International Journal Of
Industrial Electronics Control and Optimization
Print ISSN: 3517-2645
Online ISSN: 3568-2645
Homepage: https://ieco.usb.ac.ir/

Adaptive Fractional-Order Consensus Control of Cyber-Physical Power Systems in the Presence of Unbounded Perturbations

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Article Info

Article type:
Research Article

Article history:
Received: 22-May-2024
Received in revised form: 31-August-2024
Accepted: 28-September-2024
Published online:22-June-2025

Keywords:
Cyber-Physical Power Systems,
Fractional-order,
multi agent systems,
Sliding-mode control,
Adaptive control.

ABSTRACT

The consensus of Cyber-Physical Power Systems (CPPSs), where generators agree on common desired rotor angles and speeds, is vital for maintaining system stability and efficiency. This study explores this consensus using fractional-order multi-agent systems, offering advantages over traditional methods. CPPSs often encounter issues like faults, uncertainties, disturbances, and cyber-attacks. To address these, a new Adaptive Fractional-Order Sliding Mode Controller (AFOSMC) is proposed, designed to achieve consensus despite unknown nonlinear functional upper bounds characterizing system perturbations. The AFOSMC uses stable adaptive laws to determine these unknown coefficients, ensuring robust performance even under adverse conditions. It outperforms conventional Integer-Order counterparts by reducing chattering and enabling faster convergence during the initial phase of CPPS operations. The AFOSMC also ensures finite-time convergence to the sliding surface, enhancing system responsiveness and stability. The controller's stability is rigorously proven using Lyapunov's theorem. Finally, extensive simulations demonstrate the practical benefits of the AFOSMC, and comparisons with recent research highlight its superior performance in robustness and efficiency.

I. INTRODUCTION

a) Introduction and Literature review

CPPSs, introduced as a convergence of cyber-physical systems and power systems, integrate various computing devices, data acquisition instruments, and physical components through communication and transmission networks [1]. These systems leverage computing capabilities, data acquisition technologies, and physical devices to enhance the efficiency, reliability, and overall performance of power systems. The integration of sophisticated technologies in CPPSs introduces technical and security challenges [2,3]. These challenges arise from the intricate interplay between the cyber and physical components, requiring robust solutions to ensure the resilience and integrity of the power systems. Research in CPPSs spans various domains, including traffic networks and aviation electric power systems [4-7]. The emphasis on stability stems from its critical role in preventing problems such as voltage collapse and system failure. The overall goal is to leverage the benefits of CPPSs while effectively managing the associated complexities and challenges. Stability is a primary concern for CPPSs and is often addressed through the lenses of supply and demand balance and rotor angle stability [8,9]. In [10] a robust control scheme based on H_∞ performance index is presented for CPPSs which is based on attack estimation. However, the upper bound of disturbance is considered known in this paper. The supply-demand balance ensures a harmonious match between electricity production and consumption across diverse networks. On the other hand, rotor angle stability is crucial for the smooth and efficient operation of CPPSs, allowing each generator to attain a time-varying common rotor angle and speed. These stability aspects are viewed through the framework of consensus theory [11-12]. Consensus theory plays a pivotal role in addressing stability challenges in CPPSs [11-12]. By employing consensus theory, researchers aim to achieve agreement and coordination among different components within the system. This approach facilitates the attainment of a common understanding or state, ensuring the

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Publisher: University of Sistan and Baluchestan

stable and synchronized functioning of the diverse elements within CPPS. In summary, CPPSs represent an innovative paradigm in power systems, integrating advanced technologies to enhance performance [1]. The challenges and research focus on stability underscore the importance of developing robust solutions to ensure the reliable and secure operation of these complex systems.

Sliding Mode Control (SMC) stands out as a nonlinear variable structure control method known for its swift response and robustness against system uncertainties and disturbances [13]. Fractional order equations can also be used for achieving robust control. For example in [14] a variable structure of fractional-order equations is presented which is using floated pole controller in order to achieve robust approach. Using fractional order equations in SMC, in recent years, numerous studies have delved into the stabilization of nonlinear fractional-order systems, with a predominant reliance on robust SMC [13-22]. In a study by [15], a sliding mode control law was developed based on a back-stepping technique for nonlinear fractional-order systems. This innovative approach achieved external disturbances rejection through the application of a new fractional-order back-stepping technique. Another noteworthy contribution is found in [16], where a fractional-order sliding mode control, utilizing the Particle Swarm Optimization (PSO) algorithm, was designed for a specific class of nonlinear fractional-order systems. Consideration of uncertainty in the linear parts of a nonlinear fractional-order system was addressed in [17]. Stability in the face of uncertainty was guaranteed by defining an integer-order sliding surface. In [18] a robust and adaptive terminal SMC is designed for Cyber-Physical systems which are exposed to external disturbances. By considering attacks as false data injections, an even triggered adaptive SMC in also presented in [19] for controlling Cyber-Physical systems. SMC in also combined with adaptive fuzzy fractional order concept in [20] to achieve stability in an uncertain nonlinear system. The weakness of all these references is to know about disturbance or its upper bound and considering this upper bound as a simple scalar value. Output feedback sliding mode control for nonlinear fractional-order systems was the focal point in [21]. The study provided a necessary and sufficient condition for the existence of the sliding surface in the form of a Linear Matrix Inequality (LMI).

These studies collectively showcase the versatility and effectiveness of Sliding Mode Control, particularly in stabilizing nonlinear fractional-order systems amidst uncertainties and disturbances. The integration of innovative techniques, such as back-stepping, PSO algorithms, and LMI, further enriches the applicability and robustness of SMC in addressing the complexities inherent in these systems. In the study presented in [22], the focus is on a second-order system where the upper band of disturbance and its derivatives are assumed to be known. The sliding surface is carefully defined, and the stability of the sliding mode control law is subsequently demonstrated.

A distinctive approach is taken in [23] for a specific class of nonlinear fractional-order systems with nonlinear dynamics. Here, a sliding mode observer is employed to estimate the states, and a novel sliding mode control law is utilized to converge the system states to zero. A recent contribution in [24] introduces a sliding mode active disturbance rejection control for fractional-order nonlinear systems characterized by unknown states and disturbances. This innovative method combines a sliding mode extended state observer with sliding mode control, providing a comprehensive solution. In [25], an attempt is made to design SMC based Linear Quadratic Regulator (LQR) for nonlinear fractional-order systems (FOSs). This paper presents a fractional-order switching surface and some unique conditions are derived by using Lyapunov theory and linear matrix inequalities (LMIs) to guarantee the stability of FOS. An observer based distributed sliding mode secondary control is also presented in [26] to achieve consensus in DC microgrids that are under cyber-attacks. In this method, the detected false signals have been compensated using the control variables in the secondary stage control of the micro grid. The work presented in [27] introduces a fractional integral sliding surface, leveraging the Composite Learning Sliding Mode Control method to manage fractional-order nonlinear systems afflicted with actuator faults. This method integrates the benefits of fractional calculus and sliding mode control techniques to achieve effective control in the presence of actuator faults. However, dealing with modeling uncertainties and external disturbances with unknown upper bounds, especially when the upper bounds are a function of system states, is still an open problem.

b) Contribution and novelty of paper

It is crucial to highlight that, despite extensive research in the control of nonlinear fractional-order systems in the presence of uncertainties and disturbances, two key challenges persist in most studies. Firstly, many studies fail to address the stabilization problem for the general form of nonlinear affine fractional-order systems. Secondly, in most cases, the upper bounds of uncertainties are assumed to be known. However, in practical applications, the limits of model uncertainties and external disturbances are typically unknown, adding a layer of complexity to real-world systems.

In this context, the sliding mode control of the general form of fractional-order nonlinear systems, where the upper bounds of

uncertainties and external disturbances are unknown (a more realistic and practical assumption), remains a challenging problem. This specific issue is the focal point of discussion and investigation in the current paper. To illustrate the practical implications of the proposed method, the nonlinear fractional-order model of multi agent Cyber-Physical Power System (CPPS) is considered as an application. This application serves as a tangible example to showcase the effectiveness and relevance of the method in real-world scenarios, emphasizing the need for robust control strategies in the face of uncertainties and disturbances.

Therefore, the novelty and the main contribution of this paper can be summarized as achieving consensus in Cyber-Physical Power Systems in a more realistic and more applicable situation that have not been considered in previous studies:

1) Considering the fractional order model of agents in a Cyber-Physical Power System (CPPS) which is a more accurate description of the system.
2) Introducing a novel concept, postulating that the upper bound of external disturbances and model uncertainties transcends mere a known scalar value like other references, but an unknown nonlinear function of CPPS state variables.
3) Designing a new Adaptive Fractional-Order Sliding Mode Controller (AFOSMC) to achieve consensus in CPPS, in which a specific adaptive procedure is created to predict the external disturbances' functional upper bound coefficients.
4) The presented method can be easily extended for the stabilization problem of the general form of nonlinear affine fractional-order systems.

This paper is organized as follows. Section II describes the CPPS model considering cyber-attacks, uncertainties and external disturbances. The main results of the paper are presented in Section III. In this section, a new adaptive sliding mode controller is designed to guarantee asymptotic stability for the CPPS system. A chattering avoidance technique is also given in this section. Numerical examples are provided in Section IV to confirm the effectiveness of the designed controller. Finally, conclusions close the paper in Section V.

II. PROBLEM STATEMENT

a) CPPS Network Mathematical Model

In this section, we delve into the stability analysis of the rotor angle within the Cyber-Physical Power System (CPPS). The CPPS can be effectively modeled using the following formulas [28]:

$$\dot{\delta}_i = \omega_i \tag{1}$$

$$\begin{cases} \dot{\omega}_i = -\dfrac{D_i}{M_i}\omega_i + \dfrac{1}{M_i}(P_{mi} - P_{ei}) \\ \dot{E}_i' = F_i\dot{E}_i' + L_i(\delta_i)V_i + H_iE_{fi} \\ P_{ei} = V_i^T[R_i(\delta_i)V_i + T_i(\delta_i)E_i'] \\ P_{mi} = \dfrac{B_{mi}P}{A_{mi}P}u_{mi} \\ E_{fi} = \dfrac{B_{fi}P}{A_{fi}P}u_{fi} \\ V = [Y - R(\delta)]^{-1}T(\delta)E' \end{cases} \tag{2}$$

where δ_i is the rotor angle, ω_i is the rotor speed, D_i is the damping coefficient, M_i is the inertia constant, P_{mi} is the mechanical power, P_{ei} is the electrical power, E_i' is the internal bus voltage vector, V is the generator terminal voltage vector. Also, $B_{mi}P$ and $A_{mi}P$ are respectively the polynomials of the numerator and denominator of the mechanical power transfer function and $B_{fi}P$ and $A_{fi}P$ are the same case for bus voltage transfer function. Also u_{mi} and u_{fi} are respectively turbine–governor subsystem input and bus voltage input. Additionally, Y is a constant matrix and $R(\delta)$ and $T(\delta)$ are matrices constructed from elements $R_i(\delta_i)$ and $T_i(\delta_i)$ respectively. The detailed descriptions of $R_i(\delta_i)$, $T_i(\delta_i)F_i$ and $H_i$can be found in [28]. These matrices and parameters play crucial roles in characterizing the dynamics of the CPPS. A classical coherency criterion asserts that a generator pair (i, j) is deemed coherent if

$$\begin{cases} \lim\limits_{t\to\infty}\|\delta_i - \delta_j\| = 0 \\ \lim\limits_{t\to\infty}\|\omega_i - \omega_j\| = 0 \end{cases} \tag{3}$$

We can treat the rotor angle stability of the Cyber-Physical Power System (CPPS) as a consensus problem for a second-order multi-agent system. By utilizing a multi-agent system, we can delve into the study of rotor angle stability.

Considering equations (1) and (2), if the controller input is selected as follows:

$$\bar{P}_{mi} = M_iu_i' + D_i\omega_i + P_{ei} \tag{4}$$

Introducing u_i' as the new input to be determined, the differential equations governing the rotor angle and rotor speed in the Cyber-Physical Power System (CPPS) are expressed as follows:

$$\begin{cases} \dot{\delta}_i = \omega_i \\ \dot{\omega}_i = u_i' \end{cases} \tag{5}$$

Hence, the design of u_i' holds significance in this context. Subsequently, we explore the rotor angle stability of the Cyber-Physical Power System (CPPS) using a multi-agent system model.

In such multi-agent system, communication among the agents is described by a weighted directed graph G with N nodes. In

this graph, each vertex corresponds to an agent and $w_{ij} \geq 0$ denotes the weight of the edge between *i*th and *jth* agents. This weight is a value that determines the connection strength between *i*th and *j*th agents in the swarm system. Graph G is described by its adjacency matrix given as follows [29].

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & \ddots & \vdots \\ w_{N1} & \cdots & w_{NN} \end{bmatrix} \quad (6)$$

Fractional-order calculus is adopted for its generality, as an extension of the integer-order calculus. The fractional-order format of rotor equations in the Cyber-Physical Power System (CPPS) in (5) is considered as follows [30]

$$\begin{cases} D^{\alpha}\delta_i = \omega_i \\ D^{\alpha}\omega_i = u_i \quad i = 1,2,\dots,N \end{cases} \quad (7)$$

where $.\alpha \in (0,1], \delta_i, \omega_i \in R^n$. In this equation, ${}_0D_t^a$ is the fractional-order differentiator, which signifies a generalized extension of the conventional differential and integral operators, as defined in reference [31]:

$${}_0D_t^a = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & , \alpha > 0 \\ 1 & , \alpha = 0 \\ \int_0^t (d\tau)^{-\alpha} & , \alpha < 0 \end{cases} \quad (8)$$

In this context, α denotes the fractional-order, which can assume the form of a complex number. There are several definitions for general fractional and integration differentiation. Because of the congruence of initial conditions between fractional-order differential equations using Caputo derivatives and their integer-order counterparts, the Caputo fractional derivative is commonly used in fractional-order system models. As documented in references [31-32], the Caputo fractional derivative operator is expressed as follows:

$${}_0D_t^{\alpha}x(t) = \frac{1}{\Gamma(\lceil\alpha\rceil - \alpha)}\int_0^t \frac{x^{(\lceil\alpha\rceil)}(\tau)}{(t-\tau)^{\alpha-\lceil\alpha\rceil+1}}d\tau,\ 0 < \alpha \notin Z_+ \quad (9)$$

in which $\Gamma(.)$ represents the Gamma function, and $\lceil\alpha\rceil$ signifies the mapping of α to the smallest integer greater than or equal to α, as expounded in [31-33].

Now, with definitions $x_{i1} = \delta_i$ and $x_{i2} = \omega_i$, equation (6) can be expressed as follows:

$$\begin{cases} D^{\alpha}x_{i1} = x_{i2} \\ D^{\alpha}x_{i2} = u_i \end{cases} \quad (10)$$

And, considering the vector $x_i = [x_{i1}, x_{i2}] \in R^2$,we have:

$$D^{\alpha}x_i = Ax_i + Bu_i \quad (11)$$

In this expression, we have:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (12)$$

Now, let's introduce disturbances, encompassing uncertainties, disturbance modeling, cyberattacks, and input bias faults into the multi-agent system. Hence, the model of factors exposed to disturbances can be expressed as:

$$D^{\alpha}x_i = Ax_i + B_i[u_i(t) + f_{ia}(t) + f_i(t)] + \Delta f_i(x) + d_i(t) \quad (13)$$

In the provided expression, $d_i(t)$ characterizes external disturbances and is constrained by $\|d_i(t)\| < \alpha$, where α denotes the upper bound of disturbances. $f_{ia}(t)$ represents the input bias error of each factor in the system, with $\|f_{ia}(t)\| < \beta$ and $\Delta f_i(x)$ accounts for modeling uncertainties and unmodeled dynamics, assumed to be limited as $\|\Delta f_i(x)\| < \delta$. Additionally, it is posited that cyberattacks $f_i(t)$ are bounded by $\|f_i(t)\| < \xi$, where ξ signifies the maximum intensity value of cyberattacks applicable to an energy source in the CPPS.

Model (13) can be reformulated as follows:

$$D^{\alpha}x_i = Ax_i + B_iu_i(t) + h_i(x,t) \quad (14)$$

In this context, the components are defined as follows:

$$h_i(x,t) = B_if_{ia}(t) + B_if_i(t) + \Delta f_i(x) + d_i(t) \quad (15)$$

The term $h_i(x,t)$ serves as a representation of bias, collective error, cyberattacks, modeling uncertainties, and disturbances within each factor. In essence, $h_i(x,t)$ encapsulates the summation of these elements, colloquially referred to as "disturbance." The distinctive characteristic of the various components constituting this disturbance is their unknown nature and their confinement within limited upper bounds. The system represented by equation (15) is denoted as a model of distributed energy resources with disturbances in a CPPS.

The objective of controller design is to formulate a distributed control law such that, accounting for the disturbances outlined in the model of agents in the CPPS within equation (15), the following goal is achieved in terms of consensus:

$$\lim_{t\to\infty}\|x_i - x_{ref}\| = 0, \quad \forall i \neq j \quad (16)$$

b) Consensus error model in CPPS Network

After formulating the agent model, representing the distributed energy resources in a CPPS under disturbances as given in equation (15), the proposed generalized secondary control law is presented to attain consensus and drive the agents toward x_{ref}. Drawing inspiration from [26] with slight modifications, we propose the following expression, highlighting the incorporation of information from neighboring agents in the graph:

$$u_i = -\sum_{j=1}^{N} w_{ij}(x_i - x_j) - g_i(x_i - x_{ref}) + u_{a_i} \quad i\epsilon\{1,2,\dots N\} \tag{17}$$

The scalar value g_i serves as an indicator distinguishing whether the i-th node functions as a leader or a follower within the CPPS, with $g_i > 0$ for $i = 1,2,\dots,l$ denoting leader nodes and $g_i = 0$ for $i = l+1,\dots,N$ indicating follower nodes. Consequently, leader nodes are orchestrated to adhere to the reference value, while follower nodes are steered towards achieving consensus.

Upon applying the control input from equation (17) to the agent model in the CPPS outlined in equation (15), the resulting expression is as follows:

$$D^{\alpha}x_i = Ax_i - B\sum_{j=1}^{N} w_{ij}(x_i - x_j) - Bg_i(x_i - x_{ref}) + Bu_{a_i} + h_i(x,t) \tag{18}$$

By defining the difference between the state variable of each distributed energy source x_i and the reference value x_{ref}, we introduce the state variable error as $e_i = x_i - x_{ref}$. Consequently, the equation governing the error is expressed as follows:

$$\begin{aligned} D^{\alpha}e_i &= D^{\alpha}x_i - D^{\alpha}x_{ref} \\ &= Ax_i - B\sum_{j=1}^{N} w_{ij}(x_i - x_j) - Bg_i(x_i - x_{ref}) + Bu_{a_i} + h_i(x,t) - D^{\alpha}x_{ref} \end{aligned} \tag{19}$$

By incorporating both addition and subtraction of Ax_{ref} into equation (19) and taking into account the generalized error state vector in the distributed resource system as $e = [e_1,\dots,e_N]^T$, the disturbances vector as $H = [h_1(x,t),\dots,h_N(x,t)]^T$, and the secondary input vector as $U = [u_{a1},\dots,u_{aN}]^T$, the dynamics of the system and the dynamics of errors in the state variables can be expressed as follows:

$$D^{\alpha}e = (\tilde{A} - \tilde{B}(L+G))e + \tilde{B}U + H + \Phi \tag{20}$$

where L is the Laplacian graph matrix, $\Phi = Ax_{ref} - D^{\alpha}x_{ref}[1,\dots,1]^T$ and we have:

$$\tilde{B} = \begin{bmatrix} B & 0 & 0 & \cdots & 0 \\ 0 & B & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & B \end{bmatrix} \in R^{2N\times N} \tag{21}$$

$$G = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ 0 & g_2 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & g_N \end{bmatrix} \in R^{N\times N}$$

$$\tilde{A} = \begin{bmatrix} A & 0 & 0 & \cdots & 0 \\ 0 & A & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & A \end{bmatrix} \in R^{2N\times 2N}$$

By defining $\hat{A} = \tilde{A} - \tilde{B}(L+G)$, the model described in equation (16) can be expressed as follows:

$$D^{\alpha}e = \hat{A}e + \tilde{B}U + \Phi + H \tag{22}$$

This equation provides a concise representation of the error dynamics in the distributed resource system, incorporating the Laplacian graph matrix, disturbance vector, and secondary input vector.

Note 1: It should be noted that in order to obtain (22), we have used and extended the procedure that is presented in [34] for linear fractional order multiagent systems. We extended it from a linear model to a nonlinear model of agents as shown in equation (13).

c) Objective of the study

Reflect upon the forthcoming statement concerning CPPS system model uncertainty and external disturbances:

Remark 1. The model uncertainties $\Delta f_i(x)s$ are contingent on the CPPS system states. Furthermore, the situation and states of a CPPS is influenced by external disturbances and more importantly, the magnitude and how the cyber attack $f_i(t)$ is applied to the system, depend on the current state of the CPPS system. Hence, this study introduces a novel concept, postulating that the upper limit encompassing external disturbances and model uncertainties in equation (15) transcends mere scalar value but the form of a nonlinear function contingent upon CPPS state variables. Concurrent with this, stable adaptive laws will be established to estimate the coefficients governing this upper boundary function, assuming their initial unknown status. This causes the CPPS reference tracking issue to have more appropriate and practical assumptions. Therefore, consider the following assumption:

Assumption 1. Let us assume that the upper limit for the vector encompassing model uncertainties and external disturbances within the CPPS model is bounded, as illustrated in equation (22):

$$\|H(x(t),t)\| \le \alpha h(x(t),t) + \beta \tag{23}$$

in which α and β refer to the unknown values and $h(x(t),t)$ represents an arbitrary nonlinear function contingent upon the CPPS system's state variables.

It is worth mentioning that in [34], the upper bounds of modelling uncertainties and external disturbances were

considered as a linear function of system states norm as $\alpha\|x(t)\| + \beta$. However, the upper bound of the perturbations that is considered here in Assumption 1 is a more general form of what is considered in [34]. Using the assumption in (23), it can even be claimed that the perturbations may be unbounded. The nonlinear CPPS system faces external disturbances and model uncertainties, and this study seeks to build Adaptive Fractional-Order Sliding Mode Controller (AFOSMC) for it. The indicated convergences in (11) would be attained by developing the controllers as mentioned above while taking Assumption 1 into account. Additionally, specific adaptive procedures are created to predict external disturbances' functional upper limit coefficients and model uncertainties in (23). This paper's novelty lies in its approach to addressing the CPPS challenge under realistic conditions, assuming the validity of Assumption 1. In the next section, we introduce AFOSMC to address CPPS reference tracking while upholding this foundational assumption.

Before dealing with the stabilizing controller, consider the following Lemma:

Lemma 1 [33-34]: The Fractional Order system $D_t^{\alpha}x(t) = Ax(t)$, where $x(t) \in R^n$ for $t \geq 0$, $A \in R^{n\times n}$, and D_t^{α} denotes the Caputo fractional derivative operator is asymptotically stable if and only if $|arg(\lambda)| > \alpha\frac{\pi}{2}$ for each eigenvalue λ of matrix A (In this case the matrix A is called an α-Hurwitz matrix).

III. PROPOSED METHODOLOGY

Fractional-order equations enable controllers with more adjustability than traditional integer-order ones. They effectively reduce chattering in sliding mode control, as shown ahead.

In this section, we create a Fractional Order Sliding Mode surface and then introduce a new Robust-Adaptive Fractional Order Sliding Mode Controller (AFOSMC) to handle CPPS system perturbations, considering model uncertainties, cyber-attacks and external disturbances. We account for the assumption that the upper bound of model uncertainties is an unknown nonlinear function of CPPS system states.

This section also examines the convergence of CPPS system states to the switching surface and presents an innovative approach to mitigate chattering.

a) Sliding manifold and reaching law

Considering the CPPS system in (22), we define an $(N-1)d$-dimensional sliding manifold in the form

$$s(t) = C\int_0^t D_{\tau}^{\alpha}e(\tau)\,d\tau - C\int_0^t (\hat{A} + BK)\,e(\tau)d\tau \qquad (24)$$

where $s(t) = [s_1, s_2, \dots, s_{2N}] \in R^{2N}$ is the sliding manifold vector, $C \in R^{2N\times 2N}$ is an arbitrary matrix in such a way that CB is invertible and $K \in R^{2N\times 2N}$ is designed parameter matrix that will be obtained in the next subsection. According to the sliding mode approach, the switching manifold and its derivative should be equal to zero. To this end, consider the following reaching law [33-34]

$$\dot{s}(t) = -\mu s - \rho\, sgn(s) \qquad (25)$$

where $\mu, \rho \in R^+$ and $sgn(S) = [sgn(s_1), sgn(s_2), \dots, sgn(s_{2N})]^T \in R^{2N}$.

The reaching law is a stable vector first-order equation that is chosen in such a way that the sliding surface $s(t)$, which is the independent variable of this equation, converges to zero. Then, after selecting this reaching law, it is time to design the control law. The control law is chosen in such a way that the equation governing the sliding surface $s(t)$ becomes this reaching law.

As you can see from (25) left side of reaching law is $\dot{s}(t)$. Now, it is easy to take a derivative from the sliding surface in (24) to appear $\dot{s}(t)$. Then, we easily set the result of the derivative of the sliding surface equal to the right side of the reaching law and using this idea, the control law will be easily obtained as in next subsection, although with a few changes.

b) Proposed Adaptive Fractional-Order Sliding Mode Controller (AFOSMC)

The AFOSMC for stabilizing the CPPS system described in equation (15) is introduced using the fractional-order sliding surface outlined in equation (18) as follows:

$$u = (CB)^{-1}(CBKe - C\Phi - \mu s - sgn(s) \times \left(\rho + \hat{\beta}\|C\| + \hat{\alpha}\|C\|h(x(t),t)\right)) \qquad (26)$$

where $\hat{\alpha}$ and $\hat{\beta}$ come from the following adaptive rules and represent estimates of the unknown parameters in (23):

$$\dot{\hat{\alpha}} = \eta_1\|C\|s^T\, sgn(s)\, h(x,t)$$
$$\dot{\hat{\beta}} = \eta_2\|C\|s^T\, sgn(s) \qquad (27)$$

where η_1 and η_2 stand for two arbitrary, positive constant values representing adaptive gains. The following theorem expresses the primary outcome of the current study, which is the design of AFOSMC.

Theorem 1. Consider the CPPS system in (22), which is affected by external disturbances, Cyber-attacks and model uncertainty. Assuming the validity of Assumption 1, where the upper bound on model uncertainty and external disturbances depends on CPPS states with unknown coefficients, the reference tracking errors within the CPPS system (22) now asymptotically converges to zero. This is achieved using AFOSMC as defined in (26)-(27), with positive constant values $\eta_1, \eta_2 > 0$, $\mu, \rho > 0$ and constant matrix K that

satisfies $\left|\arg\left(eig(\hat{A}+BK)\right)\right| \geq \alpha\frac{\pi}{2}$.

Proof: Consider the following proposition for the Lyapunov function.

$$V = \frac{1}{2}s^T s + \frac{1}{2\eta_1}\tilde{\alpha}^2 + \frac{1}{2\eta_2}\tilde{\beta}^2 \tag{28}$$

where $\tilde{\alpha} = \hat{\alpha} - \alpha$ and $\tilde{\beta} = \hat{\beta} - \beta$ represent adaptive parameter errors. Now, differentiating (28) and substituting $\dot{s}$ from (24) yields:

$$\dot{V} = s^T\dot{s} + \frac{1}{\eta_1}\tilde{\alpha}\dot{\tilde{\alpha}} + \frac{1}{\eta_2}\tilde{\beta}\dot{\tilde{\beta}} = s^T\left(CD_\tau^\alpha e - C\left(\hat{A}+BK\right)e\right) + \frac{1}{\eta_1}\tilde{\alpha}\dot{\hat{\alpha}} + \frac{1}{\eta_2}\tilde{\beta}\dot{\hat{\beta}} \tag{29}$$

Substituting $D_\tau^\alpha e$ from the CPPS system in (22) into (29) and given that $\dot{\tilde{\alpha}} = \dot{\hat{\alpha}}$and $\dot{\tilde{\beta}} = \dot{\hat{\beta}}$, results in

$$\dot{V} = s^T\left(C\hat{A}e + CBU + C\Phi + CH - C\left(\hat{A}+BK\right)e\right) + \frac{1}{\eta_1}\tilde{\alpha}\dot{\hat{\alpha}} + \frac{1}{\eta_2}\tilde{\beta}\dot{\hat{\beta}} \tag{30}$$

Using the suggested AFOSMC from (26) in combination with the adaptive laws from (27) will results in:

$$\dot{V} = s^T\left(-\mu s - sgn(s)\left(\rho + \hat{\beta}\|C\| + \hat{\alpha}\|C\|h(x(t),t)\right) + CH\right) + \frac{1}{\eta_1}(\hat{\alpha}-\alpha)\dot{\hat{\alpha}} + \frac{1}{\eta_2}(\hat{\beta}-\beta)\dot{\hat{\beta}} \tag{31}$$

which results

$$\begin{aligned}\dot{V} = {} & -\mu\|s\|^2 - \rho s^T sgn(s) - \hat{\beta}\|C\|s^T sgn(s) \\ & -s^T\|C\|\hat{\alpha}h(x(t),t)\,sgn(s) + s^T CH(x,t) \\ & +(\hat{\alpha}-\alpha)\|C\|s^T sgn(s)\,h(x,t) \\ & +(\hat{\beta}-\beta)\|C\|s^T sgn(s)\end{aligned} \tag{32}$$

By eliminating similar terms and considering $s^T sgn(s) = \|s\|$ we obtain:

$$\begin{aligned}\dot{V} = {} & -\mu\|s\|^2 - \rho\|s\| + s^T CH(x,t) \\ & -\alpha\|C\|\|s\|h(x,t) - \beta\|C\|\|s\|\end{aligned} \tag{33}$$

In addition, using $s^T CH(x,t) \leq \|s^T CH(x,t)\| \leq \|s\|\|C\|\|H(x,t)\|$ and, then, the inequality in (23) will yields:

$$\begin{aligned}\dot{V} & \leq -\mu\|s\|^2 - \rho\|s\| + \|s\|\|C\|\|H(x,t)\| \\ & \quad -\alpha\|C\|\|s\|h(x,t) - \beta\|C\|\|s\| \\ & \leq -\mu\|s\|^2 - \rho\|s\| + \|s\|\|C\|(\alpha h(x,t)+\beta) \\ & -\alpha\|C\|\|s\|h(x,t) - \beta\|C\|\|s\| = -\mu\|s\|^2 - \rho\|s\|\end{aligned} \tag{34}$$

Consequently, with the positivity of μ and ρ as well as in accordance with Barbalat's lemma $s(t)$ converges to zero as $t \to \infty$ This, in turn, leads to the convergence of the reference error in the CPPS system to zero, as specified by the sliding surface equation in (24) and utilizing Lemma 1.

Remark 2. In the AFOSMC presented in equations (26)-(27), a sign function, which is a discontinuous function centered around $s = 0$, has been utilized. This function can lead to highly precarious situations for the CPPS system, characterized by high-frequency oscillations, commonly referred to as chattering, within the controller signal. In a sliding mode controller, chattering may be minimized in a number of methods, including by applying saturation or a hyperbolic tangent function. Although using these continuous functions to approximate the sign function may lessen chattering, the closed-loop system's controller performance may suffer. To mitigate chattering, we introduce a novel successive approximation function similar to what was outlined in [35]. Consequently, the term $\rho sgn(s)$, $\rho > 0$ is computed as follows:

$$\rho sgn(S) \cong \frac{\rho^2 S}{\rho\|S\| + \sigma(t)} \tag{35}$$

in which $\sigma(t) > 0$ represents a positively bounded function with the property $\int_0^\infty \sigma(t)dt < \infty$. An example of such a function $\sigma(t) > 0$ is as follows

$$\sigma(t) = \frac{1}{1+t^n}, n \geq 2 \tag{36}$$

The approximation in equation (35) is continuous and will continue to be continuous throughout time. Therefore, the chattering will lessen. The function $\sigma(t)$ converges to zero rapidly, rendering the approximation closer to the sign function with greater precision compared to alternative approximations such as the hyperbolic tangent function.

Lemma 2. Utilizing the reaching law as $\dot{s} = -\mu s - \rho sign(s)$ defined in equation (25), which is employed in AFOSMC, the CPPS system states reach the sliding surface in finite time t^* as follows:

$$t^* = \frac{1}{\mu}ln\left(\frac{\rho + \mu|s(0)|}{\rho}\right), \tag{37}$$

Proof: For the initial states of the sliding surface with$s(0) > 0$ or $s(0) < 0$, the differential equation governing the reaching law is derived as follows:

$$\begin{cases}\dot{s} = -\mu s - \rho, & s(0) > 0 \\ \dot{s} = -\mu s + \rho, & s(0) < 0\end{cases} \tag{38}$$

The solution to the above equations is as follows:

$$\begin{cases}s(t) = \left(s(0) + \frac{\rho}{\mu}\right)e^{-\mu t} - \frac{\rho}{\mu}, & s(0) > 0 \\ s(t) = \left(s(0) - \frac{\rho}{\mu}\right)e^{-\mu t} + \frac{\rho}{\mu}, & s(0) < 0\end{cases} \tag{39}$$

Now, given that sliding surface has $i = 1,\ldots,2N$ components, by setting $s_i(t_i) = 0$, it is found that

$$t_i = \frac{1}{\mu} ln(\frac{\rho + \mu|s_i(0)|}{\rho}), \quad (40)$$

Now, the final finite time is:

$$t^* = max_{i=1..2N}(t_i), i = 1, \ldots, 2N \quad (41)$$

This demonstrates that the system states reach the sliding surface within a finite time denoted by t^*, depending on the initial value of the sliding surface.

Remark 3. We had claimed that the method presented in this paper can be easily extended to solve the stabilization problem of a more general form meaning nonlinear affined fractional order systems i.e. $D^\alpha x = f(x) + g(x)u + H$. We first add and subtract the desired linear value Ax to above affine system and then we have:

$$D^\alpha x = f(x) + g(x)u + H + Ax - Ax \quad (42)$$

It should be said that the controller presented in Theorem 1 is designed for achieving asymptotic stability in system in (22). The point is that the fractional order nonlinear affine system in (42) can be easily transformed to system in (22), by considering the following change of variables:

$$x \to e, \; A \to \hat{A}, \; g(x) \to \tilde{B}, \quad (43)$$
$$(f(x) - Ax) \to \Phi(x)$$

IV. SIMULATION RESULTS

In this section, the effectiveness of the proposed controller is verified through numerical examples. The numerical simulations in this section use the Adams-type predictor-corrector method suggested in [36] for obtaining the solution of fractional-order differential equations.

This segment illustrates the efficacy of the main findings through a specific numerical scenario. Consider a 5-generator cyber-physical power system with 14 buses, as depicted in Figure 1. The data pertinent to this setup is sourced from the IEEE standard test system for such configurations. Each generator is linked to an agent enabling the exchange of information among them, as illustrated in Figure 2. The interconnected cyber and physical networks are connected via distinct arrow types.

With a distributed multi-agent approach, these networks are seamlessly integrated. If each agent (generator) contains two tiers of information, denoted as $N = 5, n = 2$, the rotor angle and rotor speed profiles for the cyber-physical power systems can be examined in Figures 3(a) and 3(b) respectively. Observing the plots indicates the convergence of rotor angles and speeds toward a shared dynamic value.

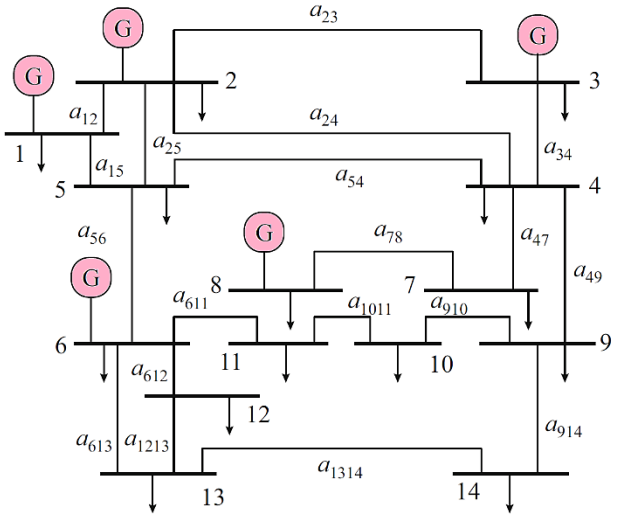


Fig. 1: A 5-generator 14-bus CPPS for simulation [30]

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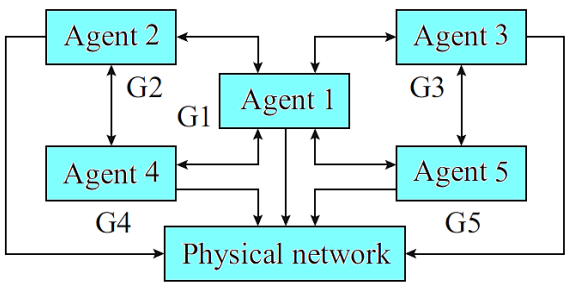
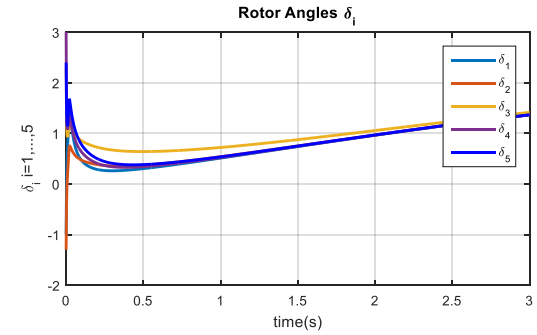


Fig. 2: Graph representation of 5-generator 14-bus CPPS in Figure 1 [30]

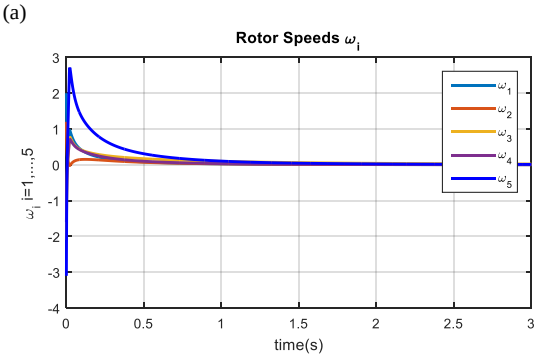
To simulate such a multi-agent cyber-physical power system, the system parameters are considered the same as those presented in [30]. The control parameters of AFOSMC control law, which should be set according to Theorem 1, are considered as follows:

$$\mu = 10, \quad \rho = 3,$$
$$\eta_1 = 2, \quad \eta_2 = 3 \quad (44)$$
$$h(x(t), t) = 5\|x\|$$

The values of sliding plates are shown in Figure 4. This figure also clearly shows that by applying the designed AFOSMC control law, the sliding surfaces converge to zero in a finite time according to Lemma 2. From that moment, the system enters the so-called sliding mode, in which the consensus error converges to zero. Adaptive parameters for employing the AFOSMC control law are presented in Fig. 5. This figure makes it evident that both adaptive parameters tend to specific bounded scalar values, demonstrating the stability of adaptive laws designed in (27).



(a)



(b)

Fig. 3: Rotor angles (a) and Rotor speeds (b) of CPPS in simulation part and achieving consensus among them

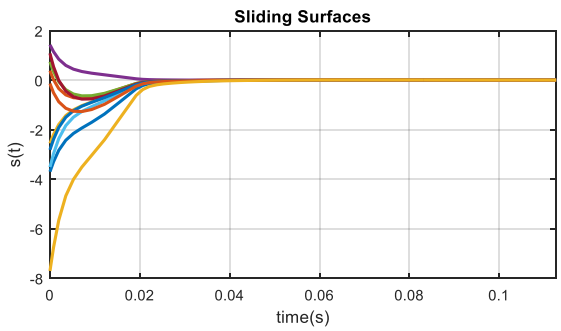
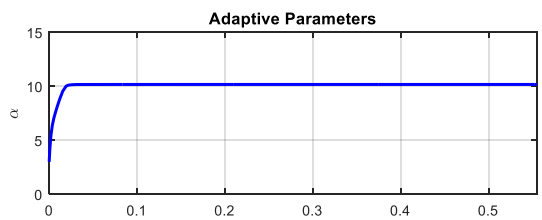
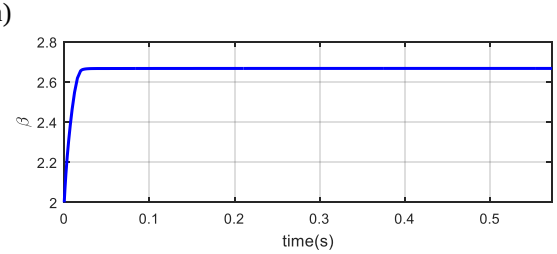


Fig. 4: Sliding surfaces of closed loop CPPs system



(a)



(b)

Fig. 5: Adaptive parameters of control law in (24)

Finally, the control input signals for all agents, i.e. rotors, are shown in Figure 6. The bounded amplitude values of these signals and the convergence to zero as expected, show that this controller has performed well in achieving consensus among the angles and speeds of the rotors in the CPPS system.

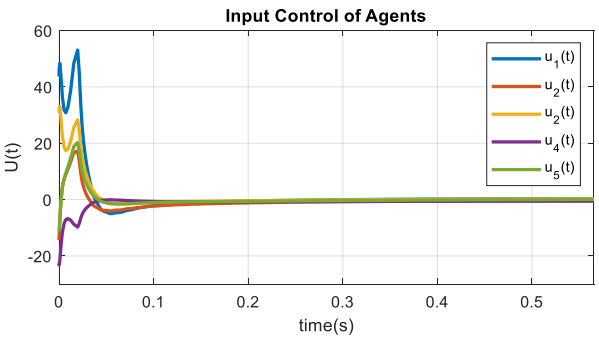
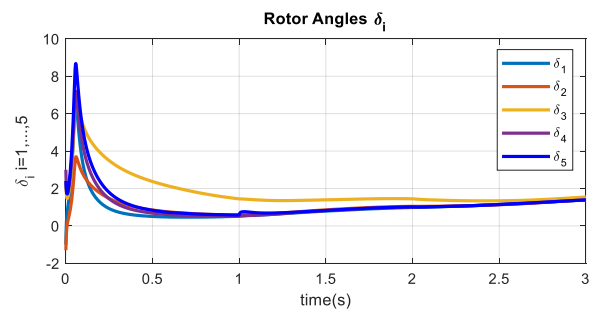
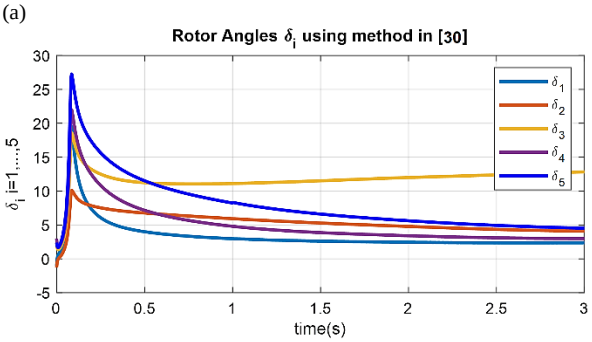


Fig. 6: Control input of rotors in closed loop CPPS system



(a)



(b)

Fig. 7. Consensus in rotor angles of CPPS system by applying: (a) AFOSMC control law designed in this paper and (b) the event triggered method presented in [30]

- *A Comparison with other reference*

In this subsection a comparison with one of the recent related references that is about using event-trigger method to achieve consensus of cyber-physical power systems based on multi-agent systems with communication constraints, is presented [30].

Figures 7 and 8 show the results of comparing the AFOSMC controller proposed in this paper with a recent controller proposed in reference [30] for achieving consensus in fractional-order cyber-physical power systems based on the event-trigger method. To do this, the disturbances, consisting of two cyber-attack signals applied to agent 1 and agent 3, are defined as follows:

$$f_1(t) = \begin{bmatrix} f_{11} \\ f_{12} \end{bmatrix}, \quad f_{11} = \begin{cases} 5, & 0.5 \le t \le 0.7 \\ 0 & otherwise \end{cases},$$
$$f_{12} = \begin{cases} 6, & 1 \le t \le 2 \\ 0 & otherwise \end{cases}, \quad (45)$$
$$f_2(t) = \begin{bmatrix} f_{21} \\ f_{22} \end{bmatrix}, \quad f_{21} = \begin{cases} 4, & 1 \le t \le 3 \\ 0 & otherwise \end{cases},$$
$$f_{22} = \begin{cases} 7, & 0.6 \le t \le 0.9 \\ 0 & otherwise \end{cases}$$

Additionally, the modeling uncertainties in the CPPS system are as follows:

$$\Delta f_i(x) = \begin{bmatrix} 6\sin(4t) + 0.01\sin(2x_{i1}) + 0.01\cos(4x_{i2}) \\ 6\sin(4t) + 0.01\cos(4x_{i1}) + 0.01\sin(2x_{i1}) \end{bmatrix}, \quad (46)$$

Both the method presented in reference [30] and our method consider uncertainties and disturbances as cyber-attacks in the CPPS system. However, Figures 7 and 8 clearly show the superiority of the method described in this paper by comparing the convergence of rotor angles and rotor speeds in achieving consensus. It is clear that the AFOSMC controller proposed in this paper has a better performance with a small overshoot and low rise time. However, in the presence of uncertainties and disturbances with unknown functional upper bounds, the event-trigger control law in [30] is unable to achieve consensus and cannot handle these uncertainties and disturbances.

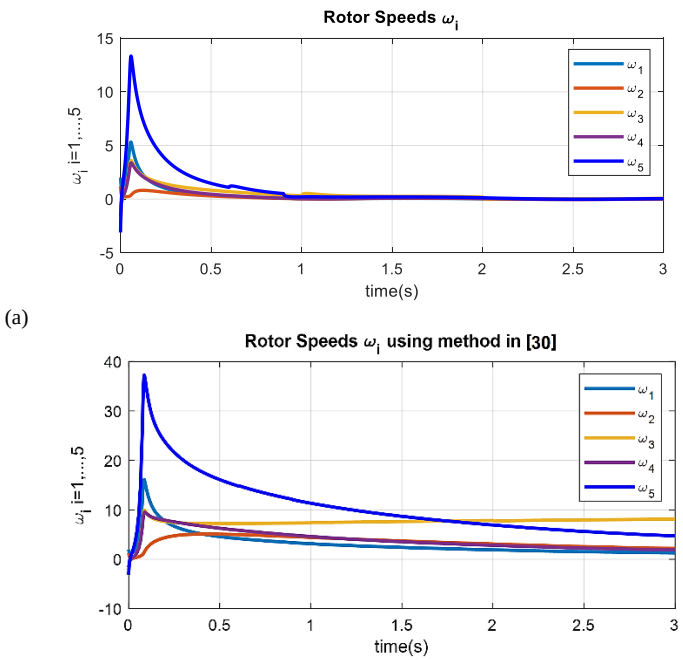


Fig. 8. Consensus in rotor speeds of CPPS system by applying: (a) AFOSMC control law designed in this paper and (b) the event triggered method presented in [30]

The comparison of transient response characteristics in the time domain between reference [30] and the proposed AFOSMC controller is also presented in Table 1. As we can see, in the convergence of both the rotor angles and the rotor speeds, by applying the control law designed in this paper (AFOSMC), both the overshoot and the settling time are less compared to the event-trigger method resented in [30]. This is, of course, next to this important feature that in our method, firstly, unlike reference [30] the upper band of disturbances is assumed to be unknown. And besides, this upper band, is not just a scalar but a nonlinear function of the system states.

TABLE 1 COMPARING THE SIMULATION RESULTS OF APPLYING CONTROL LAW IN [30] WITH OUR CONTROL LAW AFOSMC

Specifications	Controller in [30]	AFOSMC in this paper
Overshoot in Rotor Angles	28	8.7
Overshoot in Rotor Speeds	37	13
Settling Time of Rotor angle consensus	7	2.5
Settling Time of Rotor Speed consensus	10.5	0.9

V. CONCLUSION

Consensus of fractional-order linear time invariant cyber-physical power systems, in the presence of model uncertainties, disturbances and cyber-attacks in both of inherent and interactive dynamics and graph topology, was studied in this paper. In this study, an adaptive-robust fractional order sliding mode controller has been designed to achieve consensus in the presence of mentioned uncertainties and disturbances with the important constraint that the upper bound of system perturbations is a nonlinear unbounded function with unknown coefficients. To deal with these perturbations, a chattering-free fractional order AFOSMC method is presented in which the coefficients of the upper bound function are estimated by stable adaptive laws. Numerical simulation results confirm the effectiveness of the proposed controller compared to other recent references. The consensus of Cyber-Physical Power Systems in the presence of *communication time delay* among fractional order model of agents, achieving *finite time consensus* in the fractional order model of CPPS using terminal sliding mode control and designing fractional order sliding control *considering input constrains*, can be considered as three future research plans to continue this work.

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