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Binary Wyner-Ziv and CEO Problem Coding Under the Hamming Distortion Criterion

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ABSTRACT

In this paper, we present a practical encoding and decoding scheme for the binary Wyner-Ziv problem based on graph-based codes. Our proposed scheme uses low-density generator-matrix (LDGM) codes in lossy source coding part and low-density parity-check (LDPC) codes in syndrome generation and decoding part. Actually, we apply Bias-Propagation algorithm for lossy source coding or binary quantization and Sum-Product algorithm for syndrome-based channel decoding. Using appropriate degree distributions for LDGM codes and optimized degree distributions for LDPC codes, we will be able to achieve close rate-distortion performance to the theoretical Wyner-Ziv bound. Also, we extend our proposed scheme for presenting a practical coding scheme for the binary Chief Executive Officer (CEO) problem. In our scheme, encodig is based on binary-quantization and Slepian-Wolf coding using source-splitting technique. It is shown that, source-splitting technique is an efficient strategy for achieving non-corner points in Slepian-Wolf rate region. We show that, this technique along with iterative message-passing algorithms can be efficient for having close rate-distortion performance to the Berger-Tung inner bound of binary CEO problem for non-corner points too.

I. Introduction

Graph-based codes, i.e., codes with a graphical representation, are mostly utilized in channel coding, source coding, and also in joint source-channel coding schemes, because of their powerful performance in achieving theoretical bounds and low complexity. Turbo codes and LDPC codes with iterative decoding algorithms are able to achieve the capacity of the most known communication channel models [1], [2]. In addition, LDGM codes, which are the source-code dual of LDPC codes, have a good performance in achieving rate-distortion (RD) bound of the binary-symmetric source (BSS) [3] and the binary-erasure source (BES) [4].

These codes can be joined elaborately for solving network information theory problems such as Wyner-Ziv problem or CEO problem that are considered in this paper. Actually, quantization using LDGM codes and noiseless binning using LDPC codes play important role in these problems, if these part are efficiently designed then the total performance of structure will be efficient too.

Source coding with side information available at the decoder scenario, which is known as asymmetric Slepian-Wolf problem in lossless case [5], and the Wyner-Ziv problem in lossy case have own theoretical bounds [6], which determine the lowest achievable rates. These theoretical bounds depend on the structure of problem and its parameters. Using LDPC and LDGM codes elaborately, results in a structure which has good rate-distortion performance close to the Wyner-Ziv bound. Briefly, we show that when a good source code at the rate of R_s and a good channel code at the rate of R_c are utilized in the compound coding structure [7], [8], the Wyner-Ziv theoretical bound can be saturated. These codes should be selected such that the following inequalities are satisfied for arbitrarily small positive values of ε_s and ε_c ,

$$1-h(D) \leq R_s < 1-h(D)+\varepsilon_s, \quad (1)$$

$$1-h(D*p)-\varepsilon_c < R_c \leq 1-h(D*p), \quad (2)$$

where D is acceptable distortion and p relates to the side information model that is available at the decoder of the Wyner-Ziv problem model, ε_s and ε_c relate to the source and channel coding bounds, respectively.

For the multi-terminal source coding problems, we can consider different distortion criteria to evaluate their performance. The Hamming distance measure is employed in binary and discrete spaces, the mean square error criterion is employed and applicable in continuous spaces, and the logarithmic loss can be used in both discrete and continuous spaces. Under the Hamming distortion, there is no closed form expression for the rate-distortion bound of the binary CEO problem, while under logarithmic loss, we have exact rate-distortion bound of the binary CEO problem, but under this criterion, the encoding and decoding scheme are more complex than the cases that we propose in this paper.

Different source and channel codes were utilized for solving Wyner-Ziv problem, such as Convolutional codes and Turbo codes were used in [8]. Nested lattice codes were employed for coding of the Gaussian source when an Additive White Gaussian Noise (AWGN) model is considered for the correlation channel between the side information and the information source, this model is called Quadratic Gaussian problem [9]. Polar codes were also used in [10] for the Quadratic Gaussian Wyner-Ziv problem. In addition, spatially-coupled codes using the belief-propagation guided decimation were utilized in [11]. Nested parity-check codes are introduced for coding of binary sources when the correlation channel is modeled by a BSC [12]. Moreover, universal Wyner-Ziv coding is considered in [13], [14]. We are especially interested in Low-Density Graph-based codes and BSC model for the correlation channel of Wyner-Ziv problem. Our proposed method has iterative graph-based and low-complex encoding and decoding algorithms, and realizes close rate-distortion performance to the Wyner-Ziv limit.

A natural extension of the Wyner-Ziv problem can be the multi-link scenario that is called multi-terminal lossy source coding problem. One special case of multi-terminal scenarios which has increasing application in Wireless Sensor Networks (WSN), is CEO problem. In CEO problem, several noisy observations of a single source are sensed by agents. These agents desire to encode their observations and sent them to a joint decoder with close performance to the information-theoretic bounds. This practical problem is a well-known problem in network information theory [15]. If source has binary alphabet, then we will face to the binary CEO problem which is the focus of this work.

We can consider research works on binary CEO problem in two category. The first one are works that focus on presenting tight bounds with the purpose of finding exact rate-distortion region for this problem, that is now an open problem in general case. The second one are works which try to design practical and low-complex coding schemes that achieve to existing bounds. Most of works in the second category are focused on Quadratic Gaussian CEO problem [16], [17] and [18]. Our work is in the second category for binary CEO problem.

We present a successive Wyner-Ziv coding scheme for the two-link binary CEO problem based on Source Splitting idea [19]. Using this idea, achieving an arbitrary point in rate-distortion plane of two-links case (L=2) is converted to achieving corner points in rate-distortion plane of three-links case (2L-1=3).

Our practical implemented scheme uses iterative low-complex graph-based codes that have close performance to the theoretical bounds of source coding and channel coding, we apply Bias-Propagation [20] and Sum-Product [21] for these purposes, respectively. Actually, these algorithms are the basic elements of our proposed scheme. Practical lossy source coding schemes are employed in a variety of real world applications such as multi-terminal video compression algorithms [22].

The organization of this paper is as follows, in section II, we present the problem definition and preliminaries, we review existing theoretical bounds for the binary Wyner-Ziv and the binary CEO problem, in this section. In section III, our encoding and decoding schemes are described in detail. We exhibit the results in section IV, these results show rate-distortion performance of our scheme. Finally, we conclude this paper in section V.

II. The Problem Definition and Preliminaries

In this section, the binary Wyner-Ziv problem and the binary CEO problem are reviewed. Also, the concept of binary quantization, the syndrome-based decoding and source splitting technique are presented.

A. The Binary Wyner-Ziv Problem

The lossy source coding scenario with side information that is available at the decoder is illustrated in Fig. 1, this structure is so called Wyner-Ziv problem. If S has binary alphabet, then we will face to a binary Wyner-Ziv problem. In general, in each solution for this problem, there exists an encoding function $f(.)$, which maps n-vectors S emitting from the binary symmetric source to the compressed data Z_2 ($Z_2 = f(S)$). Moreover, a decoding function $g(.,.)$ using compressed data Z_2 and side information J with the length of n, decodes S to $\hat{S} = g(Z_2, J)$. Note that, our intention in solving the Wyner-Ziv problem is presenting a practical low-complex coding scheme with rate-distortion performance near the Wyner-Ziv theoretical limit.

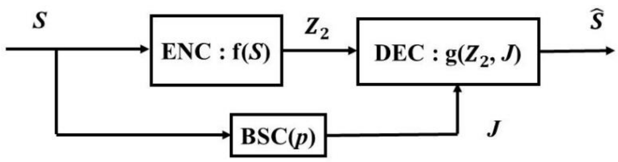

Fig. 1. The binary Wyner-Ziv problem.

According to [7], if the correlation between S and J is modeled by a BSC with the cross over probability of p and the distortion is evaluated by the Hamming distance measure, then we have the coding rate limit of (3), where D is the total distortion and D*p is the binary convolution of D and p.

$$R_{WZ} = l.c.e.\{h(D*p) - h(D), (p,0)\}, \qquad (3)$$

for $0 \le D \le p$, where $D*p=D(1-p)+p(1-D)\}$, $h(x) = -x\log_2(x) - (1-x)\log_2(1-x)$, and *l.c.e.* stands for the lower convex envelop of the term $h(D*p) - h(D)$ and the point $(p,0)$.

The total distortion in this work stems from both the encoder and decoder. The encoder distortion is basically related to the quantization of the binary source S to a codeword, with a syndrome that is necessarily zero. At the decoder side, the distortion stems from inefficiency of the decoder which will be negligible if we apply an efficient channel error correcting code, with a rate that is smaller than the capacity of the correlation channel, in fact this inefficiency is measured by Bit Error Rate (BER) value which is equivalent with the distortion. Therefore, the total average distortion is as follows:

$$\begin{aligned} D_t &= \frac{1}{n} E\sum_{i=1}^{n} d(S_i, \hat{S}_i) \\ &= \frac{1}{n} E\sum_{i=1}^{n} d(S_i, X_i) * \frac{1}{n} E\sum_{i=1}^{n} d(X_i, \hat{S}_i) \le D, \end{aligned} \qquad (4)$$

where $d(.,.)$ is the Hamming distortion measure, and D is acceptable distortion value.

Basically, the performance criterion in lossy source coding problems are the values of rate and distortion. They are compared with the theoretical limits depending on the configuration of problem. Because of this, we will evaluate performance of our proposed scheme from rate-distortion point of view and observe the gap between achieved points and theoretical bounds.

If the total rate of coding scheme in the nested structure equals to the difference $R_s - R_c$, then we will have,

$$R_{\text{total}} = R_s - R_c < h(D*p) - h(D) + \underbrace{\varepsilon_s + \varepsilon_c}_{\varepsilon}. \qquad (5)$$

Therefore, if R_s and R_c are chosen according to (1) and (2), then we can be close to $h(D*p) - h(D)$ arbitrarily, because the term $\varepsilon = \varepsilon_s + \varepsilon_c$ can be made arbitrarily small by proper choices of source and channel codes in the compound structure. Note that, the Wyner-Ziv rate-distortion theoretical bound is formulated as (3).

B. The Binary CEO Problem

The configuration of CEO problem is depicted in two-link case in the Fig. 2. As it is seen, we have two noisy observation of a single binary source. These observations are encoded independently without any collaboration between encoders, and are sent to the joint decoder that is called CEO or fusion center. Because of lossy nature of CEO problem, if encoders collaborate with each other, then the rate-distortion region will be different from our case. Indeed, collaboration of encoders expands achievable rate region in lossy source coding scenarios.

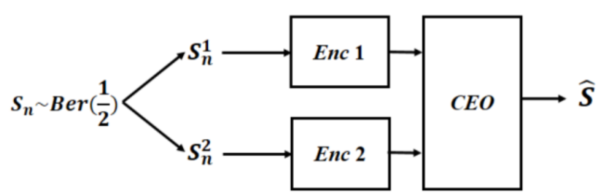

Fig. 2. Two-link binary CEO problem.

Inner and outer bounds for two-link case are provided in [23]. These bound play important role in our work, because we evaluate the performance of our scheme based on the gap value between achieved points and these bounds. In [23], binary CEO problem is reduced to a special case of binary multi-terminal source coding problem and mentioned bounds are obtained. Inner bound is actually Berger-Tung inner bound, and the outer bound is obtained using converse proof techniques. For distortions D_1 and D_2, and the link rates R_1 and R_2, if the observation noises are $Ber(p_1)$ and $Ber(p_2)$ and $\rho \triangleq p_1 * p_2$, then the inner and outer bounds are according to (6) and (7), respectively.

$$\begin{cases} R_1^i \geq h(\rho * D_1 * D_2) - h(D_1), \\ R_2^i \geq h(\rho * D_1 * D_2) - h(D_2), \\ R_1^i + R_2^i \geq 1 + h(\rho * D_1 * D_2) - h(D_1) - h(D_2), \end{cases} \tag{6}$$

$$\begin{cases} R_1^o(D_1) \geq h[\rho * h^{-1}(1 - R_2^o(D_2))] - h(D_1), \\ R_2^o(D_2) \geq h[\rho * h^{-1}(1 - R_1^o(D_1))] - h(D_2), \\ R_1^o(D_1) + R_2^o(D_2) \geq 1 + h(\rho) - h(D_1) - h(D_2). \end{cases} \tag{7}$$

C. The Compound LDGM-LDPC Construction

The compound LDGM-LDPC construction is shown in Fig. 3. This construction consists of an LDGM code $C(G, H_1)$ nested with an LDPC code $C(G, H)$, both of these notations are related to the generator matrices and we have,

$$\begin{aligned} C(G, H_1) &= G_1 G, \\ C(G, H) &= \tilde{G} G, \end{aligned} \tag{8}$$

where G_1 and $\tilde{G}$ are the generator matrices of the codes with parity-check matrices of H_1 and $H = [H_1^T, H_2^T]^T$, respectively.

In our design, we consider the encoding procedure in two steps. It is shown in [24-25] that binary quantization using LDGM codes have a performance near rate-distortion limit for a binary symmetric source. We utilize this technique in the first step of the encoding by applying the Bias-Propagation algorithm with the compound code $C(G, H_1)$, that is an LDGM code having *n* check nodes and $m - k_1$ variable nodes.

In the second step of the encoding, we exploit the Syndrome-Based decoding method by utilizing H_2. We use the side information available at the decoder in this step. Our construction for solving the Wyner-Ziv problem is illustrated in Fig. 4.

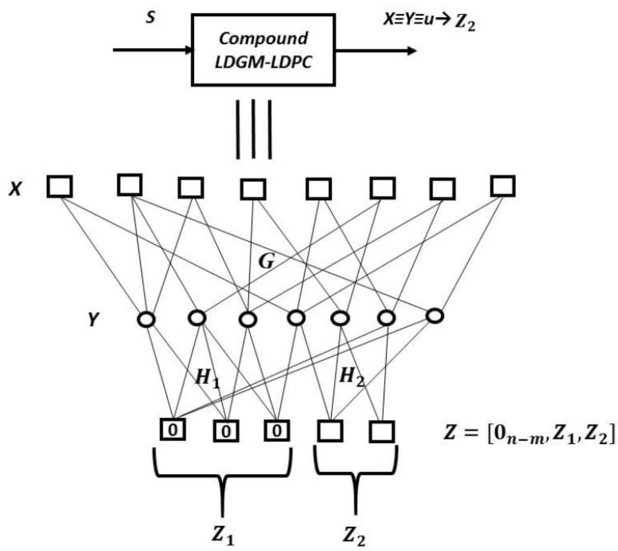

Fig. 3. The compound LDGM-LDPC code construction.

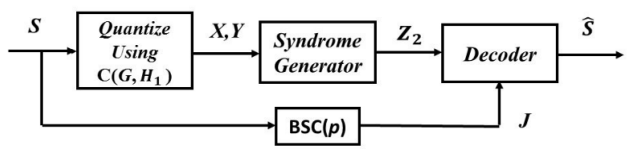

Fig. 4. Construction of the Wyner-Ziv solution.

D. The Binary Quantization

As mentioned, in our scheme much amount of distortion stems from the quantization. Since both of input and output in these quantizers are binary streams, they are called binary quantizers. Actually, the action of a binary quantizer is mapping an arbitrary $n-$vector to a codeword from a special code (for example LDGM code in our scheme) that has the minimum distance from that vector. These quantizers are the lossy source coding parts of scheme that have related rates and distortions. We can realize binary quantizations practically, using some iterative message passing algorithms, for example Bias-Propagation algorithm or Survey-Propagation algorithm. In these algorithms we will have specific compression rates and related distortions.

The Bias-Propagation Algorithm [25]:

It is shown that the Bias-Propagation algorithm has good performance in lossy compression of binary i.i.d. sources [25]. Moreover, lossy compression of binary sources was carried out using the Survey-Propagation algorithm in [26]. Suppose that an LDGM code with related bipartite graph is given, the check nodes of the graph are presented with *a*,*b*,*c*,... and the variable nodes are presented with *i,j,k,...*, as illustrated in Fig. 4.

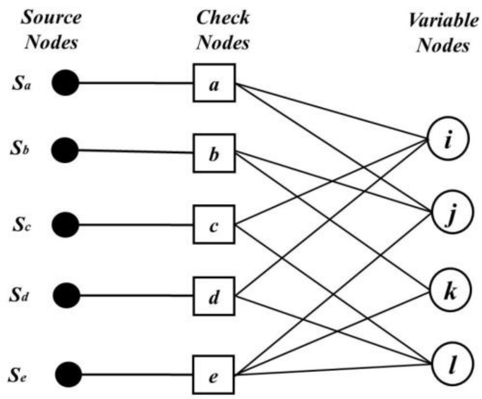

Fig. 5. Bipartite graph associated with an LDGM code for the binary Quantization.

In this algorithm, S_n is located in source nodes, which is directly connected to the check nodes. Source nodes have the bit values of the binary information source in the binary Quantization process, which are fixed and not changed during the process, while the bit values of the check nodes are changed and updated during the binary Quantization process. In each round of this algorithm, we have a constant number of iterations, the bias values in variable nodes are compared with the threshold of $0<t<1$. According to this comparison, some of the variable nodes with absolute bias values greater than t are fixed. If the bias value is positive, then the associated variable bit will be fixed to 0, and otherwise it will be fixed to 1. If there are no absolute bias values greater than t, then only a variable node with maximum absolute bias value is fixed. This procedure, continues until all the variable nodes are fixed. Finally, the compressed bit stream is obtained from the variable nodes. This algorithm is a lossy compression method. Now, we expound the update equations.

The message sent from the check node a to the variable node i in the l-th iteration of each round are as follows:

$$\phi_{a\to i}^{(l)} = \prod_{j\in \bar{N}(a)\setminus\{i\}} \theta_{j\to a}^{(l)}, \tag{9}$$

where $\bar{N}(a)$ is the set of nodes that are connected to the check node a including the source node S_a, and $\theta_{j\to a}^{(l)}$ is the message sent from the variable node j to the check node a in the l-th iteration. Moreover, the message sent from a source node S_a to the check node a is:

$$\theta_{S_a\to a}^{(l)} = (-1)^{S_a}\tanh(\gamma) \in [-1,1], \qquad \forall l \tag{10}$$

where γ is a real number depends on the LDGM code rate.

Similarly, the messages sent from the variable nodes to the check nodes in the l+1-th iteration of each round can be calculated from $\phi_{a\to i}^{(l)}$ values, according to (10),

$$\theta_{i\to a}^{(l+1)} = \frac{\prod_{b\in N(i)\setminus\{a\}}(1+\phi_{b\to i}^{(l)}) - \prod_{b\in N(i)\setminus\{a\}}(1-\phi_{b\to i}^{(l)})}{\prod_{b\in N(i)\setminus\{a\}}(1+\phi_{b\to i}^{(l)}) + \prod_{b\in N(i)\setminus\{a\}}(1-\phi_{b\to i}^{(l)})}, \tag{11}$$

where *N(i)* is the set of check nodes that are connected to the variable node i. Initial bias values are,

$$\theta(i) = \frac{|N_0(i)| - |N_1(i)|}{|N_0(i)| + |N_1(i)|}, \tag{12}$$

where,

$$N_h(i) = \{a \in N(i) \,|\, (\forall j \in \bar{N}(a)\setminus\{i\}, v_j \neq *)\& \\ (\sum_{j\in\bar{N}(a)\setminus\{i\}} v_j = h, \bmod 2)\}, \qquad h=0,1$$

$$N_*(i) = \{a \in N(i) \,|\, (\exists j \in \bar{N}(a)\setminus\{i\}, v_j \neq *)\}. \tag{13}$$

where v_j is the j-th bit of variable nodes and its initial value is chosen randomly. Finally, at the end of each round, the bias values are calculated according to (14), then they are compared with the threshold value t.

$$\theta_i = \frac{\prod_{b\in N(i)}(1+\phi_{b\to i}^{(\hat{l})}) - \prod_{b\in N(i)}(1-\phi_{b\to i}^{(\hat{l})})}{\prod_{b\in N(i)}(1+\phi_{b\to i}^{(\hat{l})}) + \prod_{b\in N(i)}(1-\phi_{b\to i}^{(\hat{l})})}. \tag{14}$$

When there are some cycles of length 4 in the graph of codes, inevitably; we can equipped the Bias-Propagation algorithm with damping operation, it means that the equation (14) will be changed according to [26].

E. The Syndrome-Based Decoding Scheme

The parity and the syndrome approaches are efficient lossy source code design methods based on the channel coding methods [27]. In these methods, the encoder sends a subset of parity or syndrome bits to the decoder as lossy compressed data. Our proposed scheme is based on syndrome approach, that is more applicable in lossless source coding problems with side information at the decoder, a binary sequence $\hat{S}$ is decoded utilizing the received syndrome and side information J with the length of n [28]. This simple coding scheme is used for the asymmetric Slepian-Wolf structure

[29]. Inspired by this syndrome decoding idea, if we set some of syndrome bits equal to zero, and send the only determined non-zero part to the decoder, we will have a lossy coding scheme that can be employed for solving the Wyner-Ziv problem.

In this work, we use these original ideas with iterative message-passing algorithms on sparse graph-based codes for obtaining a practical solution of the binary Wyner-Ziv problem and the binary CEO problem.

The Sum-Product Algorithm [29]:

It is demonstrated that the Sum-Product algorithm from the family of iterative LDPC decoders has good performance in channel error correction and generally in syndrome-based decoding schemes. We briefly restate its iterative procedure and related parameters [29]. Similar to any iterative message passing algorithm, there are messages which are passed between variable nodes and check nodes according to Fig. 6.

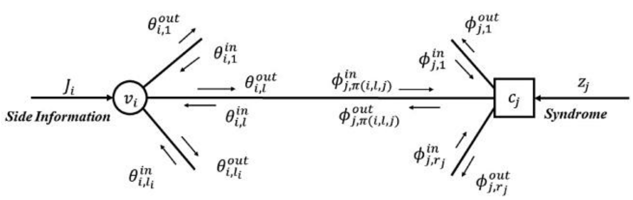

Fig. 6. Messages in Sum-Product algorithm over an LDPC code.

In each iteration, we have the following message-passing equations, the initial values of LLRs for each variable node in the corresponding bipartite graph of LDPC code are set according to:

$$\theta_{i,0} = \log \frac{\Pr[x_i = 0 \,|J_i]}{\Pr[x_i = 1 \,|J_i]} = (1-2J_i) \log \frac{1-p}{p} \qquad (15)$$

where J_i is the i-th bit of the side information J. Variable node update equation is as follows:

$$\theta_{i,l}^{out} = \theta_{i,0} + \sum_{j=1, j\neq l}^{l_i} \theta_{i,j}^{in}, \qquad l = 1,2,\ldots,l_i \qquad (16)$$

where l_i is the degree of i-th variable node. The second subscript in messages shows the number of edge which they are passed over it. The initial values of $\theta_{i,j}^{in}$s are 0. Next, it is assigned $\theta_{i,l}^{out} = \theta_{j,\pi(i,l,j)}^{in}$, for all of edges according to the connections. Check update equation is as follows:

$$\tanh(\frac{\phi_{j,r}^{out}}{2}) = (1-2z_j) \prod_{i=1, i\neq r}^{r_j} \tanh(\frac{\phi_{j,i}^{in}}{2}), r = 1,2,\ldots,r_j \quad (17)$$

where r_j is the degree of j-th check node, and z_j is the j-th bit of total syndrome $Z = [0_{n-m}\ Z_1\ Z_2]$. Now, it is assigned $\theta_{i,l}^{in} = \theta_{j,\pi(i,l,j)}^{out}$ and this process is run for a defined number of iterations. Finally, we will have the following decision making rule:

$$\hat{x}_i = \begin{cases} 0 & \theta_{i,0} + \sum_{j=1}^{l_i} \theta_{i,j}^{in} \geq 0 \\ 1 & \theta_{i,0} + \sum_{j=1}^{l_i} \theta_{i,j}^{in} < 0 \end{cases}, \quad \forall i. \qquad (18)$$

Generally, the complexity order of all message-passing algorithms is $\mathrm{O}(nc)$; where, n is the number of nodes and c is the complexity of one complete passing. Note that optimal encoding and decoding schemes have an exponential complexity order with the block length of n, while message passing algorithms which are sub-optimal reduce complexity to the linear order.

F. Source Splitting Technique.

If a solution for the Wyner-Ziv problem is successively extended to a solution for the CEO problem in a straightforward manner, then only corner points in rate region can be achieved. In other words, successive Wyner-Ziv scheme for the CEO problem cannot be efficient by itself except for the corner points of rate region. For achieving non-corner points of rate region, we exploit from source splitting technique that is firstly introduced in [19]. Using this technique, we are able to achieve non-corner points with successive refinement. The details of this technique is discussed in the next section.

III. The Encoding and the Decoding Schemes

We present the processes that are done on the information sources, from the beginning to the end of proposed scheme, including both of encoder and decoder, in detail. We discuss about the Wyner-Ziv problem and the CEO problem separately, in the following.

A. The Wyner-Ziv Problem

In this subsection, we describe our proposed encoding and decoding schemes with more details. Briefly, we apply the Bias-Propagation algorithm for binary quantization of the source sequence using an LDGM code. Thereafter, we obtain syndrome from quantized stream and send it to the decoder along with side information. At the decoder, the decoded stream is obtained using the Sum-Product algorithm. The formulation of these two iterative algorithms including their check update, variable update equations and related parameters are presented previously.

The Encoding Algorithm:

In our design, we consider the encoding procedure in two steps. It is shown in [3] that binary quantization using LDGM codes have a performance near rate-distortion limit for a binary symmetric source. We utilize this technique in the first step of the encoding by applying the Bias-Propagation algorithm with the compound code $C(G,H_1)$, that is an LDGM code having n check nodes and $m-k_1$ variable nodes. In the second step of the encoding, we exploit the Syndrome-Based decoding method by utilizing H_2. We use the side information available at the decoder in this step. Our construction for solving the Wyner-Ziv problem is illustrated in Fig. 7.

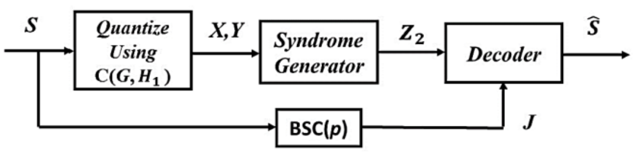

Fig. 7. Construction of the Wyner-Ziv solution.

A BSS is considered, i.e., $\Pr\{S_i=0\}=\Pr\{S_i=1\}=0.5$. As earlier mentioned, compression procedure of S_n for obtaining the syndrome consists of two steps.

Step 1: The source sequence S_n is quantized to the codeword $X_n \in C(G,H_1)$, using the compound code $C(G,H_1)$. From this step, $Y_m = u_{m-k_1} \times G_1$ is obtained, where u_{m-k_1} is information bits that is replaced in variable nodes after the quantization process, and G_1 is the generator matrix associated with H_1. Moreover, we have $X_n = Y_m \times G_{m\times n}$. This quantization is implemented by the Bias-Propagation algorithm.

Step 2: After the quantization, the syndrome of Y_m is obtained using H_2. Actually, the syndrome $Z_2 = Y_m H_2^T$, is sent to the decoder as compressed sequence. Note that the length of Z_2 is k_2, because the dimension of H_2 is $k_2 \times m$.

Note that the order of the binary Quantization and syndrome generation parts cannot be changed with each other because basically Quantization is a source encoding scheme, while the syndrome generation is used for the channel decoding.

The Decoding Algorithm:

The decoder receives side information $J = S \oplus Ber(p)$ and syndrome Z_2, then it finds nearest sequence to J in the coset with syndrome Z_2 using the sum-product algorithm that is an efficient sub-optimal LDPC decoder. This algorithm is performed by an LDPC code with the generator matrix $C(G,H)$. The formulations of this algorithm is presented, previously.

B. The Main Result on the Wyner-Ziv Solution

Our practical implemented coding scheme for binary Wyner-Ziv problem is based on an important theorem which was proven in [7]. In the following, we restate it.

Theorem. The compound LDGM-LDPC structure is able to achieve theoretical Wyner-Ziv bound if the following two conditions are satisfied:

1. An efficient lossy source coding algorithm is used for the Quantization, which achieves rate-distortion limit.
2. An efficient channel decoding algorithm is applied in the Syndrome-Decoder step, with low BER.

There are two important limitations in the proposed scheme which stem from the above conditions and should be considered. First one is that, for any quantization rate R_1, the resulting distortion d_1 should be greater than D, equivalently $R_1 = 1-h(D) > 1-h(d_1)$. Another limitation is related to the syndrome-decoding is that, the decoder rate R_2 should be smaller than the capacity of correlation channel model between quantized sequence X_n and side information J_n, which is really a BSC($d_1 * p$). In other words if $R_2 < 1-h(d_1 * p)$, then the distortion d_2 or BER in syndrome-decoding step can be arbitrary small.

These two limitations are actually theoretical. Furthermore, there are some practical limitations that should be considered in implementation and block length choosing. We express one of the practical limitations by the following lemma.

Lemma. In selecting block lengths, if $k_1 + k_2 < m < 2k_1 + k_2$ is satisfied, then the existence of at least one matrix for top layer G in the compound structure is guaranteed, given $C(G,H_1)$, $C(G,H_2)$ and matrix H in the bottom layer.

Proof. The inequality $k_1+k_2<m$ is trivial due to the dimension of matrix H, we demonstrate that $m<2k_1+k_2$. Matrix G has mn unknown elements. Total number of known elements are *n(m-{k_1})+n(m-{k_1}-{k_2})*, which are obtained from (8). Note that in these equations, G_1 and $\tilde{G}$ are known, because H is given. Therefore, if *mn>n(m-{k_1})+n(m-{k_1}-{k_2})*, then there is at least one choice for G such that (8) is satisfied for given $C(G,H_1)$, $C(G,H_2)$ and matrix H. Therefore, we will have $m<2k_1+k_2$ and this completes the proof.

Actually, the Bias-Propagation and Sum-Product algorithms are able to satisfy conditions (1) and (2) in the above theorem, respectively, and because of this, our proposed scheme works efficiently close to the existing theoretical bounds. In general, iterative message-passing algorithms are sub-optimal and there is a gap between theoretical bounds and rate-distortion performance of these algorithms. This gap can be decreased using following methods:

1. Increasing length of blocks or number of iterations in algorithms.
2. Increasing threshold value for more accuracy.
3. Using Survey-Propagation algorithm instead of Bias-Propagation, knowing that the complexity increases.
4. Using exact ODDs for each rate.

C. The Binary CEO Problem

In this section our designed construction for the encoding and decoding side of binary CEO problem is described. For the sake of simplicity, only two-link case is considered in this paper. Our design can be extended to *L*-link case, easily.

The Encoding:

In the Fig. 8, the encoding process is depicted. As it is seen, first of all, agents quantize two noisy observations to X_n and Y_n, the rates of these quantizers are $R_{11}=\frac{k_1}{n}$ and $R_{21}=\frac{k_2}{n}$ respectively. These binary-quantization steps are done using LDGM codes $(G_1)_{k_1\times n}$ and $(G_2)_{k_2\times n}$, respectively.

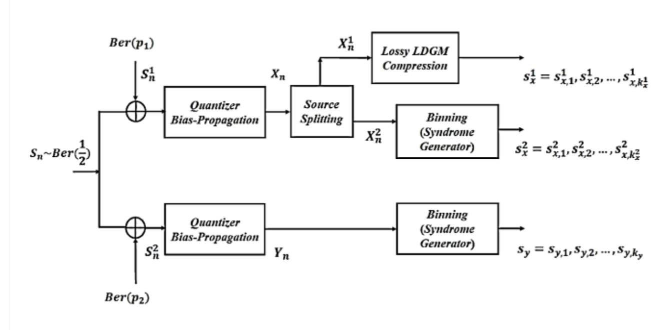

Fig. 8. The Encoding Scheme of the two-link Binary CEO Problem.

After that we split one of the quantized sources (for example X_n) to the two sources (X_n^1 and X_n^2), we use the following rule in source splitting,

$$\left.\begin{array}{l} X_i^1=\min(X_i,T) \\ X_i^2=\max(X_i,T)-T \end{array}\right\} \Rightarrow \left\{\begin{array}{l} X_i=X_i^1+X_i^2 \\ X_n \leftrightarrow (X_n^1,X_n^2) \end{array}\right. \tag{19}$$

If we suppose that the alphabet of X_n has been mapped to $\{0,1,\ldots,Q-1\}$ with $Q=2^{k_1}$, then $T\in\{1,2,\ldots,Q-2\}$. So we will have,

$$X_n^1\in\{0,1,\ldots,T\};X_n^2\in\{0,1,\ldots,Q-1-T\} \tag{20}$$

We suppose that $X_n\oplus X_n^l=Ber(\alpha_l);\ l=1,2$. The parameters α_1 and α_2 can be calculated from the source splitting rule.

Thereafter, we have syndrome generating or Slepian-Wolf encoding step. In this step, X_n^1 is compressed to the s_x^1 with the length of k_x^1 using an LDGM code with the rate R_{12} and resulted distortion d_{12}. Note that lossless compressions schemes like Entropy Coding (for example Huffman coding or Arithmetic coding) does not work properly in this step, because of their error propagation characteristic and their lossless essence. Parallel to this compression, syndromes s_x^2 and s_y is calculated from X_n^2 and Y_n respectively, using appropriate LDPC codes H_x and H_y.

$$(s_x^2)_{k_x^2}=X_n^2H_x^T, \quad (s_y)_{k_y}=Y_nH_y^T \tag{21}$$

Here, we can exploit from parity-based methods as well as syndrome-based methods [30]. Therefore, these three compressed sequences (s_x^1, s_x^2 and s_y) are sent to the CEO decoder. Total rates in links are $R_1 = \frac{k_{x_1} + k_{x_2}}{n}$, $R_2 = \frac{k_y}{n}$. Using source splitting idea, two-link structure is converted to the equivalent three-link case. We have the following rate-distortion relations in binary-quantization steps,

$$\begin{aligned} R_{11} &= \tfrac{k_1}{n} = 1 - h(d_{11}) + \varepsilon_{11}, \quad d_{11} = \tfrac{1}{n}\left(\sum_{i=1}^{n} X_i \oplus S_i^1\right) \\ R_{21} &= \tfrac{k_2}{n} = 1 - h(d_{21}) + \varepsilon_{21}, \quad d_{21} = \tfrac{1}{n}\left(\sum_{i=1}^{n} Y_i \oplus S_i^2\right) \end{aligned} . \tag{22}$$

The Decoding:

In the Fig. 9, the decoding process is depicted. As it is seen, the CEO decoder has a successive formation. In the decoder, first s_x^1 is decompressed to the $\hat{X}_1$, if Bias-Propagation or Survey-Propagation algorithm is applied in the lossy compressor using an LDGM code, then this decompressor will be a simple matrix multiplying. After that, $\hat{X}_1$ is used as side information in the Sum-Product algorithm to decode $\hat{Y}$ from the input s_y. Finally, $\hat{Y}$ and $\hat{X}_1$ are used as side information in the Sum-Product algorithm to decode $\hat{X}_2$ from the input s_x^2. By summing $\hat{X}_1$ and $\hat{X}_2$, $\hat{X}$ is obtained.

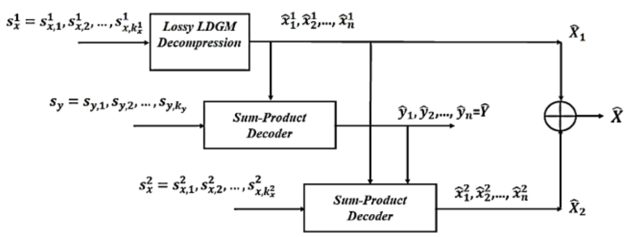

Fig. 9. The Decoding Scheme of the two-link Binary CEO Problem.

The total average hamming distortions are calculated as the following,

$$D_{t,l} = D_l * d_{l,1}, \qquad l = 1,2 \tag{23}$$

where, $D_l \equiv Ber_l$ for *l=1,2*, $Ber(D_1) = X_n \oplus \hat{X}$ and $Ber(D_2) = Y_n \oplus \hat{Y}$, if appropriate channel decoding schemes are used, then we will have $D_1, D_2 \to 0$.

Similar to equations (22), we will have,

$$R_{12} = 1 - h(d_{12}) + \varepsilon_{12} \quad , \mathrm{BSC}(d_{12}) = \hat{X}_n^1 \oplus X_n^1. \tag{24}$$

Parameters in our scheme should be designed in such a way that the following relations are satisfied. In other words, if in the first Sum-Product decoder we have,

$$\frac{n - k_v}{n} = 1 - h(A * \alpha_1) - \varepsilon_{22} \Rightarrow \frac{k_v}{n} = h(A * \alpha_1) + \varepsilon_{22} \tag{25}$$

then the Bit Error Rate of decoding can be arbitrary small, where $A \triangleq d_{11} * \rho * d_{21}$, and $BSC(A) = X_n \oplus Y_n$. Similarly, in the second Sum-Product decoder we will have,

$$\frac{n - k_u^2}{n} = 1 - h(D_2) - \varepsilon_{21} \Rightarrow \frac{k_u^2}{n} = h(D_2) + \varepsilon_{22} \tag{26}$$

because we have $\hat{Y}_n$ as side information in this decoder with distortion D_2, and $\hat{Y}_n \to \hat{X}_n \to \hat{X}_n^2$ forms a markov chain. Also, $\hat{X}_n^1$ is available at this decoder as side information.

IV. Simulation and Numerical Results and Performance Analysis

A. *Results for the Binary Wyner-Ziv Problem*

In this section, we present some simulation results which demonstrate our coding scheme performance for some special rates. We use the degree distributions which are optimized over BSC using density evolution technique. Our results are for two cases of low (p=0.3) and high (p=0.1) correlation between S_n and J_n.

In implementing the message-passing algorithms, we use time sharing between the points (p,0) and $(d_c + \varepsilon, R_c)$, where $(d_c + \varepsilon, R_c)$ is an achieved rate-distortion point. Obviously, this technique is efficient for rates in the linear part of Wyner-Ziv bound.

The block lengths which is employed in simulation, related rates and distortion values are presented in Tables 1 and 2, where d_1 and R_1 are distortion and rate of Binary-Quantization, d_2 and R_2 are distortion and rate of Syndrome-Decoding, D_t and R are the total distortion and rate, respectively. For more intuition, we represent D_{WZ} (Wyner-Ziv limit of distortion in rate R). We have $R_1 = \frac{m - k_1}{n}$, $R_2 = \frac{m - k_1 - k_2}{n}$, and $R = \frac{k_2}{n}$. Degree distributions which are utilized for producing LDGM and LDPC codes matrices are from [24] and [31]. We generate matrices according to degree distribution ($\rho(x)$, $\lambda(x)$), randomly.

We use some approximations for producing matrices from ODDs, in such a way that for some rates with unavailable ODDs, we round them to the rates with available ODDs and use them instead. We run the encoding and decoding process for 100 randomly generated source sequence S_n. We run 25 iterations in each round of the Bias-Propagation algorithm, we set t =0.8 and $\gamma \approx 2R_{\text{LDGM}}$ in our simulations. Moreover, we run 80-100 iterations in the Sum-Product algorithm.

In Figures 10 and 11, rate-distortion performance of our proposed scheme for two different side information models are illustrated. As it is seen in these figures, rate-distortion performance of our scheme is very close to the theoretical Wyner-Ziv limit. This ability of compound codes has been proved in [7]. It is obvious that when time sharing is utilized, it is possible of being more close to the Wyner-Ziv limit for small rates in comparison with the regions where time sharing is not utilized (high rates).

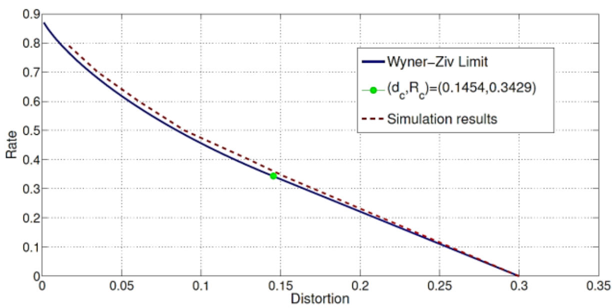

Fig. 10. Rate-distortion performance of the proposed scheme for p=0.3.

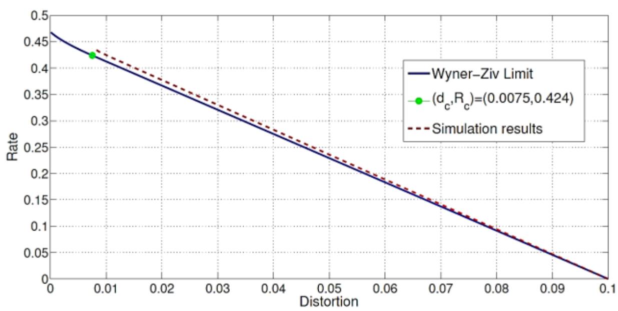

Fig. 11. Rate-distortion performance of the proposed scheme for p=0.1.

In these figures, the border points on Wyner-Ziv limit for p=0.3 and p=0.1 are $d_c = 0.1454$ and $d_c = 0.0075$, respectively. These points separate the Wyner-Ziv limit curve into two parts, rates in the high rate region are in the formation of $h(p * D_t) - h(D_t)$ for distortion D_t and for low rates the curve has linear formation.

B. Results for the Binary CEO Problem

Here, we present simulation and numerical results and related parameters that are used in implementation of iterative message passing algorithms. We apply Bias-Popagation algorithm in binary-quantization and lossy compression blocks using LDGM codes. Also, we apply Sum-Product decoder using LDPC codes in CEO decoders. Degree distributions for producing low-density matrices are from [25] and [31], these degree distributions are optimized over Binary Symmetric Channel using Density Evolution technique.

We set $(p_1, p_2) = (0.0625, 0.1)$, therefore we will have $\rho = p_1 * p_2 = 0.15$. Our results are for

$S_n \sim Ber(\frac{1}{2}), n = 10^4$, and $D_{t,1} = D_{t,2} = 0.005$. Theoretical bounds related to these parameters are shown in Fig. 12 according to [23].

Resulted values for parameters is presented in Tables 3 and 4 in details. Note that, in corner points, the binary CEO problem is reduced to a single binary Wyner-Ziv problem; in this case, one of the links should be lossless transmission link.

If we change the rule of two links with each other, i.e. $X \leftrightarrow Y$, then we achieve three new points that are symmetric with three achieved points that are described in tables. In other words we can achieve the points (R_1, R_2) =(0.85,0.71), (R_1, R_2) =(0.9,0.66) and (R_1, R_2) =(1,0.59) in a similar way. These six points obtained from simulation along with outer bound, Berger-Tung inner bound and Slepian-Wolf bound are shown in Fig. 12. Note that Slepian-Wolf bound is related to the lossless case, i.e. when we have $D_{t,1} = D_{t,2} = 0$.

The reason of the near the existing theoretical bounds performance returns to the intelligently selecting the block lengths in the parts of the binary Quantization and the syndrome decoding. In this paper, we divide the theoretical rate-distortion bound expression of the binary Wyner-Ziv and the binary CEO problem in such a way that a point-to-point source coding bound expression and a binary channel capacity expression appear. Then, in the design procedure, we choose block lengths such that those bound be achievable. Finally, by combining them, the general theoretical bounds of the binary Wyner-Ziv and the binary CEO problem will be achievable.

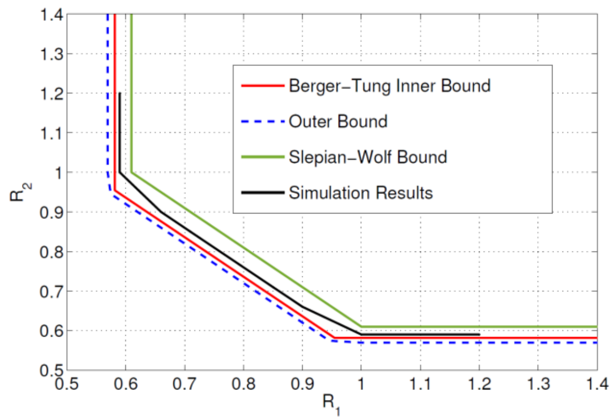

Fig. 12. Rate-distortion performance comparison of the proposed scheme.

Calculating Parameter α_1: We have the following relations in source splitting part of our proposed scheme,

$$R_y = H(Y|X_1) \tag{27}$$

$$R_x = \underbrace{H(X_1)}_{R_{X_1}} + \underbrace{H(X_2|X_1,Y)}_{R_{X_2}} = H(X,Y) - R_y \tag{28}$$

As it is mentioned $Ber(\alpha_1) = X_n \oplus X_n^1$. In the continue, we calculate the relation between α_1 and T using Gaussian approximation. For more intuition, we present some special cases as the following,

$$\begin{aligned} &T = 2 \Rightarrow \alpha_1 \approx 0.5 \\ &T = \tfrac{Q}{2} \Rightarrow \alpha_1 \approx 0.25 \\ &T = Q - 2 \Rightarrow \alpha_1 \approx 0 \end{aligned} \tag{29}$$

We can suppose $k_1 \approx n$. So, X_n^1 can possess all binary streams with length n. There are two possible states for X_n^1,

$$\begin{aligned} &X_n \le T \Rightarrow X_n^1 = X_n \Rightarrow X_n^1 \oplus X_n = 0 \\ &X_n > T \Rightarrow X_n^1 = T \Rightarrow X_n^1 \oplus X_n \ne 0 \end{aligned} \tag{30}$$

Therefore, α_1 is approximately average of term $d_H(X_n^1, X_n)$ over all of $X_n > T$, where $d_H(.,.)$ represents Hamming distance between two streams. So with a good approximation, the minimum possible value for α_1 is as follows,

$$\alpha_1 \approx \frac{1\binom{n}{1} + 2\binom{n}{2} + \ldots + k\binom{n}{k}}{n \times 2^n} \tag{31}$$

the integer k is selected such that we have,

$$\binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{k} \approx 2^n - (T+1) \tag{32}$$

the number of X_n s such that $X_n > T$. Because $i\binom{n}{i} = n\binom{n-1}{i-1}$, we will have,

$$\alpha_1 \approx \frac{\binom{n-1}{0} + \binom{n-1}{1} + \ldots + \binom{n-1}{k-1}}{2^n} \tag{33}$$

If $N \triangleq n-1$ and $K \triangleq k-1$, then,

$$2\alpha_1 \approx \sum_{i=0}^{K} \binom{N}{i} \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right)^{N-i}. \tag{34}$$

Using Gaussian approximation $\binom{n}{i} p^i (1-p)^{n-i} \approx \frac{1}{\sqrt{2\pi np(1-p)}} e^{-\frac{(i-np)^2}{2np(1-p)}}$, we can calculate K for a known α_1 and vice versa from (34). From (34) and Gaussian approximation, we will have,

$$\sum_{i=0}^{K} \binom{N}{i} \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right)^{N-i} \approx \int_{-\infty}^{B} \frac{1}{\sqrt{0.5\pi N}} e^{-\frac{(t-\frac{N}{2})^2}{0.5N}} dt \tag{35}$$

where $B = \frac{20\sigma}{N} \times K + (-10\sigma + \mu)$, which $\sigma = \frac{\sqrt{N}}{2}$ and $\mu = \frac{N}{2}$. For standard Gaussian distribution, Q and G functions is defined as follows, and their table is available for required calculations,

$$Q(z) = \int_{z}^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = 1 - \int_{-\infty}^{z} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = 1 - G(z), \tag{36}$$

where, $z \sim N(0,1)$ is standard normal distribution. Using (36), we can write (35) as follows,

$$G\left(z=\frac{B-\mu}{\sigma}\right)=1-Q\left(z=\frac{B-\mu}{\sigma}\right) \tag{37}$$

The results of this work are based on these calculations. There are some hints in our proposed scheme that should be noted, they are listed in the following,

1. The degree distributions which are optimal in channel coding aren't necessarily optimal in source coding problems, but they have better performance, compared to regular degree distributions.

2. There is no necessity that the side information should be used only in variable nodes of the LDPC code; actually, the side information can be used in different ways; for example, it can be multiplied or added with received data and etc (e.g. other computational or combinatorial functions).

3. The output of Binary Quantization can be supposed to be uniform, this uniformity causes the calculating of α_1 has more accuracy. Also, the parameter α_2 don't affect to our design and calculations.

4. All of iterative message-passing algorithms are sub-optimal (with linear complexity), because optimal solution is really exhaustive search for finding nearest codeword (with exponential complexity), these iterative algorithm are done using a low-density graph-based code.

5. Source splitting idea for designing a practical coding scheme for the binary CEO problem results in a successive decoding scheme, this successive decoding scheme is in contrast with joint decoding scheme.

V. CONCLUSIONS

In this paper, we implemented a practical coding scheme for solving Wyner-Ziv problem, employing compound LDGM-LDPC construction, which is really a nested coding structure. When efficient source and channel coding schemes with low complexity are elaborately used in this construction, this structure will have the capability of coming close to the Wyner-Ziv limit with any arbitrary precision. We applied Bias-Propagation algorithms for binary quantization using LDGM codes and Sum-Product algorithm for syndrome-decoding using LDPC codes. Employing these algorithms and time sharing technique, the numerical and simulation results confirm close rate-distortion performance of proposed scheme to the Wyner-Ziv theoretical limit.

Furthermore, we proposed an encoding scheme based on binary quantization and source splitting idea in the Slepian-Wolf coding part for the binary CEO problem, in this paper. Our CEO decoder has successive structure and decoded sequence in each link is used as side information in the next links. All of binary quantization, lossy compressors and link decoders are graph based and utilize iterative message-passing algorithms. Because of good performance of these major components, the total rate-distortion performance of our proposed scheme is close to the theoretical bounds of CEO problem.

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