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Adaptive Fuzzy Fractional-order Fast Terminal Sliding Mode Control for a Class of Uncertain Nonlinear Systems

Amir Razzaghian[1], Reihaneh Kardehi Moghaddam[2,†] and Naser Pariz[3]

[1,2] Department of Electrical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran
[3] Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ABSTRACT

The paper introduces a novel adaptive fuzzy fractional-order (FO) fast terminal sliding mode control procedure for a class of nonlinear systems in the presence of uncertainties and external disturbances. For this purpose, firstly, using fractional calculus, a new FO nonlinear sliding surface is proposed and then, the corresponding FO fast terminal sliding mode controller (FOFTSMC) is designed to satisfy the sliding condition in finite time. Next, to eliminate the chattering phenomenon, a fuzzy system is constructed to design a continuous switching control law. The finite-time stability of the proposed adaptive fuzzy FOFTSMC (AFFOFTSMC) is proved using the concept of Lyapunov stability theorem. Finally, to illustrate the effectiveness of the proposed AFFOFTSMC, three examples are given as case studies. The numerical simulation results confirm the superiority of the proposed controller, which are the better robust performance, faster convergence, finite-time stability of the closed-loop control system, and a chattering free control effort compared to other mentioned control methods.

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I. INTRODUCTION

In fractional calculus, non-integer and complex orders are applied in definitions of derivative and integral which are called the FO derivative and FO integral, respectively [1]. FO derivatives are widely-applied in both pure and applied mathematics. Recently, some new extensions on FO derivatives are introduced in [2]. Fractional calculus widely used in control engineering because of useful properties to improve the performance of integer-order (IO) controller. Using fractional calculus, provides more accuracy in system modeling and describing real phenomena behaviors [3]. Hence, in [4], a FO model has presented for the hybrid power source system. [5], has proposed a novel FO hyperchaotic economic system. It is also proved that some FO dynamical systems show chaotic behaviors. Many articles investigate FO chaotic systems [6]. The authors in [7] have proposed a FO model for COVID-19. The results show that the FO model has less root-mean-square error than the classical one, and in [8], FO model using the Caputo–Fabrizio derivative is introduced for the same purpose.

Various FO controllers have been introduced by researchers for different purposes such as FOPID [9], FO optimal control [10], FO adaptive control [11] and FO sliding mode control [12]. Stability analysis of FO systems with some new features is investigated in [13-15].

Sliding mode control (SMC) is a well-known and useful controller with many benefits such as simple design, robustness against uncertainties, and disturbance rejection which has been widely used for control of nonlinear systems in the presence of structured and unstructured uncertainties [16]. Convectional SMC designed based on a linear switching manifold which can guarantee the asymptotic stability of the closed-loop control system. To improve the convectional SMC, terminal sliding mode control (TSMC) was developed to

[†]**Corresponding Author:** r_k_moghaddam@mshdiau.ac.ir
Department of Electrical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

obtain finite-time stability and faster closed-loop control behavior [17]. However, TSMC suffered from singularity problem. To avoid this problem, nonsingular terminal sliding mode controller (NTSMC) is proposed [18].

Chattering phenomenon is another problem that exists in both the conventional SMC and TSMC. These undesirable high-frequency oscillations can stimulate the unmodelled high-frequency dynamics of the system.

To reduce or eliminate the chattering, various techniques have been proposed, such as higher-order SMC [19], Dynamic SMC [20], and boundary layer method which defines a boundary layer around the sliding surface and using a continuous control within the boundary layer [21]. Although, this method reduces the chattering, but it no longer drives the system state to the origin, and steady-state error will be increased. Recently, intelligent approaches such as fuzzy-based SMC [22-24] and neural network-based SMC [25-26] have been designed for reducing chattering.

To deal with structured and unstructured uncertainties, SMC-based techniques have utilized two main approaches: disturbance observer-based SMCs [27-28] and intelligent SMCs [29-30].

Many researches have been using the fractional calculus in the design of FO sliding mode control methods for uncertain nonlinear systems to improve the performance of them [31-34]. However, proposed methods demonstrate robustness against structured and unstructured uncertainties, unknown upper bound of lumped uncertainty forces the designer to choose a large switching gain that results chattering phenomenon. In order to overcome this problem, a FO adaptive neuro-fuzzy sliding mode H_∞ controller is proposed in [35] that neuro-fuzzy system used to estimate the matched uncertainty. In [36], an adaptive neural network FO nonsingular TSMC is introduced. The proposed method uses RBF neural network to estimate lumped uncertainty. An adaptive fuzzy neural network FO sliding mode control technique for micro gyroscope has been presented in [37]. A new multi-layer fuzzy neural network is introduced to improve the control performance and then, the proposed controller is derived to guarantee the trajectory tracking performance. In [38], adaptive FO NTSMC based on fuzzy wavelet neural networks for mobile robots has been investigated. The proposed method uses a fuzzy wavelet neural network to estimate the model uncertainty of the system. The effectiveness of the proposed technique is verified by simulation results. The author in [39], has designed a fuzzy neural network-based FO robust control approach based on the nonlinear SMC for exoskeleton robots. The article has designed a fuzzy neural network compensator to approximate the lumped uncertainty. The simulation results confirm the superiority of the proposed intelligent robust controller. Although, the proposed methods show high robustness in the presence of lumped uncertainty, but in most of them, high computational complexity and guarantee the closed-loop control system stability is a challenging problem.

Finally, several disturbance observer-based FOSMCs for nonlinear systems are proposed in various articles [40-43]. In [40], a FO command filtered backstepping SMC based on a FO nonlinear disturbance observer for the nonlinear systems has been introduced. The controller applied a FO disturbance observer to obtain fast and improved disturbance estimation performance. FO disturbance observer-based adaptive SMC for an uncertain fractional-order nonlinear time-delayed system has investigated in [41]. The numerical simulations have verified the effectiveness of the proposed method. The authors in [42-43] have studied disturbance observer-based FO SMC for a class of nonlinear systems in the presence of the matched and mismatched disturbances. In [42], a finite-time disturbance observer is introduced to estimate the matched and mismatched disturbances and then a FO nonsingular TSMC based on the disturbance observer is designed. The simulation results have demonstrated the superiority of the proposed control method. Also in [43], a FO nonlinear SMC based on an extended nonlinear disturbance observer for the uncertain FO nonlinear systems is presented. The simulation results have confirmed the effectiveness of the proposed controller in the presence of both of the matched and mismatched uncertainties.

Motivated by the above discussion, in this research, based on Lyapunov stability concept and universal approximation theorem using fuzzy systems, we design a novel AFFOFTSMC for a class of uncertain nonlinear systems. The proposed AFFOFTSMC has some superiorities such as: robust performance in the presence of uncertainties, finite-time convergence, faster response with higher accuracy, coping with singularity problem, the chattering elimination in control signal, and higher precision control performance using FO nonlinear sliding surface instead IO one.

The rest of this paper is organized as follows: Preliminaries of fractional calculus are introduced in Section 2. Section 3, presents the problem formulation. In Section 4, the FOFTSMC is introduced. Section 5, designs the proposed AFFOFTSMC for a class of uncertain nonlinear systems. In Section 6, the effectiveness of the proposed controller is illustrated by the several numerical simulations and the results are compared with FOFTSMC and integer-order terminal sliding mode controller (IOTSMC) methods. Finally, conclusions are given in Section 7.

II. PRELIMINARIES

In this section, some basic definitions of fractional calculus are given. Three of the most commonly used definitions of fractional-order derivatives are Caputo, Riemann–Liouville and Grünwald–Letnikov. In this paper, we use the Caputo definition and the simplified notation D^α to denote the Caputo fractional-order derivative.

Definition 1. [44] The αth-order fractional integral of function $f(t)$ is defined as follows

$$ {}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad \alpha \in \mathbb{R}^+ \tag{1} $$

where $\Gamma(.)$ is the Gamma function, which is defined as $\Gamma(r) = \int_0^{\infty} e^{-\tau} \tau^{r-1} d\tau$.

Definition 2. [44] The αth-order Caputo fractional derivative of function $f(t)$ is defined as follows

$$ D^{\alpha} f(t) = {}_{t_0}^{C}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1 < \alpha < m \\ \frac{d^m}{dt^m} f(t), & \alpha = m \end{cases} \tag{2} $$

where m is the smallest integer number, larger than or equal to α.

Property 1. [44] The following equality holds for fractional-order integral for $m = 1$.

$$ D^{\alpha}\left({}_{t_0}I_t^{\alpha} f(t)\right) = f(t) \tag{3} $$

Property 2. [44] The following equality holds for the Caputo fractional-order derivative for $m = 1$.

$$ {}_{t_0}I_t^{\alpha} f(t)(D^{\alpha} f(t)) = f(t) - f(t_0) \tag{4} $$

Property 3. [45] Let $f(t)$ is $p \in \mathbb{N}$ times continuously differentiable. If there exists $l \in \mathbb{N}$ such that $l \le p$ and $\beta, \alpha + \beta \in [l-1, l]$, then $D^{\alpha}\left(D^{\beta} f(t)\right) = D^{\alpha+\beta} f(t)$ (5)

Lemma 1. [46] Let $y(t) \in \mathbb{R}^n$ be a differentiable vector. Then, for any time instant $t \ge 0$

$$ \frac{1}{2} D^{\alpha}(y(t)^T P y(t)) \le y(t)^T P D^{\alpha} y(t), \forall\ \alpha \in (0,1] \tag{6} $$

where $P \in \mathbb{R}^{n \times n}$ is a positive definite constant matrix.

Lemma 2. [47] Let $x = 0$ be an equilibrium point of the following non-autonomous fractional-order system

$$ D^{\alpha} x(t) = f(x,t) \tag{7} $$

where $\alpha \in (0,1)$ and $f(x,t)$ satisfies the Lipschitz condition with Lipschitz constant $M > 0$. Suppose that there exists a Lyapunov function $V(t, x(t))$ and a class-K functions ι_1, ι_2 and ι_3 satisfying

$$ \iota_1(\|x\|) \le V(t,x) \le \iota_2(\|x\|) \tag{8} $$

$$ D^r V(t,x) \le -\iota_3(\|x\|) \tag{9} $$

where $r \in (0,1)$. Then the equilibrium point of system (7) is asymptotically stable.

III. System Description

Consider the following class of uncertain high-order nonlinear systems

$$ \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(\boldsymbol{x},t) + b(\boldsymbol{x},t)u(t) + d(\boldsymbol{x},t) \end{cases} \tag{10} $$

where $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^T$ indicates the system state vector, $f(\boldsymbol{x},t) = f_n(\boldsymbol{x},t) + \Delta f(\boldsymbol{x},t)$ and $\Delta f(\boldsymbol{x},t)$ is an uncertain term representing the un-modeled dynamics or structural variation of the system, $b(\boldsymbol{x},t) = b_n(\boldsymbol{x},t) + \Delta b(\boldsymbol{x},t)$ and $\Delta b(\boldsymbol{x},t)$ is the input uncertainty, $u(t)$ is the control input, and $d(x,t)$ is the external disturbance.

Assumption 1. The matrices $f_n(\boldsymbol{x},t)$ and $b_n(\boldsymbol{x},t)$ are known, smooth nonlinear functions and $b_n(\boldsymbol{x},t) \neq 0$ for of all possible $\boldsymbol{x}$ and $t \ge 0$.

Assumption 2. $L(x,t)$ called the lumped uncertainty and defined as

$$ L(\boldsymbol{x},t) = \Delta f(\boldsymbol{x},t) + \Delta b(\boldsymbol{x},t)u(t) + d(\boldsymbol{x},t) \tag{11} $$

It is supposed that $L(\boldsymbol{x},t)$ is bounded $\|L(\boldsymbol{x},t)\| \le \sigma$, where σ is a known positive constant.

The controlling aim is to design a novel chattering free FO fast terminal sliding mode controller using switching fuzzy systems for stabilizing state variables of the system in the presence of uncertainties in finite time.

IV. FOFTSMC Design

The novel fractional-order fast terminal sliding surface is defined as

$$ s(t) = D^{1-\alpha} x_n + \gamma_n \mathrm{sig}(x_n)^{\zeta_n} + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \dots + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1} \tag{12} $$

where $s = [s_1, \dots, s_n]^T$, γ_i $(i = 1, 2, \dots, n)$ are designed positive constants and choose such that the polynomial $\Lambda^n + \gamma_n \Lambda^{n-1} + \dots + \gamma_2 \Lambda + \gamma_1$ is Hurwitz.

$$ \zeta_{i-1} = \zeta_i \zeta_{i+1} / (2\zeta_{i+1} - \zeta_i)\ (i = 2, \dots, n), \zeta_{i+1} = 1, \zeta_n = \zeta_0 \in (1-\epsilon, 1), \epsilon \in (0,1), \text{ and } \mathrm{sig}(.)^{\zeta_i} = |.|^{\zeta_i}\mathrm{sign}(.) \tag{13} $$

[18].

$$ D^{\alpha} s = \dot{x}_n + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \dots + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1}) = f_n(\boldsymbol{x},t) + b_n(\boldsymbol{x},t)u(t) + L(\boldsymbol{x},t) + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \dots + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1}) $$

Theorem 1. For the nonlinear system (10) with the fractional-order nonlinear sliding surface (12), if control law is designed as

$$ u(t) = -b_n^{-1}(f_n + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \dots + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1}) + \sigma + k_1 s + k_2 \mathrm{sign}(s)) \tag{14} $$

where k_1 and k_2 are known positive constants, then the system states will converge to zero in finite time.

Proof. Let the Lyapunov function is selected as

$$ V_1(t) = \frac{1}{2} s^T s \tag{15} $$

Taking the FO derivative of $V_1(t)$ and based on Lemma 1, one obtains

$$D^{\alpha}V_1(t) \le sD^{\alpha}s \tag{16}$$

Substituting (13) into (16) yields

$$\begin{aligned} D^{\alpha}V_1(t) \le s\Big(& f_n(\boldsymbol{x},t) + b_n(\boldsymbol{x},t)u(t) + L(\boldsymbol{x},t) \\ & + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} \\ & + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \cdots \\ & + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1})\Big) \end{aligned} \tag{17}$$

Applying the control law (14) into (17) results in

$$\begin{aligned} D^{\alpha}V_1 &\le s(L(\boldsymbol{x},t) - \sigma - k_1 s - k_2 \mathrm{sign}(s)) \\ &\le s(|L(\boldsymbol{x},t)| - \sigma - k_1 s \\ &\quad - k_2 \mathrm{sign}(s)) \\ &\le s(-k_1 s - k_2 \mathrm{sign}(s)) \\ &\le -k_1 s^2 - k_2|s| \\ &\le -k(s^2 + |s|) \le -k|s| \end{aligned} \tag{18}$$

where $k = \min\{k_1, k_2\}$

Consequently, the system states will converge to sliding surface $s(t) = 0$ asymptotically. Next, we show that convergence occurs in finite time.

Let the reaching time, the time for the system states to reach the sliding surface, be t_r. Using Property 2 and taking the fractional integral of both sides of (18) from 0 to t_r, one obtains

$$V_1(t_r) - V_1(0) \le -\frac{k}{\sqrt{1/2}\,\Gamma(\alpha)} \int_0^t {V_1}^{1/2}(\theta)(t-\theta)^{\alpha-1} d\theta \tag{19}$$

Since $V_1(t_r) - V_1(0) = \int_0^{t_r} \dot{V}_1(\theta)d\theta$, we can write

$$\int_0^{t_r} \frac{\dot{V}_1(\theta)}{{V_1}^{1/2}(\theta)} d\theta \le -\frac{k}{\sqrt{1/2}\,\Gamma(\alpha)} \int_0^{t_r} (t-\theta)^{\alpha-1} d\theta \tag{20}$$

$$2{V_1}^{1/2}(t)\Big|_0^{t_r} \le -\frac{k t_r^{\alpha}}{\sqrt{1/2}\,\Gamma(\alpha+1)} \tag{21}$$

Noting that $V_1(t_r) = 0$, it yields

$$t_r \le \left(\frac{2\sqrt{1/2}\,\Gamma(\alpha+1){V_1}^{1/2}(0)}{k}\right)^{1/\alpha} \tag{22}$$

Thus, it can be concluded that the system states will converge to sliding surface $s(t) = 0$ in finite time and the proof is completed.

V. Main Results

In this section, a fuzzy system is used to estimate the switching control signal $k_2\mathrm{sign}(s)$. Therefore, the switching term in the control law becomes continuous, and chattering phenomenon can be alleviated.

Based on universal the approximation theorem, fuzzy systems with singleton fuzzifier, center average defuzzifier, and product inference engine, can approximate every function in the domain. First, we define fuzzy sets $\psi_i^{\varsigma_i}$, $\varsigma_i = 1,2,\dots,\rho_i$ for input vector x_i $(i = 1,2,\dots,n)$. Next, use $\prod_{i=1}^{n}\rho_i$ fuzzy rules to construct fuzzy system $\hat{h}(\boldsymbol{x}|\omega_h)$, where j-th fuzzy rule is:

R^j: If x_1 is $\psi_1^{\varsigma_1}$ and … and x_n is $\psi_n^{\varsigma_n}$ then $\hat{h}$ is $B^{\varsigma_1,\dots,\varsigma_n}$

Hence, the output of fuzzy system is

$$\hat{h}(\boldsymbol{x}|\omega_h) = \frac{\sum_{\varsigma_1=1}^{\rho_1}\cdots\sum_{\varsigma_n=1}^{\rho_n} \bar{y}_h^{\varsigma_1\cdots\varsigma_n}\left(\prod_{i=1}^{n}\mu_{\psi_i^{\varsigma_i}}(x_i)\right)}{\sum_{\varsigma_1=1}^{\rho_1}\cdots\sum_{\varsigma_n=1}^{\rho_n}\left(\prod_{i=1}^{n}\mu_{\psi_i^{\varsigma_i}}(x_i)\right)} \tag{23}$$

where $\mu_{\psi_i^{\varsigma_i}}(x_i)$ is the Gaussian membership function of x_i and defined as

$$\mu_{\psi_i^{\varsigma_i}}(x_i) = \exp\left(-\frac{(x_i - c_i)^2}{2\varrho_i^2}\right) \tag{24}$$

where c_i and ϱ_i are the center and width of the i-th fuzzy set $\psi_i^{\varsigma_i}$, respectively.

If $\bar{y}_h^{\varsigma_1\cdots\varsigma_n}$ select as the free parameter, and put it in the set $\hat{\omega} \in R^{\prod_{i=1}^{n}\rho_i}$. Fuzzy basis function column vector $\xi(\boldsymbol{x})$ in introduced such as

$$\hat{h}(\boldsymbol{x}|\omega_h) = \hat{\omega}_h^T \xi(\boldsymbol{x}) \tag{25}$$

where

$$\xi(\boldsymbol{x}) = \frac{\prod_{i=1}^{n}\mu_{\psi_i^{\varsigma_i}}(x_i)}{\sum_{\varsigma_1=1}^{\rho_1}\cdots\sum_{\varsigma_n=1}^{\rho_n}\left(\prod_{i=1}^{n}\mu_{\psi_i^{\varsigma_i}}(x_i)\right)} \tag{26}$$

Now, we use fuzzy system $\hat{\eta}(s|\omega_\eta)$ in the form of (25) to approximate the $k_2\mathrm{sign}(s)$.

Replacing $k_2\mathrm{sign}(s)$ by $\hat{\eta}(s|\omega_\eta)$, the control law is rewritten as

$$\begin{aligned} u(t) = -b_n^{-1}\Big(& f_n + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} + \\ & \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \cdots + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1}) + \sigma + \\ & k_1 s + \hat{\eta}(s|\omega_\eta)\Big) \end{aligned} \tag{27}$$

$$\hat{\eta}(s|\omega_\eta) = \hat{\omega}_\eta^T \phi(s) \tag{28}$$

where $\hat{\eta}(s|\omega_\eta)$ is the output of the fuzzy system and $\phi(s)$ is fuzzy vector.

Let the define the optimal parameters of fuzzy system as

$$\omega_\eta^* = \arg\min_{\omega_\eta \in \Omega_\eta}\left[\sup_{x\in R^n}|\hat{\eta}(s|\omega_\eta) - k_2\mathrm{sign}(s)|\right] \tag{29}$$

where Ω_η is constraint set of ω_η, and can be defined as

$$\Omega_\eta = \{\omega_\eta \in R \mid |\omega_\eta| \le M_\eta\} \tag{30}$$

where M_η is a positive constant.

The ideal $\hat{\eta}(s|\omega_\eta)$ is

$$\hat{\eta}(s|\omega_\eta^*) = k_2\mathrm{sign}(s) \tag{31}$$

The block diagram of proposed adaptive fuzzy fractional-order fast terminal sliding mode controller with switching fuzzy system is shown in Fig. 1.

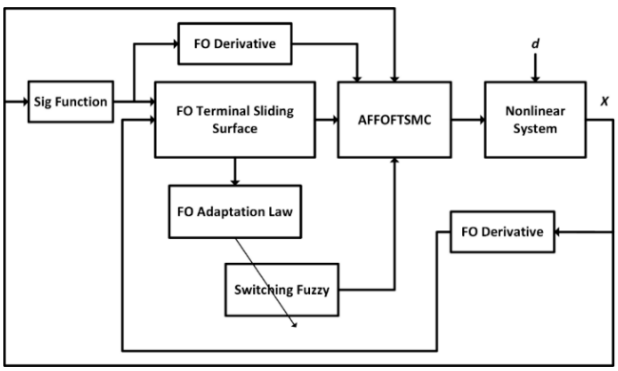


Fig. 1. Block diagram of the proposed AFFOFTSMC

Theorem 2. For the nonlinear system (10) with the fractional-order nonlinear sliding surface (12), if control law is chosen as (27) and adaptive law is designed as (32), then the system states will converge to zero in finite time.

$$D^{\alpha}\omega_{\eta} = \frac{1}{\delta}s\phi(s) \tag{32}$$

where $\delta > 0$.

Proof. Let the Lyapunov function is selected as

$$V_2(t) = \frac{1}{2}s^T s + \frac{1}{2}\delta\tilde{\omega}_{\eta}^T\tilde{\omega}_{\eta} \tag{33}$$

where $\tilde{\omega}_{\eta} = \omega_{\eta}^{*} - \omega_{\eta}$.

Taking the FO derivative of $V_2(t)$ and based on Lemma 1, one obtains

$$D^{\alpha}V_2(t) \le sD^{\alpha}s + \delta\tilde{\omega}_{\eta}^T D^{\alpha}\tilde{\omega}_{\eta} \tag{34}$$

Substituting (13) into (33) yields

$$\begin{aligned} D^{\alpha}V_2(t) \le s\Big(& f_n(\boldsymbol{x},t) + b_n(\boldsymbol{x},t)u(t) + L(\boldsymbol{x},t) \\ & + D^{\alpha}(\gamma_n \mathrm{sig}(x_n)^{\zeta_n} \\ & + \gamma_{n-1}\mathrm{sig}(x_{n-1})^{\zeta_{n-1}} + \cdots \\ & + \gamma_1\mathrm{sig}(x_1)^{\zeta_1})\Big) + \delta\tilde{\omega}_{\eta}^T D^{\alpha}\tilde{\omega}_{\eta} \end{aligned} \tag{35}$$

Applying the control law (27) into (35) results in

$$\begin{aligned} D^{\alpha}V_2 \le s\Big(& L(\boldsymbol{x},t) - \sigma - k_1 s - \hat{\eta}\big(s|\omega_{\eta}\big) \\ & + \hat{\eta}\big(s|\omega_{\eta}^{*}\big) - \hat{\eta}\big(s|\omega_{\eta}^{*}\big)\Big) \\ & + \delta\tilde{\omega}_{\eta}^T D^{\alpha}\tilde{\omega}_{\eta} \\ \le s\Big(& L(\boldsymbol{x},t) + \tilde{\omega}_{\eta}^T\phi(s) - \sigma \\ & - k_1 s - \hat{\eta}\big(s|\omega_{\eta}^{*}\big)\Big) \\ & + \delta\tilde{\omega}_{\eta}^T D^{\alpha}\tilde{\omega}_{\eta} \\ \le \delta\tilde{\omega}_{\eta}^T\Big(& \frac{1}{\delta}s\phi(s) - D^{\alpha}\omega_{\eta}\Big) \\ & + s\Big(L(\boldsymbol{x},t) - \sigma - k_1 s \\ & - \hat{\eta}\big(s|\omega_{\eta}^{*}\big)\Big) \end{aligned} \tag{36}$$

where $D^{\alpha}\tilde{\omega}_{\eta} = -D^{\alpha}\omega_{\eta}$.

Using the adaptive law (32), we have

$$\begin{aligned} D^{\alpha}V_2 &\le s(L(\boldsymbol{x},t) - \sigma - k_1 s - k_2\mathrm{sign}(s)) \\ &\le s(|L(\boldsymbol{x},t)| - \sigma - k_1 s \\ &\quad - k_2\mathrm{sign}(s)) \\ &\le s(-k_1 s - k_2\mathrm{sign}(s)) \\ &\le -k_1 s^2 - k_2|s| \\ &\le -k(s^2 + |s|) \le -k|s| \end{aligned} \tag{37}$$

where $k = \min\{k_1, k_2\}$

Therefore, the system states converge to zero asymptotically. Similar to Theorem 1, it is proved that the system states converge to zero in finite time and the proof is completed.

VI. Simulation Results

In this section, in order to verify superiority of the proposed AFFOFTSMC, three examples are given: a second-order dynamical system, a third-order dynamical system and a chaotic system. Also, the proposed method is compared with FOFTSMC, and IOTSMC in [19].

A comparative analysis based on the integral absolute value (IAV) index using different controllers for each example is presented in Tables I-III, respectively.

$$IAV = \int_0^t |V(t)|\,dt.$$

Example 1. Consider the dynamical equation of an inverted pendulum as follows [48]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\boldsymbol{x}) + b(\boldsymbol{x})u + d \end{cases} \tag{38}$$

where $f(\boldsymbol{x}) = \frac{\mathrm{g}\sin x_1 - (mlx_2\cos x_1 \sin x_1)/(m+m_c)}{l(4/3 - m\cos^2 x_1/(m+m_c))}$,

$b(\boldsymbol{x}) = \frac{\cos x_1/(m+m_c)}{l(4/3 - m\cos^2 x_1/(m+m_c))}$,

$\boldsymbol{x} = [x_1 \quad x_2]$, x_1 and x_2 are the angular position and velocity of the pole, respectively, u is the control input, $\mathrm{g} = 9.8\ \mathrm{m/s^2}$, $m_c = 1$ kg and $m = 0.1$ kg are the mass of the vehicle and the mass of the pendulum, respectively, $l = 0.5$ m is the half length of the pendulum, and $d = 1 + \sin \pi t/2$ is an external disturbance where $|d| \le 2$.

From (12) and (27), the sliding surface and the control effort are determined as:

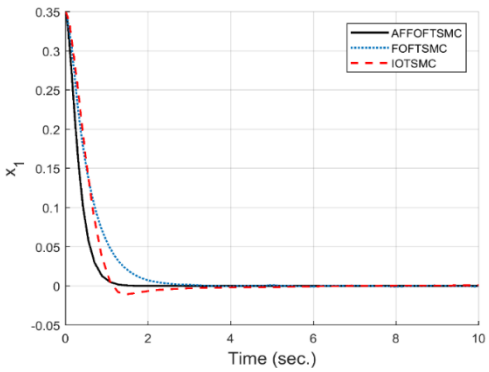
$$s(t) = D^{1-\alpha}x_2 + \gamma_2\mathrm{sig}(x_2)^{\zeta_2} + \gamma_1\mathrm{sig}(x_1)^{\zeta_1} \tag{39}$$

$$\begin{aligned} u(t) = -b^{-1}\Big(& f + D^{\alpha}(\gamma_2\mathrm{sig}(x_2)^{\zeta_2} + \gamma_1\mathrm{sig}(x_1)^{\zeta_1}) \\ & + \sigma + k_1 s + \hat{\eta}\big(s|\omega_{\eta}\big)\Big) \end{aligned} \tag{40}$$

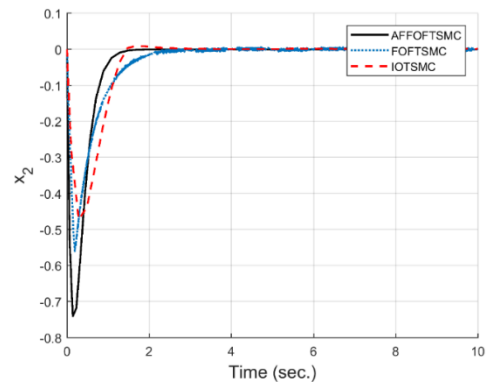
The membership function of the sliding surface defined as

$$\begin{cases} \mu_{NM}(s) = \dfrac{1}{\big(1 + \exp\big(5(s+3)\big)\big)} \\ \mu_{ZO}(s) = \dfrac{1}{(1 + \exp(-s^2))} \\ \mu_{PM}(s) = \dfrac{1}{\big(1 + \exp\big(5(s-3)\big)\big)} \end{cases} \tag{41}$$

The control parameters are given as: $\gamma_1 = 10$, $\gamma_2 = 7$, $\zeta_1 = 9/23$, $\zeta_2 = 9/16$, $\sigma = 2$, $\alpha = 0.6$, $k_1 = 2$, $\delta = 120$, and in FOFTSMC, $k_2 = 3$. The initial condition is $\boldsymbol{x}(0) = [\pi/9 \quad 0]^T$.



(a)



(b)

Fig. 2. State responses of inverted pendulum under the mentioned controllers

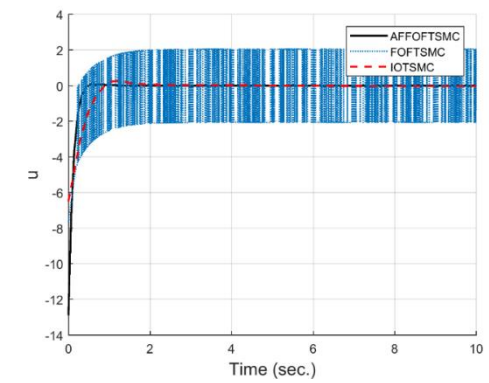


Fig. 3. The time responses of control signals for inverted pendulum

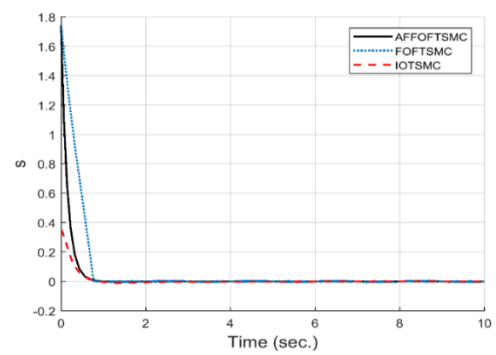


Fig. 4. The time responses of sliding surfaces for inverted pendulum

TABLE I
INTEGRAL OF THE ABSOLUTE VALUES FOR INVERTED PENDULUM

Controller	x_1	x_2	u
AFFOFTSMC	0.119	0.289	1.428
FOFTSMC	0.145	0.454	2.375
IOTSMC	0.194	0.369	2.361

In Figs. 2-4, the state responses, control inputs and sliding surfaces of the inverted pendulum system under the maintained controllers are shown, respectively. Based on Fig. 2, the state responses of the inverted pendulum system under the proposed controller have faster convergence and smaller reaching time respect to the other mentioned controllers.

It is observed from Fig. 3 that the chattering phenomenon is completely removed by the proposed AFFOFTSMC due to applying switching fuzzy, while the sign function in FOFTSMC produces serious chattering. However, control signal in IOTSMC can significantly reduce chattering due to the continuous control law, but has an overshoot and longer reaching time.

As it is seen in Fig. 4, sliding surface of AFFOFTSMC converge to the zero quicker than other sliding surfaces.

Example 2. Consider the nonlinear system as follow [49]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -(1+0.3\sin t)x_1^2 - (1.5+0.2\cos t)x_2 \\ \quad -(1+0.4\sin t)x_3 + (3+\cos x_1)u + d \end{cases} \tag{42}$$

where disturbance d is a random noise with a mean value of 0.5 and $|d| \le 0.1$.

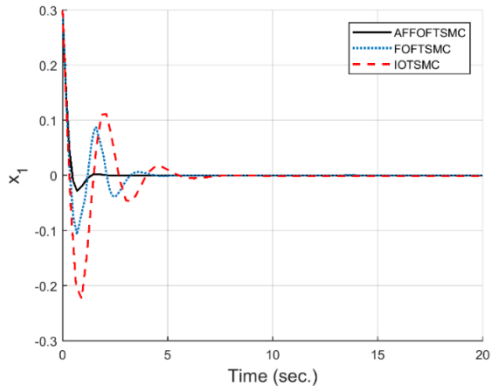
From (12) and (27), the sliding surface and the control effort are determined as:

$$s(t) = D^{1-\alpha}x_3 + \gamma_3 \mathrm{sig}(x_3)^{\zeta_3} + \gamma_2 \mathrm{sig}(x_2)^{\zeta_2} + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1} \tag{43}$$

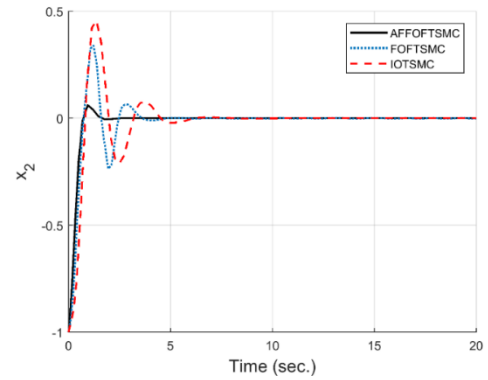
$$\begin{aligned} u(t) = (3+\cos x_1)^{-1}\Big(& (1+0.3\sin t)x_1^2 \\ & + (1.5+0.2\cos t)x_2 \\ & + (1+0.4\sin t)x_3 \\ & - D^{\alpha}(\gamma_3 \mathrm{sig}(x_3)^{\zeta_3} \\ & + \gamma_2 \mathrm{sig}(x_2)^{\zeta_2} + \gamma_1 \mathrm{sig}(x_1)^{\zeta_1}) \\ & - \sigma - k_1 s - \hat{\eta}\big(s|\omega_\eta\big)\Big) \end{aligned} \tag{44}$$

The membership function of the sliding surface is defined as (41).

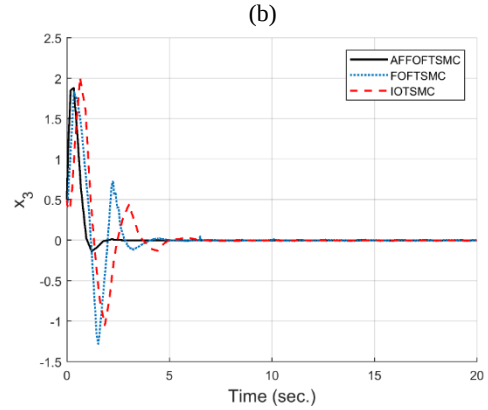
The control parameters are given as: $\gamma_1 = 80$, $\gamma_2 = 66$, $\gamma_3 = 15$, $\zeta_1 = 7/16$, $\zeta_2 = 7/13$, $\zeta_3 = 7/10$, $\sigma = 0.1$, $\alpha = 0.35$, $k_1 = 3$, $\delta = 150$, and in FOFTSMC, $k_2 = 5$. The initial condition is $\boldsymbol{x}(0) = [0.3 \quad -1 \quad 0.5]^T$.



(a)



(b)



(c)

Fig. 5. State responses of the nonlinear system under the mentioned controllers

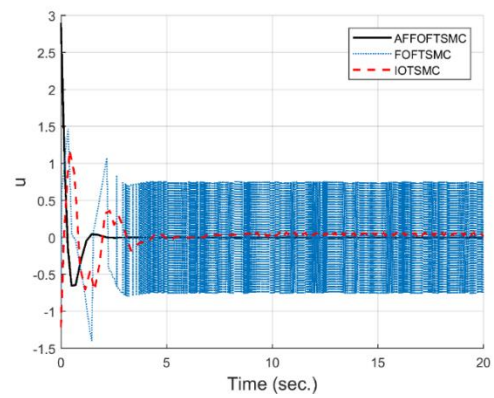


Fig. 6. The time responses of control signals for the nonlinear system

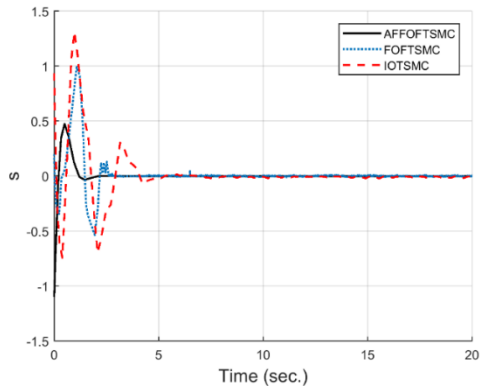


Fig. 7. The time responses of sliding surfaces for the nonlinear system

TABLE II

INTEGRAL OF THE ABSOLUTE VALUES FOR THE SYSTEM 2

Controller	x_1	x_2	x_3	u
AFFOFTSMC	0.069	0.361	1.135	0.789
FOFTSMC	0.166	0.674	1.865	1.087
IOTSMC	0.361	1.142	2.547	1.596

The time responses of system states, control inputs and sliding surfaces are shown in Figs. 5-7, respectively.

Based on Fig. 5, using proposed AFFOFTSMC, the state variables of the system have faster convergence and high robustness against uncertainties compared to other mentioned controllers. Also, state response curves of FOFTSMC and IOTSMC methods have large overshoots and undershoots, and much longer reaching times compared to the proposed method.

Fig. 6, illustrates that control input obtained by the proposed AFFOFTSMC is smooth and free of chattering. It is obvious, though chattering existing in the control input obtained by IOTSMC is significantly lesser compared to that in FOFTSMC, but as it observes in Fig. 5, the time that the system states converge to zero is much longer.

It can be seen from Fig.7 that the proposed sliding surface converge to zero faster than other sliding surfaces. On the other hand, sliding surface in IOTSMC is oscillated around the origin.

Example 3. Consider the uncertain chaotic Genesio system described as [50]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -6x_1 - 2.92x_2 - 1.2x_3 + x_1^2 + \Delta f(\boldsymbol{x}, t) + u + \end{cases} \quad (45$$

The uncertain term $\Delta f(\boldsymbol{x}, t) = 0.1\sin(\pi x_1)\sin(2\pi x_2)\sin(3\pi x_3)$, where $|\Delta f(\boldsymbol{x})| \leq 0.1$ and $d = 0.1\cos(3t)$, where $|d| \leq 0.1$.

From (12) and (27), the sliding surface and the control effort are determined as:

$$s(t) = D^{1-\alpha}x_3 + \gamma_3 \text{sig}(x_3)^{\zeta_3} + \gamma_2 \text{sig}(x_2)^{\zeta_2} + \gamma_1 \text{sig}(x_1)^{\zeta_1} \quad (46)$$

$$u(t) = 6x_1 + 2.92x_2 + 1.2x_3 - x_1^2 - \sigma - D^{\alpha}(\gamma_3 \text{sig}(x_3)^{\zeta_3} + \gamma_2 \text{sig}(x_2)^{\zeta_2} + \gamma_1 \text{sig}(x_1)^{\zeta_1}) - k_1 s - \hat{\eta}(s|\omega_\eta) \quad (47)$$

In (40), the membership function of the sliding surface is described.

The control parameters are given as: $\gamma_1 = 80$, $\gamma_2 = 66$, $\gamma_3 = 15$, $\zeta_1 = 7/16$, $\zeta_2 = 7/13$, $\zeta_3 = 7/10$, $\sigma = 0.2$, $\alpha = 0.85$, $k_1 = 5$ and $\delta = 200$, and in FOTSMC, $k_2 = 10$. The initial condition is $\boldsymbol{x}(0) = [-1 \quad 1 \quad 0]^T$.

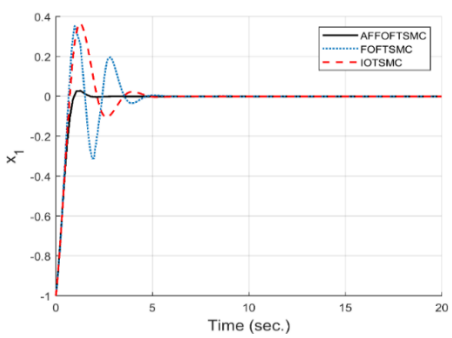
The state responses of the chaotic Genesio system are shown in Fig. 8. It is evident that the state trajectories of the chaotic system under the proposed AFFOFTSMC reach the equilibrium point in finite time. Although, other mentioned controllers can suppress chaos and stabilize the system states around the equilibrium point, but it is occurring in much longer time.

Fig. 9, shows the time response of the control inputs and the time response of the sliding surfaces are depicted in Fig. 10. Clearly, the proposed control method eliminated chattering while achieving small reaching time. Finite-time and smaller sliding surface of the proposed AFFOFTSMC confirmed excellent performance of the method compared with the other mentioned controllers.

TABLE III

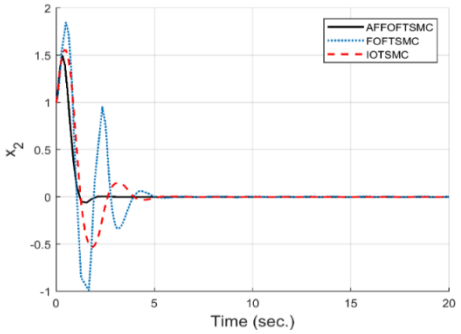
INTEGRAL OF THE ABSOLUTE VALUES FOR THE GENESIO SYSTEM

Controller	x_1	x_2	x_3	u
AFFOFTSMC	0.416	1.069	2.139	9.877
FOFTSMC	0.642	1.856	4.365	17.19
IOTSMC	0.804	1.997	3.558	10.64

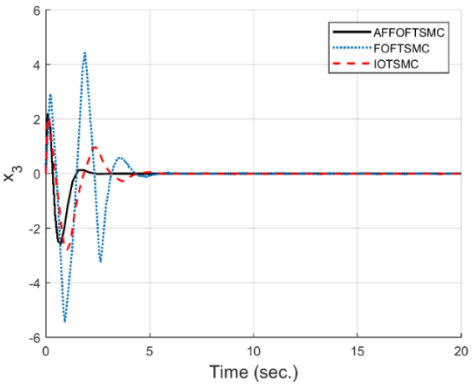


(a)

Fig.8. Continued



(b)



(c)

Fig. 8. State responses of chaotic Genesio system under the mentioned controllers

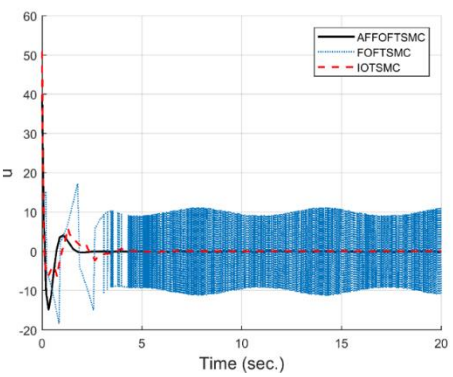


Fig. 9. The time responses of control signals for chaotic Genesio system

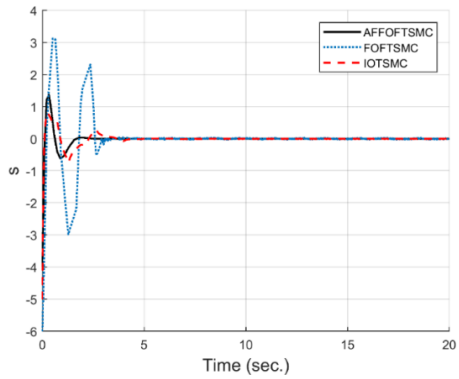


Fig. 10. The time responses of sliding surfaces for chaotic Genesio system

VII. CONCLUSIONS

This paper studies a procedure for the design of a FO robust controller using a combination of sliding mode control technique and fuzzy systems to stabilize the state variables and the finite-time convergence of a class of nonlinear systems in the presence of model uncertainty and external disturbances.

The finite-time stability of the closed-loop control system using Lyapunov stability theorem has been proved and the fuzzy system through the online adaptive learning laws is utilized to approximate the switching control term. Unlike integer-order fast terminal sliding mode controller, the FO controller has more degree of freedom to increase control performance accuracy. Furthermore, applying the fuzzy system helps to achieve a chattering free control signal.

The proposed control method was successfully applied to the three nonlinear systems. The numerical simulation results have been confirmed the superior performance of the proposed control method comparing to the other mentioned controllers.

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Amir Razzaghian received his M.Sc. degree in Control Engineering from the Department of Electrical Engineering, Islamic Azad University of Mashhad, Iran in 2015. He completed his Ph.D. in Control Engineering at Islamic Azad University of Mashhad, Iran in 2020. His main research interests include Nonlinear Control, Fractional-order Systems, Fuzzy Systems and Neural Networks.



Reihaneh Kardehi Moghaddam received her B.Sc. degree in Bioelectric Engineering from Amirkabir University of Technology, Tehran, Iran in 2001. She received her M.Sc. and Ph.D. degrees in Control Engineering from Ferdowsi University of Mashhad, Iran in 2004 and 2010, respectively. Currently, she is an Associate Professor at Islamic Azad University of Mashhad, Iran. Her main research interests include Nonlinear Control and Optimal Control.



Naser Pariz received his B.Sc. and M.Sc. degrees in Electrical Engineering from Ferdowsi University of Mashhad, Iran in 1988 and 1991, respectively, and Ph.D. degree from the Department of Electrical Engineering, Ferdowsi University of Mashhad, Iran in 2001. From 1991 to 1995, he was a Lecturer at Ferdowsi University of Mashhad, where he is currently a Full Professor. His main research interests include Nonlinear Control, Hybrid Systems and Applied Mathematics.

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