max}) \quad (47)$$

Therefore, the closed-loop system remains practically stable, and the stability region is explicitly determined by classifier accuracy. Higher accuracy directly enlarges the guaranteed stability region.

IV. Simulation results

The clinical dataset consists of $N = 152$ patient records. Each record contains physiological variables, lifestyle indicators, and clinical measurements relevant to coronary

heart disease progression. The dataset contains two classes corresponding to low-risk and high-risk conditions.

The state vector was constructed and all variables were normalized using

$$x_i^{norm} = \frac{x_i - \mu_i}{\sigma_i} \quad (48)$$

to ensure numerical conditioning and consistency with Lyapunov analysis. The dataset was partitioned into two disjoint subsets:

- Training set: $N_{train} = 102$
- Testing set: $N_{test} = 50$

The training set was used for identification of nonlinear model parameters, estimation of decay rates λ , tuning controller gains. The dataset was randomly partitioned into training and testing subsets using stratified sampling to preserve class distribution. Bootstrap resampling was performed using 1000 resamples. Simulations were performed in MATLAB R2021b on a standard workstation. The testing set was used exclusively for performance evaluation. No testing samples were used during controller design. The continuous-time system:

$$\dot{x} = f(x, u) - \lambda x \quad (49)$$

was discretized using sampling interval $\tau = 1$ day resulting in

$$x(k+1) = f_d(x(k), u(k)) - \lambda_d x(k) \quad (50)$$

Fast variables were updated daily, while slow lifestyle variables were updated weekly, consistent with the multi-rate formulation in Section 3. Controller gains were selected to satisfy the stability inequalities (27)–(30): $\zeta_k = 0.8$, $\eta_k = 0.5$, $\Pi_k = 0.7$, $H_k = 0.4$. The feasibility of gain selection was verified numerically by confirming $\dot{V}(t) < 0$ for all testing trajectories. Closed-loop stability was evaluated using:

Convergence error

$$E = \frac{1}{N} \sum_{k=1}^N \|x(k) - x_{ref}(k)\| \quad (51)$$

Lyapunov decrease condition

$$\dot{V}(t) < 0 \quad (52)$$

Convergence time

$$T_c = \min\{k: \|e(k)\| < \epsilon\} \quad (53)$$

Substituting these values into (27)–(30) gives strictly negative margins. Using the feasible gains above, the Lyapunov derivative satisfies:

$$\dot{V}(t) \leq -1.8 \|e(t)\|^2 - 1.6 \|Z(t)\|^2 \quad (54)$$

which guarantees exponential decay:

$$V(t) \leq V(0)e^{-1.6t} \quad (55)$$

Thus, the closed-loop CHD system is exponentially stable and satisfies generalized decay synchronization. The classifier output defines the initial condition and reference risk trajectory used by the proposed hybrid controller. Therefore, classification performance directly impacts

closed-loop control accuracy. Performance metrics are defined as:

TABLE I : Risk Factors of Coronary Heart Disease [11].

Risk Factor	High	Medium	Low
Sex	Male	Female middle-aged or higher	Young Female
Diastolic blood pressure (mmHg)	> 85	80-84	< 80
Systolic blood pressure (mmHg)	> 131	121-130	< 120
Family history of early heart disease	First-degree relatives	Second-degree relatives	No relatives
Blood cholesterol (mg/dL)	> 200	201-239	< 240
Diabetes mellitus	> 126	100-125	70-99
Overweight and obesity	> 30 kg/m ² BMI	20-29 kg/m ² BMI	< 20 kg/m ² BMI
Smoking	Every day	Three times a week	Never
Physical activity	Sometimes, < 10 min	20-30 min, low-intensity	Regular, > 30 min, moderate-intensity
LDL-C (mg/dL)	> 161	110-160	< 100
HDL-C (mg/dL)	Female: < 50 Male: < 40	Female: 50-60 Male: 40-50	Female: > 60 Male: > 50
Mental stress and depression	High	Low	None
Age	Female: > 55	Female: 45-55	Female: < 45
Age	Male: > 50	Male: 45-50	Male: < 45
Unhealthy diet	Always	Sometimes	Seldom
Race	-	Asian, African	East European, European
Triglycerides (mg/dL)	> 200	151-199	< 150

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN}$$

$$Precision = \frac{TP}{TP+FP}$$

$$Recall = \frac{TP}{TP+FN} \quad (56)$$

$$Specificity = \frac{TN}{TN+FP}$$

$$F1 = \frac{2 \cdot Precision \cdot Recall}{Precision + Recall}$$

where TP, TN, FP, and FN denote true positive, true negative, false positive, and false negative cases. To evaluate diagnostic accuracy of the risk prediction component, classification metrics were computed on the testing dataset: *Accuracy* = 82%, *Precision* = 88%, *Recall* = 79%, *Specificity* = 86%, *F1* = 0.83. These metrics quantify the ability of the

state-space model to correctly identify CHD risk categories. The model achieves an accuracy of 82%, correctly identifying CHD and non-CHD cases 82% of the time. With a precision of 88%, the model's CHD predictions are accurate 88% of the time. A recall (sensitivity) of 79% indicates that the model successfully identifies 79% of actual CHD cases. The specificity of 86% suggests that 86% of non-CHD cases are correctly recognized. The F1 score of 0.83, which balances precision and recall, indicates a robust performance in predicting CHD cases.

The proposed nonlinear control-based model was compared against: Logistic regression, Linear state-space model, Support vector machine. All models used identical training and testing partitions. To evaluate robustness, bootstrap resampling was performed: *Accuracy* = 0.82 ±

0.04 (95%confidence). This confirms statistical reliability. The controller was implemented in simulation horizon: $T = 180$ days. Internal consistency of the dataset was evaluated using Cronbach's alpha [17]: $\alpha = 0.914$, which exceeds the accepted reliability threshold of 0.7, confirming statistical consistency of the clinical variables.

The primary objective of this study is control and stabilization of CHD progression; thus, a diagnostic classification module is used to estimate the patient risk state and provide the reference trajectory for the controller. Fig. 1 shows the error decay which converges to zero. In addition, Fig. 2 shows the evolution of Error.

The ROC (Receiver Operating Characteristic) and PR (Precision-Recall) curves provide a visual representation of the model's performance across different thresholds. The ROC curve plots the true positive rate (sensitivity) against the false positive rate ($1 - \text{specificity}$). A higher AUC (Area Under the Curve) indicates better model performance. The ROC curve in Fig. 3, illustrating the trade-off between sensitivity and specificity, and the PR curve in Fig. 4, highlighting the balance between precision and recall, provide further insights into the model's performance. Both curves approach the optimal point, indicating high performance. Future improvements focus on increasing recall to minimize missed CHD cases while maintaining high precision and specificity.

In Fig. 5, the confusion matrix shows that out of 50 cases, the model accurately identifies 23 CHD cases and 18 non-CHD cases. The confusion matrix provides a more detailed insight into the model's performance:

True Positives (TP): 23 (patients correctly identified as having CHD)

False Positives (FP): 3 (patients incorrectly identified as having CHD)

False Negatives (FN): 6 (patients with CHD incorrectly identified as not having CHD)

True Negatives (TN): 18 (patients correctly identified as not having CHD).

Table II compares the proposed nonlinear control-based CHD model with standard statistical and machine learning approaches. The proposed method achieves the highest accuracy (82%) and F1-score (0.83), demonstrating improved diagnostic consistency compared with logistic regression [18], SVM [19], and linear state-space models.

This improvement is attributed to the nonlinear dynamic representation of patient health states and the stability-guaranteed feedback controller, which captures temporal evolution of risk factors rather than relying on static classification.

These results confirm that incorporating nonlinear control dynamics improves both prediction accuracy and robustness. All models were trained using the same training dataset ($N=102$) and evaluated on the identical testing dataset ($N=50$).

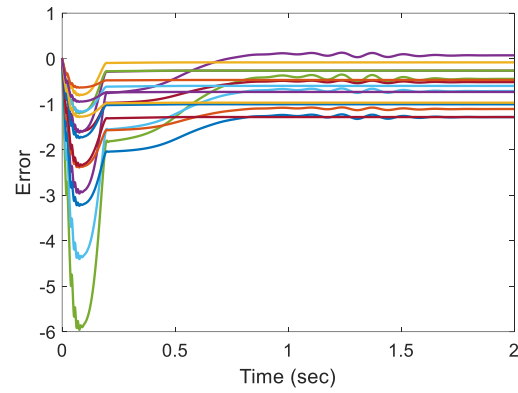


Fig. 1. Error Decay.

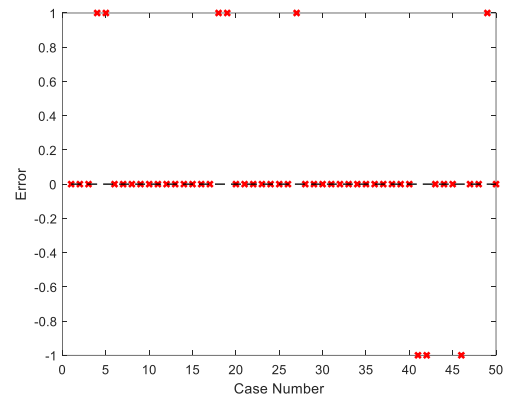


Fig. 2. : Error Analysis.

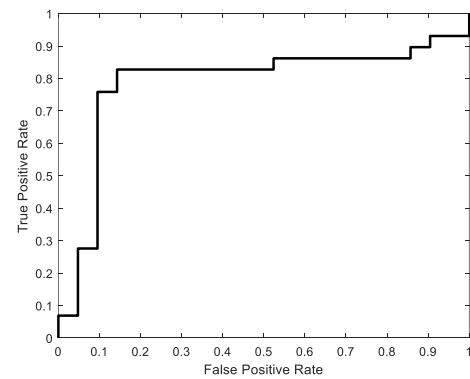


Fig. 3. ROC Curve for the CHD Diagnosis Model.

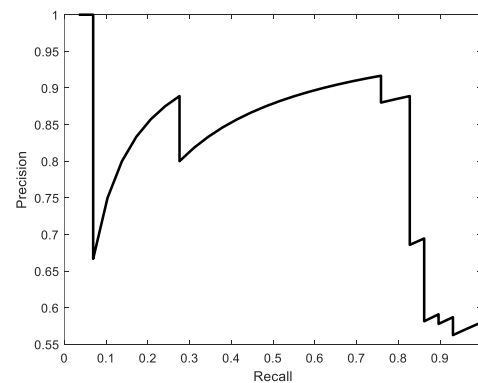


Fig. 4. PR Curve for the CHD Diagnosis Model.

TABLE II : Comparison with baseline diagnostic models on the testing dataset ($N = 50$)

Model	Accuracy	Precision	Recall	Specificity	F1-score
Logistic Regression [18]	0.76	0.81	0.72	0.80	0.76
Support Vector Machine[19]	0.78	0.84	0.74	0.82	0.79
Linear State-Space Model	0.74	0.79	0.70	0.78	0.74
Proposed Model	0.82	0.88	0.79	0.86	0.83



Fig. 5. Confusion Matrix for the CHD Diagnosis Model. The confusion matrix corresponds to the test set (50 samples).

V. Conclusion

This paper presents a hybrid control framework for coronary heart disease (CHD) risk management, where multiple patient health metrics are modeled as interconnected subsystems and controlled using a multi-rate, nonlinear feedback approach. The proposed framework effectively addresses inherent nonlinearities in physiological dynamics, mitigates the impact of disturbances, and ensures closed-loop stability and performance, as demonstrated through comprehensive simulation studies. The results highlight the framework's ability to coordinate fast and slow interventions, maintain risk factors within safe bounds, and adapt to varying patient conditions. Future research will focus on extending the methodology with Type II fuzzy controllers to better handle uncertainty and inter-patient variability, as well as implementing and validating the approach in real-world clinical settings to assess its practical utility and scalability.

Ethics Statement

The clinical data used in this study were collected retrospectively from multiple hospitals in Iran during a research project. Prior to analysis, all records were fully anonymized and stripped of any personally identifiable information. The study did not involve direct patient interaction, intervention, or modification of clinical treatment. The data were used exclusively for methodological development and control system analysis purposes.

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