Solving Economic Load Dispatch by a New Hybrid Optimization Method
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This paper presents a new solution method to efficiently handle non-convexity stemmed from valve points in the economic load dispatch problem. The proposed solution technique integrates both the advantage of fast solution algorithms of linear programming and powerful solution techniques of nonlinear programming to find the global solution. In the first step of the proposed solution framework, non-convex terms are replaced by some linear segments and the new linear model solved by modern fasts algorithms. In the second step, a nonlinear programming algorithm as a powerful local search algorithm solves the original non-convex model to improve the solution obtained in the previous step. By exploiting the main strength of linear and nonlinear programming algorithms, the proposed solution approach can quickly converge to nearly the global solution method. By experimental results on three test cases with different sizes, we show that the presented method outperforms the other algorithms published in the literature in the quality of the solution.

I. INTRODUCTION

Among different operational tools provided to efficiently operate power systems in dispatch centers, economic load dispatch (ELD) has special importance as it determines final generation levels of generators. Economic dispatch optimally assigns output power of generators while satisfies power system demand. To this end, an optimization problem with a cost function, as the objective function, and a set of practical constraints is defined [1]. The cost function shows the operational or just fuel cost of power plants and usually models as a quadratic term.

Nonetheless, it is shown that the cost function can better be represented by taking a rectified sine into account as it more precisely models the effect of opening several valves, in a power plant, on cost function [2]. However, from an optimization viewpoint, the considered rectified sinusoidal term changes the ELD problem into a non-smooth non-convex optimization problem. A wide range of solution algorithms has been published in the literature to tackle the optimization problem and can be categorized into two gradient and artificial intelligence-based (AI) methods. The gradient-based methods can include Lagrangian relaxation [3], modified lambda-iteration [4], linear programming [5], and quadratic programming [6]. The traditional gradient-based methods rely on derivatives of objective and constraints in optimization problems and as a result, they can not effectively manage the non-differentiable absolute function induced by rectified sinusoidal term. Moreover, they are local optimizer in nature and their performances dramatically depend on the starting point. Therefore, they have poor performances in the no-convex problem with multiple local minimums [7].
Applications of AI algorithms have demonstrated in some studies [8]-[10]. AI based algorithms particularly in the ELD subject may consist of bat algorithm [11], gravitational search algorithm [12], symbiotic organisms search algorithm [13], firefly algorithm [14], cuckoo search algorithm [15], cooperative search algorithm [16], phasor particle swarm optimization [17], hybrid optimization framework [18], hybrid PSO–SQP [19], and hybrid GA–PS–SQP [20]. The algorithms do not require a differentiable search space. Moreover, they work with a set of solutions, so-called population, swarm, etc., compared to the gradient-based methods use a single solution. Hence, they may find the global solution using the parallel search. However, the computation burdens of these algorithms are high. Moreover, the performances of these methods severely depend on algorithm parameters limiting their application in practice.

In this paper, a new hybrid solution method is employed to tackle with the non-convexity of economic load dispatch accommodating the valve point effects. In the first step, the proposed method decomposes the non-convex problem to the set of linear problem method. The best solution among these linear problem methods can be found by a strong branch and bound algorithm. In the second step of the presented approach, the original problem is solved using a nonlinear programming (NLP) algorithm considering non-convex terms using the starting point, obtained in the first step, most likely located near the global solution depending on the quality of the linear segments approximation. Thus, NLP may converge to the global solution by providing an effective starting point.

The rest of the paper is organized as follows. The economic load dispatch problem formulation accounting for valve loading points is modeled in section II. Section III presents the proposed solution framework for solving non-convex ELD. Experimental results to show the effectiveness of the proposed solution technique is provided in section IV. Section V concludes the paper.

II. PROBLEM STATEMENT

The economic load dispatch tool aims to minimize total operation cost considering the physical and operational limits of power systems. From a formulation point of view, the aims and the limits can be represented by an objective function and set of equality and inequality constraints in an optimization model as follows [2]:

Minimize \[
\sum_{j} \left( a_j P_j^2 + b_j P_j + c_j + e_j \sin \left( f_j \left( P_{j}^{\text{min}} - P_j \right) \right) \right) \] (1)

\[\quad P_j^{\text{min}} \leq P_j \leq P_j^{\text{max}}, \quad j = 1, 2, \ldots, n \] (2)

\[\quad \sum_{j=1}^{n} P_j = D \] (3)

In the formulation, \( j \) shows an index of generators from 1 to \( n \) (number of generators). \( a_j, b_j, c_j, e_j, f_j \) are the coefficient cost of generator \( j \). The decision variable \( P_j \) stands for the generation level of generator \( j \). The minimum and maximum power limits of generator \( j \) are represented by \( P_j^{\text{min}} \) and \( P_j^{\text{max}} \) respectively. The power system demand is shown by \( D \).

The objective function in (1) indicating the total cost of generators includes two convex and non-convex components. The quadratic term \( a_j P_j^2 + b_j P_j + c_j \) is a convex function while the term \( e_j \sin \left( f_j \left( P_{j}^{\text{min}} - P_j \right) \right) \) is a non-convex and non-smooth function added due to valve loading effects. The non-convex term changes the ELD problem to non-convex one with many local optimal solutions.

In the next section, a two layers method is proposed to cope with the non-convex and non-smooth space of the presented ELD problem.

III. THE PROPOSED ELD SOLUTION METHOD

As mentioned before, the ELD accommodating valve point effects is a non-convex problem. Here, to tackle with the non-convexity, the non-convex cost functions replaced with some of the linear segments finally producing a mixed integer linear problem. Subsequently, a mixed integer programming (MIP) solution technique is used to solve the new model. As the mixed inter algorithms are very powerful at present, the new model can be solved very fast. The obtained solution of mixed inter then used as the starting solution of the original non-convex ELD problem solved by a nonlinear programming (NLP) approach. The NLP can remove the approximation error and converge to the nearly global solution as it currently uses the high-quality starting point obtained in the first step. The mixed integer problems usually solved by powerful branch and bound (B&B) algorithms [21]. In the B&B algorithm, firstly, all integer values relaxed and they can adopt fractional values. The resulting relaxed LP model is solved. In the B&B algorithm, a so-called tree is generated composed of a root (the first LP model) with some branches, nodes, and leaves as shown in figure 1. Nodes Correspond to some integer variables that have fractional values in the solution of the LP model. In each node, a decision is made about rounding up or down the fractional values of the integer variable shown by corresponding branches. Subsequently, two new nodes in two branches ending of the node show the two new models with fixing the fractional to integer values. The nodes that have been not branched yet called leaves. The two new models may be solved in two new nodes and other new nodes can be generated. The procedure continues until some stopping criteria satisfied. Consequently, the solution space of the original non-convex model is approximated by some probably
smaller linear models with a more efficient solution algorithm.

A mixed integer linear model of a nonlinear function can be found as described below.

Consider that the \( B+1 \) breakpoints or equivalently \( B \) segments are used to linearly approximate a nonlinear function and \( x^\text{min} = x_0 < x_1 < \cdots < x_B = x^\text{max} \) stand for the breakpoints.

Let us show the non-convex function with its \( k \)th linear segment approximation as follows:

\[
y_k = m_k \hat{x}_k + d_k, \quad x_{k-1} \leq \hat{x}_k \leq x_k
\]

Where \( x_{k-1} \) and \( x_k \) show the beginning and ending points of the segments respectively. Here, in the ELD problem, \( \hat{x}_k \) and \( y_k \) can be interpreted as the generation level and associated variable \( x_k \). Based on (7) and (8) only one continuous variable \( \hat{x}_k \) can be non-zero that is determined based on the corresponding non-zero binary variable \( z_k \).

Relying on the technique, the MIP model for ELD problem can be described as follows:

\[
\text{Minimize } \sum_{j=1}^{m} F_j
\]  
\[
P_j^\text{min} \leq P_j \leq P_j^\text{max}, \quad j = 1, 2, ..., n
\]  
\[
\sum_{j=1}^{m} P_j = D
\]  
\[
F_j = \sum_{k=1}^{B} (m_{j,k} \hat{P}_{j,k} + d_{j,k}), \quad j = 1, 2, ..., n
\]  
\[
P_j = \sum_{k=1}^{B} \hat{P}_{j,k}, \quad j = 1, 2, ..., n
\]  
\[
p^*_j \leq \hat{P}_{j,k} \leq p^d_j \quad \forall k = 1, 2, ..., B, \text{ and } j = 1, 2, ..., n
\]  
\[
\sum_{k=1}^{B} z_{j,k} = 1, \quad j = 1, 2, ..., n
\]  
\[
z_{j,k} \in \{0,1\}, \quad \forall k = 1, 2, ..., B, \text{ and } j = 1, 2, ..., n
\]

\[
\text{IV. NUMERICAL RESULTS}
\]

This section deals with performance evaluation of the proposed solution technique for solving non-convex smooth economic load dispatch. To show the efficiency of the presented method three case studies with different size are selected as follows:

Case I: A small scale 3 unit test system.

Case II: A medium scale 13 unit test system.

Case III: A large scale 40 unit test system.

The proposed solution technique is implemented using...
GAMS software. We used a laptop with CPU Core i3, 2.4GHZ clock frequency and 4 GB RAM in all simulations. The mixed integer model is solved by CPLEX and the nonlinear model using CONOPT. It is noted that the optimality of obtained results is not sensitive to the selected solver.

Application results of the proposed hybrid approach on these case studies are reported in the following. Moreover, the performance of the presented technique is compared with other solution algorithms in the literature to evaluate the capabilities of the proposed method more precisely.

Case I: 3 unit case study
The load demand of the three units test case is 850MW. All of its units have vale loading effects in their cost function. The required data for the test case can be found in [23].

For Case I, result of the presented algorithm and other ELD solution techniques, which are adopted from their published paper, are provided in Table I. As the table shows, different algorithms nearly have the same optimal solutions due to relatively a simple ELD problem in this small test case.

Nonetheless, most of the solution techniques present a noticeable fluctuation in results among different algorithms runs shown by best, average and worst costs. However, the proposed solution method always converges to a unique solution showing its robustness in solving the nonconvex ELD problem. Note that the optimal cost obtained by the proposed method is lower than the mean cost and worst costs of all the reported algorithms in table I illustrating its merit in solving the ELD problem in practice. Table II demonstrates the generation levels of units of case I for the optimal solution of the proposed method.

The outcome of MIP, i.e. the approximate objective function as (9), for the case I is 8233.64$. The solution of MIP is employed as the initial point of the nonlinear programming solver to solve the original non-convex model, here CONOPT solver. The final solution achieved by the CONOPT is 8233.64$ showing that the main part of the provided solution method is actually the presented MIP model.

**TABLE I:** Comparison Of Solution Algorithms For Solving Eld In Case I

<table>
<thead>
<tr>
<th>Methods</th>
<th>Best cost ($)</th>
<th>Mean cost ($)</th>
<th>Worst cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFEP[23]</td>
<td>8234.08</td>
<td>8234.71</td>
<td>8241.80</td>
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<td>CEP[23]</td>
<td>8234.07</td>
<td>8235.97</td>
<td>8241.83</td>
</tr>
<tr>
<td>FEP[23]</td>
<td>8234.07</td>
<td>8234.24</td>
<td>8241.78</td>
</tr>
<tr>
<td>IFEP[23]</td>
<td>8234.07</td>
<td>8234.16</td>
<td>8243.54</td>
</tr>
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<td>FA[24]</td>
<td>8234.07</td>
<td>8234.08</td>
<td>8241.23</td>
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<tr>
<td>PS*[25]</td>
<td>8234.05</td>
<td>8453.00</td>
<td>8352.41</td>
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<tr>
<td>Proposed hybrid algorithm</td>
<td>8234.07</td>
<td>8234.07</td>
<td>8234.07</td>
</tr>
</tbody>
</table>

*Violation of demand constraint

**TABLE II:** Generation Levels, And Associated Cost In Optimal Solution Of Case I

<table>
<thead>
<tr>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300.2669</td>
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<td>2</td>
<td>149.73310</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>400.0000</td>
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</table>

Case I: 13 unit case study
The generator characteristics of thirteen test system are adopted from [23]. The demand for this test case is D=1800MW. The cost functions of all 13 units test case have vale loading effects. Table III shows the outcome pertaining to the application of the proposed hybrid solution method and results taken from other published approaches in the literature to solve the economic load dispatch problem for case II. The presented method outperforms most of the other methods in finding the better optimal solution, as the table III shows. Again, while the solution of the proposed piecewise technique is unique, dissimilar final solutions of other techniques challenge their applicability in real-word problems. Table IV illustrates the unit dispatch as the solution of the suggested method in solving ELD in test system II. For the case II, the output of the MIP solver is 17959.455$ finally reaches to 17960.366$ by the nonlinear solver.

**TABLE III:** Generation Levels, And Associated Cost In Optimal Solution Of Case II

<table>
<thead>
<tr>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Cost ($)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>628.319</td>
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<td>2</td>
<td>149.6</td>
<td>9</td>
<td>109.867</td>
<td></td>
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<td>3</td>
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<tr>
<td>4</td>
<td>109.867</td>
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<td>40</td>
<td></td>
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<td>5</td>
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<td>55</td>
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<td>6</td>
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<td></td>
</tr>
<tr>
<td>7</td>
<td>109.867</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE IV:** Comparison Of Solution Algorithms For Solving Eld In Case I

<table>
<thead>
<tr>
<th>Methods</th>
<th>Best cost ($)</th>
<th>Mean cost ($)</th>
<th>Worst cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEP [23]</td>
<td>18048.21</td>
<td>18190.32</td>
<td>18404.04</td>
</tr>
<tr>
<td>MFEP [23]</td>
<td>18028.09</td>
<td>18192.00</td>
<td>18416.89</td>
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<td>FEP [23]</td>
<td>18018.00</td>
<td>18200.79</td>
<td>18453.82</td>
</tr>
<tr>
<td>IFEP [23]</td>
<td>17994.07</td>
<td>18127.06</td>
<td>18267.42</td>
</tr>
<tr>
<td>SOMA [26]</td>
<td>17967.42</td>
<td>17985.32</td>
<td>18017.62</td>
</tr>
<tr>
<td>MDE [2]</td>
<td>17960.39</td>
<td>17967.19</td>
<td>17969.09</td>
</tr>
<tr>
<td>CSOMA [26]</td>
<td>17960.36</td>
<td>17967.87</td>
<td>17970.83</td>
</tr>
<tr>
<td>Proposed hybrid algorithm</td>
<td>17960.36</td>
<td>17960.366</td>
<td>17960.366</td>
</tr>
</tbody>
</table>

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Case III: 40 unit case study

The forty unit test case demand is 10500MW. The valve loading effects appear in the cost function of all of its units. The data for the test case are taken from [23].

Table V compares the obtained results of the proposed solution technique and reported outcomes of other methods in the literature in solution quality of case III. The first point that should be highlighted is the considerable difference between reported solutions due to the large scale nonconvex ELD problem in this test case. For the same reason, more solution oscillations can be seen in multiple runs of the reported algorithms. Nevertheless, the proposed method converges to the best solution and without any change showing its ability to solve the large scale nonconvex ELD problem. Optimal dispatch for test case III found by the proposed algorithm is illustrated in Table VI. While the returned cost by the MIP solver in the case III is 121413.57$, the final optimal solution is 121412.53$ given by the CONOPT nonlinear solver.

### TABLE V: Comparison Of Solution Algorithms For Solving ELD In Case I

<table>
<thead>
<tr>
<th>Methods</th>
<th>Best cost ($)</th>
<th>Mean cost ($)</th>
<th>Worst cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEP [23]</td>
<td>122679.71</td>
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<td>MFEP[23]</td>
<td>122647.57</td>
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<td>IFEP[23]</td>
<td>122624.35</td>
<td>123382.0</td>
<td>125740.63</td>
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<td>ESO [27]</td>
<td>122122.16</td>
<td>122558.4</td>
<td>123143.07</td>
</tr>
<tr>
<td>MDE [2]</td>
<td>121414.79</td>
<td>121418.4</td>
<td>121466.04</td>
</tr>
<tr>
<td>HS [28]</td>
<td>121544.51</td>
<td>121761.0</td>
<td>122113.96</td>
</tr>
<tr>
<td>HIS [28]</td>
<td>121560.53</td>
<td>121796.4</td>
<td>121963.31</td>
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<td>GHS [28]</td>
<td>121624.32</td>
<td>121870.9</td>
<td>122272.68</td>
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<td>SAHS [28]</td>
<td>121516.94</td>
<td>121694.4</td>
<td>121900.42</td>
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<td>ACHS [28]</td>
<td>121414.85</td>
<td>121510.5</td>
<td>121655.66</td>
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<tr>
<td>Θ-MBA [29]</td>
<td>121412.53</td>
<td>121412.7</td>
<td>121412.95</td>
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<td>Proposed hybrid algorithm</td>
<td>121412.53</td>
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<td>121412.53</td>
</tr>
</tbody>
</table>

### TABLE VI: Generation Levels, And Associated Cost In Optimal Solution Of Case III

<table>
<thead>
<tr>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Unit</th>
<th>Generation (MW)</th>
<th>Unit</th>
<th>Generation (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>11</td>
<td>94</td>
<td>21</td>
<td>523.3</td>
<td>31</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>110.8</td>
<td>12</td>
<td>94</td>
<td>22</td>
<td>523.3</td>
<td>32</td>
<td>190</td>
</tr>
<tr>
<td>3</td>
<td>97.4</td>
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<td>23</td>
<td>523.3</td>
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<td>4</td>
<td>179.7</td>
<td>14</td>
<td>394.3</td>
<td>24</td>
<td>523.3</td>
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<td>164.8</td>
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<td>87.8</td>
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<td>394.3</td>
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<td>8</td>
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<td>30</td>
<td>87.8</td>
<td>40</td>
<td>511.3</td>
</tr>
</tbody>
</table>

Cost ($) 121412.5355

Although the presented hybrid technique establishes a nearly global solution and virtually outperforms the other ELD solution methods published in the literature, its computational burden also should be evaluated for the practical application. Because of the different software and hardware used in this paper compared with other techniques presented in the literature, comparisons of computation times are fairly invalid. However, to illuminate the application of the hybrid method, the elapsed time of the suggested technique for each experiment, for all the three test cases, are shown in Table VII. As can be seen from Table VII, the computation times of the presented hybrid algorithm in all the three experiments are less than 2 seconds showing the fast convergence of the method and its application for practical test cases.

### TABLE VII: The Computation Burden Of The Proposed Technique In All The Three Test Cases

<table>
<thead>
<tr>
<th>Test case</th>
<th>Computation time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I: 3 units test system</td>
<td>0.41</td>
</tr>
<tr>
<td>Case II: 13 units test system</td>
<td>1.18</td>
</tr>
<tr>
<td>Case III: 40 units test system</td>
<td>1.35</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

To handle the nonconvexity appears in the ELD problem with valve point effects, we propose a hybrid solution method of powerful local and global searching ability. We describe how the solution method exploits both the branch and bound algorithm as the global search technique and NLP solvers as the local search algorithm to converge to unique high-quality solutions. Comparison results of solution algorithms of the nonconvex ELD problem show the considerable advantage of the presented hybrid technique concerning the other algorithms published in the literature both in the solution quality and robustness of the solution in the multiple
algorithms runs. It is noticed that the performance of the AI-based algorithms highly depends on their parameters. However, the proposed technique outperforms the other methods without a trial and error parameter tuning mechanism. Another crucial factor the presented solution method is the low computational burden allowing to be used in practical applications.

REFERENCES


Hossein Sharifzadeh is an assistant professor of electrical engineering at Hakim Sabzevari University (HSU). He mainly works on application of modern optimization techniques and uncertainty handling on power systems operation. His current research interest is application of global optimization techniques to power system problems.