

Designing an Indirect Adaptive Multiple Controller for LTI Systems with Large Time-Varying and Unknown Delays in Control Input Based on Online Estimation of Delay by Kalman Filtering

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The present study presents an adaptive controller for LTI systems with an unknown and time-variant input time delay with the purpose of tracking. Due to the wide area considered for time delay variations, the structure of the proposed controller is considered to be in the form of Multiple Adaptive Control (MAC). The presented adaptive control system is of indirect type, i.e., at any moment, one band of time delay is first identified using a proposed estimator, and then the main control signal, which is a linear combination of multiple controllers output, is formed with a switching rule in the supervisory subdivision. In fact, each multiple controller in the MAC structure with fit weights participates in forming the main control signal. The multiple controllers used in this study are of PID type. It should be noted that the parameters for the individual multiple controllers, for the system under control, are adjusted offline and proportional to its corresponding time-delay sub-band using a genetic algorithm. Finally, simulation results show the relatively desirable performance of the proposed control system and observer in facing with wide unknown and time-variant delays.

Article Info

Keywords:

Adaptive control, Delay estimator, Input delay, Kalman filtering, LTI Systems, Satellite telecommunications, Time-Variant uncertainty.

Article History:

Received 2020-01-14

Accepted 2020*-08-31

I. INTRODUCTION

The control of delayed systems is among the issues of the day in the world of Control Engineering. The existence of delays in most real and manmade systems has attracted the attention of many researchers to this issue. The importance of this issue arises from the fact that delay in a control loop is accompanied by the system divergence from the desirable performance, and if the necessary preparations are not considered, it may even lead to system instability. Since inherent delays are an inseparable component of a delayed

system, considering delays in the controller design process is the only strategy for encountering this issue. Even if an open-loop system could be modeled without any delay, this point should be kept in mind that the closed-loop control process itself, i.e. measurement, decision-making, and reaction of the controller, causes a delay in the plant input; a delay that can also be affected by the varying environment. Time delay systems introduce a class of systems with an infinite dimension, which is related to mechanisms such as dispersion, transmission, traffic, exhaustion, flexibility, inertia, and other factors that are created after time drop-off. From the perspective of time delay certainty and variability, delayed systems are generally categorized into four groups: (1) systems with a certain and time-invariant delay, (2)

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systems with a certain and time-variant delay, (3) systems with an uncertain and time-invariant delay and, (4) systems with an uncertain and time-variant delay.

All these four types may appear in different systems. Concerning the fourth group, it is of great importance to discuss a control algorithm that can fulfill the stability or tracking performance. In general, a delay may exist in a state vector or an input. In this article, time delay means the delay in control input. In most real systems, time delay affects the system performance either inherently or under the impression of environmental conditions. The problems that mainly emerge in a closed-loop control process of delayed systems are caused by the following factors [1]:

- 1- The effect of disturbances is not felt until a considerable time has elapsed.
- 2- The effect of the control action takes some time to be felt in the controlled variable.
- 3- The control action that is applied based on the actual error tries to correct a situation that has originated some time before.
- 4- Since system equations are nonlinear in relation to time delay parameters, the sensitivity of control characteristics to these parameters is relatively high, so researchers usually look for a robust design given the uncertain nature of time delay.
- 5- Changes in an open-loop transfer function time delay in closed-loop control systems lead to phase margin change and gain margin indirect change too; this may lead to closed-loop system instability or may decrease relative stability, and hence it may cause transient response characteristics to become undesirable.

Considering that the system behavior is nonlinear to the delay factor, designing techniques for making appropriate control reaction gets more complicated by delay change over time because the designer will also face a time-variant system (certain or uncertain). When a delay is time-varying and unknown, it is often dealt with as a parameter with uncertainty, so the designing robustness against any changes in this parameter should be discussed. Robust control always provides appropriate control strategies for parametric uncertainties [2]-[3], but if these uncertainties change slowly, adaptive control would definitely be a more appropriate option. In particular, the largeness of the variation area for an uncertain parameter always persuades the designer to use a specific area in adaptive control, named Multiple Adaptive Control (MAC). This study assumes that any change in an unknown parameter, i.e., time delay, is in the LTI system, so the MAC structure is used to control this system. In this structure, the wide uncertainty band is partitioned into smaller sub-bands, and for each selected band, a suitable controller is designed. The controller at any moment is selected based on the delay estimation at that very moment. Also, regarding time delay estimation, this paper presents a

new method with a rather high accuracy and speed. In the multiple control structure, the supervisory subdivision is responsible for switching between controllers. Switching between the designed controllers, due to its discrete nature, may by itself be a reason for instability. Picking a suitable switching rule decreases the undesired effects of the controller variation in the control loop and the control signal effort. Using combined methods, in which the control signal is a weighted combination of multiple controllers output, is accompanied by the risk of instability because of the increase in control signal gain. Therefore, choosing a suitable switching rule is as important as appropriately designing the controllers of each band. Using modern optimization methods, in addition to providing an appropriate stability margin for switching operation, may provide optimal conditions for controlling a delayed system. In this study, it has been tried to use an optimal approach in the structure of a MAC to reduce the switching effect of controllers and optimally adjust the parameters of the controllers related to each delay band in order to provide a rather desirable performance for systems with wide variable delay. Further, Subsection A provides a brief history of what has been done in this regard so far. Subsection B focuses on the challenges of the current control methods. Section II introduces the structure of the plant, and then Section III presents the proposed estimator for estimating a time-variant delay. Section IV provides a brief description of the MAC control structure and the proposed controller. Finally, Section V presents the results of the simulations for a sample system and Section VI analyzes the performance of the proposed controller and concludes the paper.

A. Research Literature

The difficulty of time delay identification comes from the fact that the process model is nonlinear in relation to the delay parameter. Based on the literature studied in Ref. [4], the delay identification methods are divided into four groups: (a) time delay approximation methods, (b) time delay explicit parameter methods, (c) area and moment methods, and (d) higher-order statistics (HOS) methods. Several methods of this category have been presented in papers [4]-[11] such that in case of adaptive control and real-time control, methods of groups (a) and (b) are mainly used. Particularly, for time-varying delay estimation in LTI systems, many articles have been published [12]-[19].

Generally, a time delay in open-loop systems is a non-minimum phase factor. It has been proven in many studies that the stability of control loops, in the presence of delay, is too sensitive to time delay parameters, so this has made researchers try hard to present reliable criteria for creating a safe stability margin in the area of designing controllers for such systems [20]-[26].

Also particularly for LTI systems with uncertainty in time delay factor, a lot of efforts have been put on presenting criteria

for guaranteeing robust stability [27]-[29].

The importance of controlling systems with a time-varying delay has provoked researchers to design controllers that are acceptably robust and adaptable against variations of the sensitive time delay parameter in order to satisfy the supreme control purposes such as reference input tracking.

Some of these controllers are as follows: robust controllers [30]-[32], fuzzy controllers [33], predictive controllers [34]-[37], sliding mode controllers [38]-[39], and optimal controllers [40]-[42]. In particular, the presented controllers are mostly of adaptive type for LTI systems with an unknown and time-varying delay [43]-[48].

B. Challenges of the Current Control Methods

Pondering upon the works done in the area of controlling systems with a time-varying delay, it seems that they have some problems and challenges. Generally, in all studies on delayed systems, the band of time delay variations and uncertainty are assumed to be rather small. Although robust controllers and sliding mode show rather appropriate response in small uncertainty bands, since the nature of these controllers is unit and constant, the control system response in great perturbations of the nominal model, even with precise modeling of uncertainties, would be of undesired transient characteristics, especially if the variations in time delay parameters is rapid. Particularly, in sliding mode controllers, large and fast time delay variations result in chattering and hence control effort.

Predictive and optimal controllers are often presented for systems in which the time delay is known (constant or time-variant) because if the delay parameter is time-varying and unknown, these controllers perform weakly, since both of these controllers have an optimization operation, and if the delay is time-varying and unknown, the designer should deal with a time-variant and unplanned optimization problem, the solution of which, due to the computational complications, may in turn cause time delay in the control process of real-time mode [49].

Also, intelligent controllers, such as fuzzy and neural ones, partly have lower spreading speed in comparison to real time due to the computational and decision making complications of the network, which, in turn, results in time delays in control loops.

Eventually, the adaptive controllers presented for systems with a time-varying and unknown delay are mainly assumed to have a small band of uncertainty. The reason for this is that the large band of uncertainty causes a decrease in the estimation speed, hence a decrease in the adaptability of the controller and a decrease in the speed of control reactions, which by themselves lead to the emergence of undesired transient mode errors in response to the control system.

II. PROBLEM STATEMENT

The plant in this research is an LTI and a stable system

(BIBO Stable). The state-space equations and the Laplace transfer function of the assumed system are as follows:

$$\begin{cases} \dot{X}(t) = A_g X(t) + B_g u_c(t - d(t)) \\ y(t) = C_g X(t) \end{cases} \quad (1)$$

$$P(s, t) = G(s)e^{-d(t)s} \quad (2)$$

in which t is the variable of real-time domain, $X(t) \in R^n$ is the state vector of the system, scalars $u_c(t), y(t)$ are control input and plant output, respectively, and A_g, B_g, C_g are the constant matrices with appropriate dimensions. Also, s is the variable of the Laplace domain and $P(s, t)$ is the system transfer function at the time t , in which $G(s)$ is obtained from Eq. (3).

$$G(s) = C_g(sI - A_g)^{-1}B_g \quad (3)$$

In addition, the $e^{-d(t)s}$ factor is the Laplace transfer function of the time delay. In Eq. (1) and (2), $d(t)$ is the same input time delay of the system, which is assumed to be time-variant and unknown in this study, and it is also assumed that the considered system, except for t from a time delay factor, is a linear and time-invariant system. In general, this study makes the following assumptions for the time delay [50]:

- 1- $0 \leq d_{min} \leq d(t) \leq d_{max} \leq \infty$
- 2- $-\infty < \dot{d}(t) < 1$

The main purpose of this research is to design a tracking controller in the presence of a time-variant and unknown time delay, in which the band of the time delay uncertainty is rather wide.

III. PROPOSED DELAY ESTIMATOR

In general, an estimator is a dynamical system that estimates one or more unknown parameter(s) of the real system using the real system input and output. Particularly, in this study, one of the important goals is to design an estimator system in that the only unknown and time-variant parameter of the system is identified at an online mode and with an acceptable speed using the input and output of the system with Eq. (2). A simple block diagram of the system and delay estimator is shown in Fig. 1. As can be seen in Fig. 1, the estimator inputs are the same input and output of the real system, and the only output of the estimator is an estimation of the time delay ($\hat{d}(t)$).

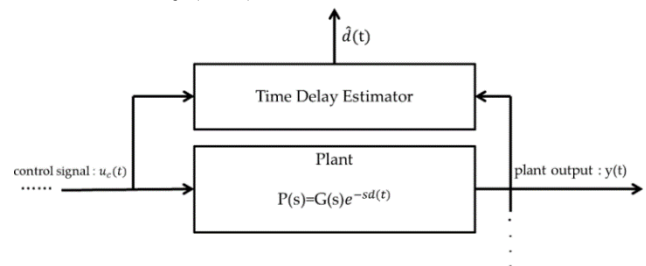


Fig. 1. The general diagram of the time delay estimator.

According to Eq. (2), a delayed system transfer function includes a time delay factor (exponential and non-rational). In this research, to design the proposed estimator, instead of a time delay exponential factor, a pade approximation is first used to rationalize the system transfer function. In this case, the new transfer function, which is an approximation of the main transfer function of the system, will include a time delay parameter (time-variant). After writing a state-space realization from the mentioned transfer function, and by regarding the time delay parameter as an extra state variable parameter, a system with nonlinear state equations will be established. Eventually, using a Kalman filter (linear or extended for linearized or non-linear state equations, respectively), the states of this system, e.g. the system time delay, are predicted.

In this research, to further describe the estimator design in detail, a first-order pade approximation is used as an alternative for the time delay factor. Obviously, higher-order pade approximations can be used to improve the estimation accuracy. Eq. (4) is indicative of a first-order pade approximation for the time delay factor in the Laplace area [1].

$$C'(s) = \frac{1 - \frac{d}{2}s}{1 + \frac{d}{2}s}, \quad \hat{d} = \hat{d}(t) \quad (4)$$

Substituting Eq. (4) in Eq. (2), the system transfer function will be as follows:

$$P(s) = G(s) \frac{1 - \frac{d}{2}s}{1 + \frac{d}{2}s} \quad (5)$$

Considering that the $G(s)$ transfer function is of n -order, the $P(s)$ transfer function will definitely be of $(n+1)$ -order. If the delay is assumed a constant parameter, in general, A_p, B_p, C_p matrixes, for a realization of $P(s)$ system, will be a function of delay parameter, i.e. d . Therefore, $P(s)$ system state equations could be written as Eq. (6).

$$\begin{cases} \dot{Z}(t) = A_p(d).Z(t) + B_p(d).u(t) \\ y(t) = C_p(d).Z(t) \end{cases} \quad (6)$$

Where $Z(t) \in R^{n+1}$ is the $P(s)$ system state variables vector, and $A_p(d), B_p(d), C_p(d)$ are the matrixes with appropriate dimensions, which are dependent on the time delay parameter. Also, $y(t)$ and $u(t)$ scalars are the input and output of $P(s)$ system, respectively. Given that the parameter d is constant, the equations in Eq. (6) will be relevant to an $(n+1)$ -order LTI system. Considering that $d(t)$ is, in reality, an unknown and time-variant parameter, it could be regarded as an extra state variable in Eq. (6). In this case, Eq. (6) will turn into a group of nonlinear state-space equations, whose order is changed from $(n+1)$ to $(n+2)$ due to the addition of a new state variable (time delay). State-space equations for this system will conform to Eq. (7).

$$\begin{cases} \dot{w}_1(t) = z_1(t) = f_1(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t)) + \\ g_1(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t), u(t)) + v_1(t) \\ \vdots \\ \dot{w}_{n+1}(t) = z_{n+1}(t) = f_{n+1}(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t)) + \\ g_{n+1}(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t), u(t)) + v_{n+1}(t) \\ \dot{w}_{n+2}(t) = d(t) = f_{n+2}(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t)) + \\ g_{n+2}(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t), u(t)) + v_{n+2}(t) \\ y(t) = h(w_1(t), w_2(t), \dots, w_{n+1}(t), w_{n+2}(t)) + n(t) \\ \begin{cases} \dot{W}_{(t)} = F(W_{(t)}) + G(W_{(t)}, u(t)) + V_{(t)} \\ y(t) = h(W_{(t)}) + n(t) \end{cases} \end{cases} \quad (7)$$

Comparing Eq. (7) and Eq. (8), $G(\cdot)$ and $F(\cdot)$ matrix functions and also $V_{(t)}$ vector in the above equation are identified. After computing the system state space equations in the structure of Eq. (7), all the system state variables, including the system time delay, should be estimated using a state observer in the next step. At this point, all system states can be estimated with a Kalman filter (linear or extended) [51]. Further details on the design of this estimator are provided in Ref. [18].

IV. PROPOSED CONTROLLER

In general, controlling the system based on the estimated situation is an adaptive control issue, in which the controller parameters are adjusted either directly or indirectly. Experience shows that using adaptive control in cases where the band of uncertainty for one or more parameter(s) is rather large, large transient state errors occur because the convergence speed of controller parameters (direct or indirect adjustment) towards the desired parameters is low. In such cases, an alternative method is to use MACs to control an unknown system; a method that is considered a higher level of adaptive control. The advantages of this control method, as mentioned in the literature, are high speed, ability to control systems with rather wider parametric uncertainties, and smaller control effort (in case of optimal designing for a safe and soft switching) [52]-[55].

In general, a multiple adaptive controller can be used in two structures, direct and indirect. In both control structures, an essential element in the MAC controller is the supervisory and switching section of the controller. Thus, presenting better methods in the switching section to improve stability and transient behavior of the plant has always been among the concerns of researchers while designing these controllers [52].

Fig. 2 presents an indirect multiple adaptive controller (IMAC), in which, an estimation of the plant's unknown parameters is first obtained by an estimator block, and then, considering the band in which the estimated parameters exist, each of the multiple controllers, with specific weights, take part in forming the main control signal [52].

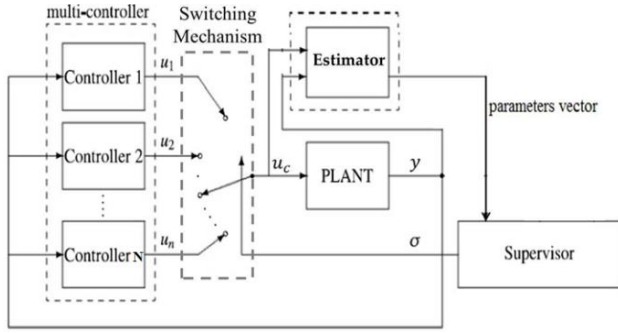


Fig. 2. The proposed structure of the IMAC controller.

Given the challenges and difficulties of control methods for systems with a varying delay, presented in Section I (subsection B), and also taking into account the advantages of multiple adaptive controllers, this study uses an IMAC control structure to control LTI systems with an input time-varying delay. For this purpose, the considered time delay of the plant is an unknown and variable parameter. The control process will be such that first, according to Fig. 3, the area of the time delay variations will be partitioned into smaller sub-bands (N sub-bands). Then, depending on the band in which the system delay has been estimated, in the IMAC structure, multiple controllers with fit weights will enter the control loop. In Fig. 3, M_i shows the center of the i^{th} sub-band.

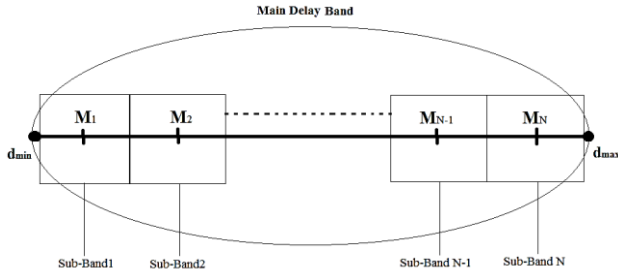


Fig. 3. How to split the main band of the delay into N equal sub-bands.

In this study, assuming that the control structure is the one presented in Fig. 2, a number of PID controllers in the multiple controllers section are used [56]. The parameters of each multiple controller (proportional, integral, and derivative coefficients) are adjusted at offline mode and by solving an optimization problem. The minimization method used in this paper is a genetic algorithm [57]. It should be said that the parameters of each of the multiple controllers are optimized independently for its corresponding delay sub-band. In this regard, the fitness function counted for the optimization problem is considered to be affected by the control signal effort, and the error between plant output and desired output (reference input). Eq. (9) expresses the mentioned fitness function.

$$\int (\alpha(y(t) - u_R(t))^2 + \beta(u_c(t + \Delta t) - u_c(t))^2) dt \quad (9)$$

Where α and β are positive and real constants that express the importance of minimization for tracking error and control effort, respectively. These are assumed to be equal in this article. In this study, a suitable method for adjusting controllers weights have been used to avoid the adverse effects of discrete switching of the controllers (hard switching) such that, according to Eq. (10), the main control signal, i.e. $u_c(t)$, is a linear and weighted combination of the output of all multiple controllers.

$$\begin{aligned} u_c(t) &= A^T \times U(t) = \sum_{i=1}^N a_i u_i(t) \\ A &= [a_1 \ a_2 \ \dots \ a_N]^T, \quad U(t) = [u_1(t) \ u_2(t) \ \dots \ u_N(t)]^T \end{aligned} \quad (10)$$

Where $u_i(t)$ is the i^{th} controller output at time t , N is the number of multiple controllers, and a_i is the weight of the i^{th} controller, which is obtained from Eq. (11):

$$a_i = \frac{e^{-b_i}}{\sum_{j=1}^N e^{-b_j}} \quad (11)$$

Where b_i , at any moment of time, is obtained from Eq. (12).

$$b_i = |K(\hat{d}(t) - M_i)| \quad (12)$$

Assuming that the main uncertainty band for a time delay is partitioned into N sub-bands, in Eq. (12), M_i will be the center of the i^{th} sub-band. Thus, as seen in the above equations, the weight of the controllers is a function of the absolute value of the error between the average value of delay bands and the value of the estimated delay. It is obvious that, at any moment, the closer the delay estimation is to the average value of the delay in a sub-band, the more weighted the controller corresponding to that band will participate in creating the main control signal, and also at any moment, the sum of the weights of all controllers is equal to one, and this factor always prevents the increase in the open-loop dc gain. Parameter $K \geq 0$ in Eq. (12) is a designing parameter. In this equation, K determines the type of switching such that if $K = 0$, switching will be of soft and continuous type (no switching operation), and each controller with a share of $\frac{1}{N}$ will always participate in forming the main control signal, and by an increase in K from zero to infinite, the type of switching tends to be discrete and hard. More details of the design will be presented in Section V for a specific system in a numerical example.

V. SIMULATION RESULTS

In this section, to implement different simulations and assess the proposed estimator and controller, an LTI system with the following open-loop transfer function is selected. It should be noted that the phase margin (PM) for this system is 1.7068 [rad] and the delay margin is 2.6364 [sec]. This transfer function relates to a direct current (DC) servo motor system, which expresses the relationship of the motor output (rotation angle) to its input (electricity current). Since the motor in a telecommunication satellite is responsible for

adjusting the orientation of an antenna, there is always a time-varying unknown delay between sending the control signal from the earth station and receiving it in the space station [58].

$$P(s, t) = \frac{3(1+s)}{(1+2s)(1+3s)} e^{s \times d(t)} \quad (13)$$

In which $d = d(t)$ is an unknown and time-variant delay. To design the delay estimator, the band of delay variations should be specified, so it is assumed that $d_{min} = 0.65[s] \leq d \leq d_{max} = 4.65[s]$. Given that Eq. (4) is used for time delay factor approximation, $P(s)$ could be rewritten as Eq. (14).

$$P(s, t) = \frac{-0.5s^2 + (\frac{1}{d} - 0.5)s + \frac{1}{d}}{s^3 + (\frac{2}{d} + 0.83)s^2 + (\frac{1.66}{d} + 0.17)s + \frac{0.33}{d}} \quad (14)$$

By writing an observer canonical realization for the above system, the state space equations of this system will be obtained as follows:

$$\begin{cases} \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \\ \dot{z}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-0.33}{d} \\ 1 & 0 & -(\frac{1.66}{d} + 0.17) \\ 0 & 1 & -(\frac{2}{d} + 0.83) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{d} \\ \frac{1}{d} - 0.5 \\ -0.5 \end{bmatrix} u(t) \\ y(t) = [0 \quad 0 \quad 1] \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} \end{cases} \quad (15)$$

Considering $d(t)$ as the fourth state variable ($d = w_4(t)$), and defining $w_1(t) = z_1(t)$, $w_2(t) = z_2(t)$, $w_3(t) = z_3(t)$, nonlinear state space equations for this system are obtained according to Eq. (16):

$$\begin{cases} \dot{w}_1(t) = \frac{-0.33w_3(t)}{w_4(t)} + \frac{u(t)}{w_4(t)} + v_1(t) \\ \dot{w}_2(t) = w_1(t) - (\frac{1.66}{w_4(t)} + 0.17)w_3(t) + (\frac{1}{w_4(t)} - 0.5)u(t) + v_2(t) \\ \dot{w}_3(t) = w_2(t) - (\frac{2}{w_4(t)} + 0.83)w_3(t) - 0.5u(t) + v_3(t) \\ \dot{w}_4(t) = v_4(t) \\ y(t) = w_3(t) + n(t) \end{cases} \quad (16)$$

It is obvious that these equations are used to design the extended Kalman filter (EKF); and to implement the delay estimator, based on a linear Kalman filter (LKF), the system with equations presented in Eq. (16) should be linearized around a fixed operating point.

In this research, the results of the simulations will be presented along with two main objectives; the first is to design a time delay estimator for the system with Eq. (13), and the second is to design a tracking adaptive controller for this system. In all simulations, the reference input signal for the system has been considered a periodic pulse wave; a two-level periodic pulse wave with a periodicity of 40 seconds and a pulse width of 20 seconds (levels with the values of 10 and 0, respectively). It should be noted that all measurements are in the presence of noise.

The results of the simulations are presented below to estimate the system delay with Eq. (13). It should be noted that this estimate is in the loop and in the presence of the controller. Fig. 4 shows the performance of the proposed estimators in estimating sinusoidal delay. In this figure, the

real delay of the system equals $d(t) = 2.65 + 2 \sin(0.025\pi t)$. To accurately compare the performance of these estimators, the mean of squares of estimation error (MSE) for all two estimators have been calculated, which are shown in Table I. According to what is presented in Fig. 4 and Table I, it could be said that the accuracy of the second proposed estimator is higher than that of the first proposed estimator.

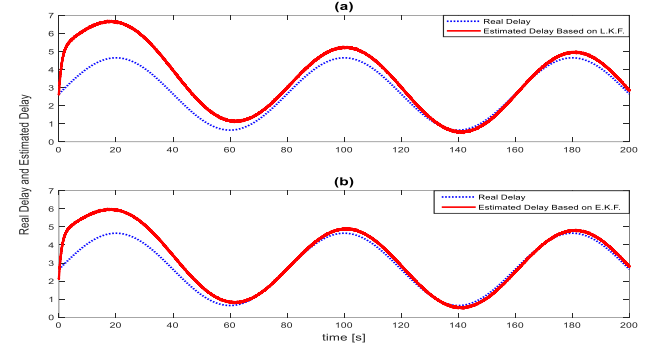


Fig. 4. The comparison of the two proposed estimators in the sinusoidal time delay estimation problem.

(a) Delay estimator based on LKF, (b) Delay estimator based on EKF.

TABLE I

THE COMPARISON OF THE ACCURACY OF THE PROPOSED ESTIMATORS IN SINUSOIDAL TIME DELAY ESTIMATION PROBLEM.

Delay Estimator	Mean of Squares of Estimation Error
Proposed Method 1 (P.M.1)	0.68
Proposed Method 1 (P.M.2)	0.32

Next, the design of a tracker controller in accordance with the proposal in Section IV will be the goal so that the output of the closed-loop system will follow the reference input signal with a minimum error. For this purpose, four PID controllers are considered as multiple controllers ($N = 4$); that, in an offline optimization problem, with the cost function according to Eq. (9), the parameters of these controllers have to be calculated. It should be noted that the interval of delay variations should also be divided into the following four sub-intervals:

$$\begin{cases} 0.65 \leq d < 1.65 & \rightarrow M_1 = 1.15 \\ 1.65 \leq d < 2.65 & \rightarrow M_2 = 2.15 \\ 2.65 \leq d < 3.65 & \rightarrow M_3 = 3.15 \\ 3.65 \leq d \leq 4.65 & \rightarrow M_4 = 4.15 \end{cases} \quad (17)$$

As was mentioned in Section IV, M_i is the center of the i^{th} sub-frame of delay. It should be noted that each of the multiple controllers corresponds to a sub-band of the time delay. The corresponding transfer functions for multiple controllers are as follows:

$$\begin{cases} PID_1(s) = K_{P1} + K_{I1} \frac{1}{s} + K_{D1}s \\ PID_2(s) = K_{P2} + K_{I2} \frac{1}{s} + K_{D2}s \\ PID_3(s) = K_{P3} + K_{I3} \frac{1}{s} + K_{D3}s \\ PID_4(s) = K_{P4} + K_{I4} \frac{1}{s} + K_{D4}s \end{cases} \quad (18)$$

Where $PID_i(s)$ is the i^{th} controller transfer function (for $i = 1, \dots, N = 4$). In Eq. (18) K_{Pi} , K_{Di} and K_{Ii} are proportional, derivative, and integral coefficients for the i^{th} controller, respectively. Also according to Eq. (10)-(12) in Section IV, the supervisory rule for switching the above controllers will be as displayed in Fig. 5 where it is assumed that $K = 1$ is assumed.

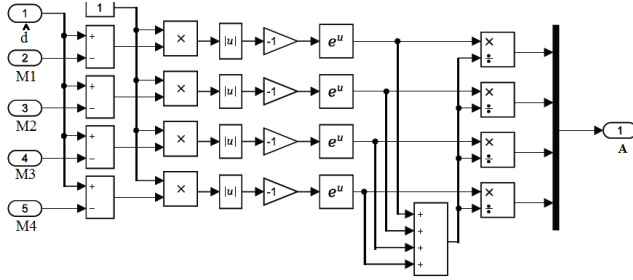


Fig. 5. The block diagram of the switching mechanism of multiple controllers.

After solving four optimization problems in the 3D space by a genetic algorithm, the parameters of the multiple controllers are set as follows:

$$\begin{cases} PID_1: K_{P1} = 0.74, K_{I1} = 0.11, K_{D1} = 0.12 \\ PID_2: K_{P2} = 0.55, K_{I2} = 0.08, K_{D2} = 0.25 \\ PID_3: K_{P3} = 0.25, K_{I3} = 0.06, K_{D3} = 0.11 \\ PID_4: K_{P4} = 0.17, K_{I4} = 0.04, K_{D4} = 0.02 \end{cases} \quad (19)$$

By specifying the delay estimator, switching, and multiple controllers mechanisms, simulation results for the proposed controller performance in several scenarios are presented below. To understand the importance of controlling the system specified in Eq. (13), the Nyquist diagram of this system for various time delays (available in the main band of delay variations) is shown in Fig. 6. As can be seen in this figure, the closed-loop system becomes unstable for delays of more than $DM=2.6364$ seconds because the Nyquist plot for the open-loop system has encircled clockwise the critical point -1 . In such a situation, due to the variability of the time delay of the system, by fluctuating the delay signal around its critical value, the closed-loop system experiences various states of stability and instability, making it important to control these types of systems.

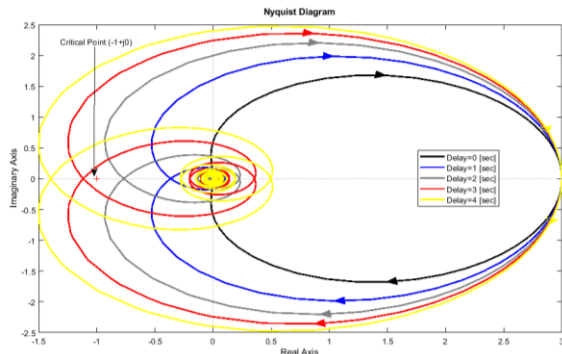


Fig. 6. The Nyquist diagrams of the open-loop system, $d = 0, 1, 2, 3, 4$.

Fig. 7 illustrates the closed-loop system response in various conditions. In part (a) of Fig. 7, the closed-loop system response is observed in the presence of a PID controller whose parameters are set for the minimum delay ($d = 0.65$). Also In part (b) of this figure, the closed-loop system response is observed in the presence of a PID controller, but this time the parameters of this controller are set for the maximum delay ($d = 0.65$).

The parameters of these controllers, adjusted for minimum and maximum delay, are given in Eq. (20) and (21), respectively.

$$PID \text{ for } d_{min}: K_P = 0.68, K_I = 0.20, K_D = 0.10 \quad (20)$$

$$PID \text{ for } d_{max}: K_P = 0.19, K_I = 0.04, K_D = 0.01 \quad (21)$$

In part (c) of Fig. 7, the closed-loop system response is plotted in the presence of a multiple adaptive controller, which is actually the proposed controller. It should be noted that for this controller, a hard switching for the proposed controller is considered. That is, the constant of switching for the supervisory subsystem is assumed to be $K = 100$. Fig. 8 also shows the control effort signal corresponding to each of the above three control systems.

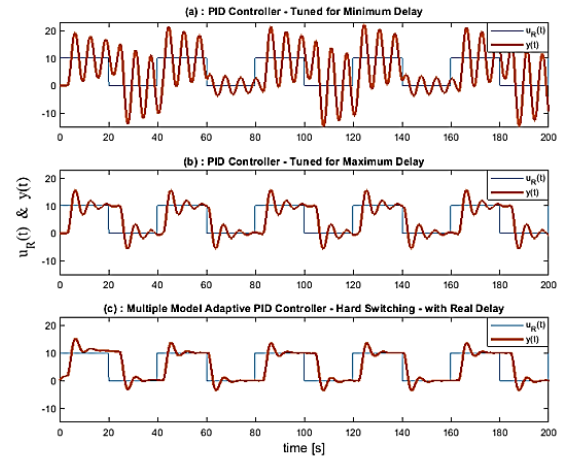


Fig. 7. The closed-loop system response depending on different controllers.

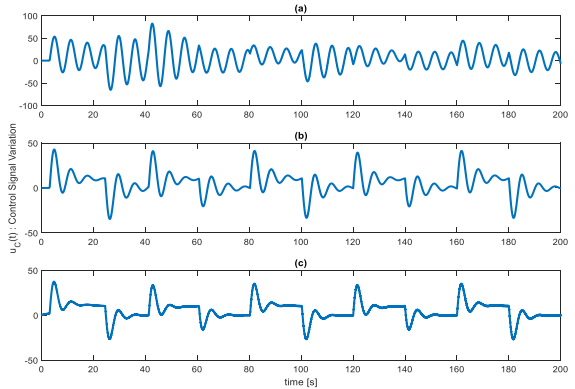


Fig. 8. The variations of the control signal for different control systems.

Fig. 9 shows the closed-loop system response in the presence of the proposed controller. In parts (a) and (b) of this figure, the first and second proposed estimators (based on LKF and EKF) have the task of estimating the delay in the control system, respectively. Fig. 10 also shows the control effort corresponding to each of these two control systems above.

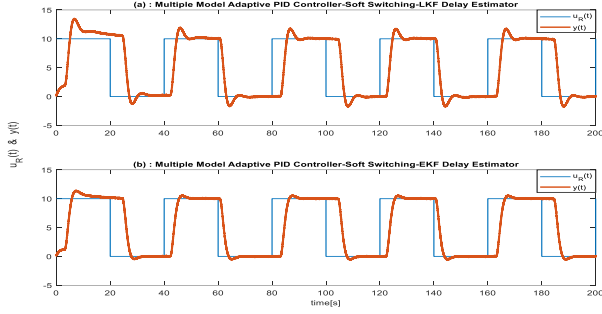


Fig. 9. The closed-loop system response for the proposed controller, with soft switching, in the presence of the two proposed estimators.

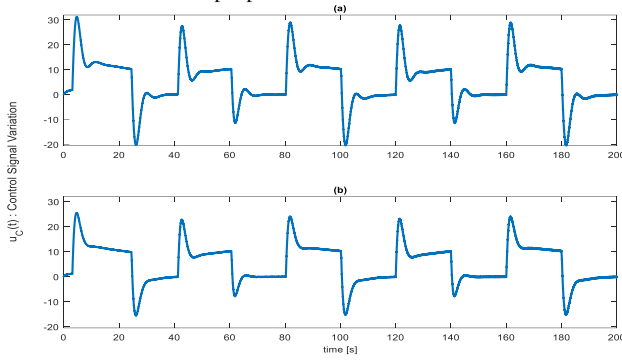


Fig. 10. The variations of the control signal for the proposed controller, with soft switching, in the presence of the two proposed estimators.

To accurately compare the performance of the above controllers, two important control criteria in these systems are evaluated:

1- Mean of squares of tracking error (MSTE): This criterion is used to measure the traceability of a controller and its exact value for a control system is derived from Eq. (22).

$$MSTE = \frac{1}{n+1} \sum_{k=0}^n (y(kT) - u_R(kT))^2 \quad (22)$$

2- Mean of squares of control signal variations (MSCV): This criterion is to measure the effort of the control signal and its exact value in a control system can be calculated using Eq. (23).

$$MSCV = \frac{1}{n} \sum_{k=1}^n (u_c(kT) - u_c((k-1)T))^2 \quad (23)$$

In which n is the number of samples recorded for measuring the aforementioned criteria and T is the sampling time for data recording. Tables II and III present MSTE and MSCV for all control systems included in this study, respectively.

TABLE II

THE COMPARISON OF TRACEABILITY

Controller type	MSTE
PID controller set for minimum delay time	58.28
PID controller set for maximum delay time	2265
MAC-hard switching- certain equivalent estimated delay	20.02
MAC-soft switching- LKF delay estimator	17.49
MAC-soft switching- EKF delay estimator	15.62

TABLE III

THE COMPARISON OF THE CONTROL SIGNAL EFFORT

Controller type	MSCV
PID controller set for minimum delay time	0.0002
PID controller set for maximum delay time	0.00003
MAC-hard switching- certain equivalent estimated delay	0.02
MAC-soft switching- LKF delay estimator	0.006
MAC-soft switching- EKF delay estimator	0.004

Finally, the input-output stability for the proposed control system should be analyzed in the BIBO sense [56]. Since the delay estimator and the switching mechanism are two essential parts of the control system, the stability of the closed-loop system should be examined in the presence of the proposed switching mechanism with different amounts of delay estimation (in the main band of delay changes). For this purpose, the stability of the proposed control system is examined for different values of $\hat{d}(t)$. Fig. 11 and 12 represent bode and Nyquist diagrams of the open-loop system for the proposed control system, respectively. As can be seen in these figures, the system is BIBO-stable at a relatively safe margin for different estimates of the time delay. It should be noted that these diagrams have been plotted assuming the presence of multiple controllers and a supervisory mechanism in the control loop. It can be seen in Fig. 11 that, for the largest estimated delay, the closed-loop system with a gain margin of 7.44dB and a phase margin of 48.76deg is input-output stable. It is also apparent in Fig. 12 that, for the largest amount of delay, the closed-loop system is stable with a relatively safe margin.

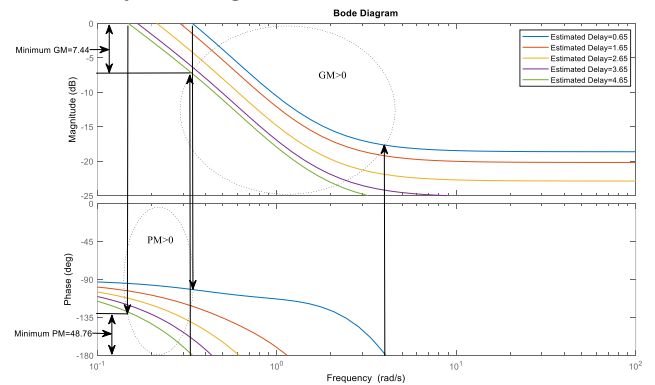


Fig. 11. The Bode diagrams for the open-loop system in the proposed control system for different values of estimated delay.

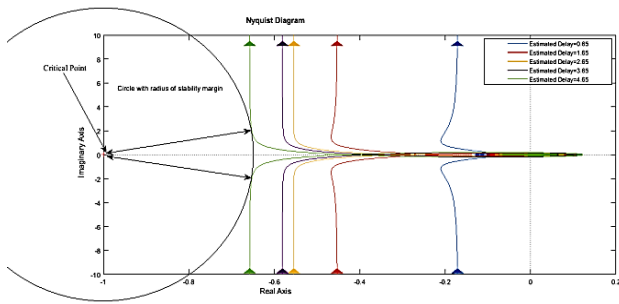


Fig. 12. The Nyquist diagrams for the open-loop system in the proposed control system for different values of estimated delay.

VI. CONCLUSIONS

This study presented a controller in MAC structure for a specific class of LTI systems with an unknown and time-varying delay. The most important reason that encouraged designers into using MAC structure is that the time delay variation band (uncertainty band) was large. In this designing process, the used adaptive controller is of the indirect type. It is obvious that an essential element in this type of controllers is the estimation mechanism of the plant parameters. In this paper, a novel method for designing a fast and accurate estimator was presented. The main idea behind this method is based on the performance of the Kalman filter observer (linear and extended) in estimating the states of linear and nonlinear systems. Particularly, considering the subject mentioned in Section II of this paper, the plant time delay was estimated using the proposed observer at an online mode and with acceptable precision. Multiple controllers used in this study are considered four PID's corresponding to the four sub-bands of the delay. It should be noted that the parameters for each of the multiple controllers, for the system under control, were calculated offline and proportional to its corresponding time-delay sub-band, from solving several independent optimization problems in a 3D space using a genetic algorithm. Also in this study, a soft and continuous switching mechanism was presented in the adaptive subdivision to reduce the undesired effects of discrete switching in the MAC control structure, especially to reduce control effort. The simulation of different types of reference input signal and time delay signal were made on a sample system. The simulation results presented in Section V are indicative of a rather desired tracking performance for the proposed controller, a decrease in control effort in the proposed control system, a very low estimation error for the proposed estimator, as well as the closed-loop system stability. Two important points that are definitely effective in performance improvement of the proposed controller and estimator are as follows:

1- In designing the estimator, using higher-order pade approximations for the time delay factor will definitely increase estimation precision.

2- In designing the controller, an increase in the number of sub-bands for time delay, and hence an increase in the number of multiple controllers, will definitely decrease tracking error and control effort.

Also, it should be noted that in case of optimal designing, the use of other controllers such as Smith controller and fuzzy PID, as well as using other continuous switching rules such as TSK fuzzy switching, in the proposed control structure may improve the efficiency of the proposed controller.

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