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Robust Decentralized Model Predictive Control for a Class of Interconnected systems

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ABSTRACT

This paper presents a robust decentralized model predictive control scheme for a class of discrete-time interconnected systems subject to state and input constraints. Each subsystem is composed of a nominal LTI part and an additive time-varying perturbation function which presents the interconnections and is generally uncertain and nonlinear, but it satisfies a quadratic bound. Using the dual-mode MPC stability theory and Lyapunov theory for discrete-time systems, a sufficient condition is constructed for synthesizing the decentralized MPC's stabilizing components; i.e. the local terminal cost function and the corresponding terminal set. To guarantee robust asymptotic stability, sufficient conditions for designing MPC stabilizing components are characterized in the form of an LMI optimization problem. The proposed control approach is applied to a system composed of five coupled inverted pendulums, which is a typical interconnected system, in a decentralized fashion. Simulation results show that the proposed robust MPC scheme is quite effective and has a remarkable performance.

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I. INTRODUCTION

Recently, decentralized control of interconnected systems [1] has received considerable interest in control community within academia and industry [2]-[4]. Interconnected systems can be found in transportation systems [5], water management networks [6], and electric power grids [7]-[9]. These systems are composed of several subsystems. To simplify the design procedure, they are often decomposed into small-order subsystems [10] and the interconnections are often neglected [11]. So, interconnected systems are regularly controlled by decentralized control schemes, in which there is no need to information exchange between the subsystems. On the other hand, the interconnections which can influence the dynamics of each subsystem are considered as a model uncertainty and if they are not sufficiently weak, asymptotic stability of the overall closed-loop system may not be satisfied.

Among the possible solutions for decentralized control problem, the methods based on Model-based Predictive Control (MPC) [12]-[13] have received considerable attention. Due to its conceptual simplicity and ability to deal with constrained multivariable systems, MPC has gained a lot of success in theory and application in comparison with the conventional methods of multivariable control [14]-[15]. Furthermore, MPC as a feedback control strategy, has some degrees of inherent robustness, which have been analyzed by several researchers [16]. Therefore, for interconnected systems with weak interconnections, the simplest way to reach robustness is to ignore the interconnections and to rely on the inherent robustness of the model predictive control applied to the isolated subsystems [17]-[18].

However, one of the major issues in designing decentralized model predictive control (DMPC) strategies is to ensure robust stability of the overall closed-loop system despite the strong

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interconnections among the subsystems that are not sufficiently small in the process. Unfortunately, only few contributions can be found in the context of DMPC scientific literature, mainly due to the fact that using the optimal cost function as a Lyapunov function, is not an easy task [19]. Nevertheless, synthesis of DMPC controllers for dynamic interconnected systems is investigated in some studies. In [20] interactions among subsystems are considered as disturbances to be rejected, where Input to State Stability [21] is used to guarantee stability. In [22]-[23] a linear system structured into physically coupled subsystems is considered and a decentralized control strategy by using tube-based MPC strategy [24] is proposed to guarantee asymptotic stability and satisfaction of constraints on system inputs and states. A DMPC design approach for dynamically coupled processes is proposed in [25]-[26], where linear interconnections are considered. In [27] the problem of decentralized model predictive control for constrained large-scale linear systems subject to additive bounded disturbances is considered. In some studies, a class of nonlinear discrete-time systems is considered in which subsystems are dynamically decoupled while they are affected by the neighbouring subsystems through the cost function of MPC [28]. It is assumed that each subsystem has local input and state constraints. In [2] an optimal control approach based on learning algorithms is suggested to attain the decentralized guaranteed cost controller design for a class of continuous-time nonlinear systems with bounded interconnection.

It should be noted that most of the studies consider the linear subsystems to develop a DMPC strategy. On the other hand, some real-world interconnected systems exhibit either nonlinear or uncertain interconnections, which leads to a special class of interconnected systems [29]. Also, the problem of increasing the interconnection bounds which can be tolerated by each subsystem is not paid attention to in the aforementioned studies. Therefore, studying the problem of constrained control of these systems is of practical importance. Some examples of the interconnected processes that can be modelled in this way include the turbine/governor control system [30]-[31], power systems with exciter control [32] and Lotka–Volterra models in biology [33] or chemical processes [34]. It is worth mentioning that, so far, this type of interconnected systems has not been focused in the context of DMPC.

The main contribution of this paper is to develop a constrained Robust DMPC (RDMPC) strategy to establish robust decentralized stabilization for these interconnected systems and, at the same time, maximizes the bound on the perturbation function which can be tolerated by each subsystem and models the interconnections between subsystems. The main advantage of this approach is that, it determines a linear DMPC law which stabilizes the overall interconnected system. Moreover, using this control law, the robust stability of overall system is guaranteed.

The remaining sections of this paper are organized as follows. In section 2, a class of discrete-time interconnected systems is introduced. In Section 3, the standard nominal MPC problem for an isolated subsystem is reviewed. The RDMPC law for the considered class of interconnected systems is presented in section 4. In section 5, the suitability and validity of the proposed RDMPC scheme is demonstrated by applying it to a typical interconnected system. Finally, some concluding remarks are provided in section 6.

Notation. In this paper, the short-hands $\mathbb{M}$ and $Z = diag_{i\in\mathbb{M}}(Z_i)$ are exploited to denote the set $\{1,\dots,M\} \subseteq \mathbb{N}$ and a block-diagonal matrix S with blocks S_i, where $i \in \mathbb{M}$, respectively. Similarly, $A \times B$ denotes the cartesian product of the sets A and B. A function $f(\cdot): R^+ \to R^+$ is of class κ if it is continuous, strictly increasing and if $f(0) = 0$. It belongs to class κ_∞ if additionally it holds that $\lim_{z\to\infty} f(z) = \infty$.

II. Problem Formulation

Consider a class of interconnected systems, which can be decomposed into an ordered set $\mathbb{M} = \{1,\dots,M\}$ of M interconnected non overlapping subsystems coupled in the state, in which each subsystem $i \in \mathbb{M}$ is presented as

$$x_i(\mathrm{k}+1) = A_i x_i(\mathrm{k}) + B_i u_i(\mathrm{k}) + h_i\big(k, x_1(k), \cdots, x_M(k)\big), \quad i \in \mathbb{M}, \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$ are the state vector and the control input of i^{th} subsystem, respectively. $h_i: \mathbb{R}^{n+1} \to \mathbb{R}^{n_i}$, $i \in \mathbb{M}$, is the perturbation function which presents the interconnection function, which may be uncertain or nonlinear in general. $n = \sum_{i=1}^{M} n_i$. The following assumptions will be used through this paper.

Assumption 1. Both the local control inputs u_i and states x_i are constrained as

$$u_i \in \mathrm{U}^i, \; x_i \in \mathcal{X}^i, \; \forall i \in \mathbb{M}, \tag{2}$$

where $\mathrm{U}^i \subseteq \mathbb{R}^{m_i}$ and $\mathcal{X}^i \subseteq \mathbb{R}^{n_i}$ are convex sets and both of them include origin.

Assumption 2. $h_i: \mathbb{R}^n \to \mathbb{R}^{n_i}$ satisfies the following quadratic bound,

$$h_i^T(k,x) h_i(k,x) \leq \alpha_i^2 x^T H_i^T H_i x, \forall\, (k,x) \in \mathbb{Z}^+ \times \mathbb{R}^\mathrm{n}, \tag{3}$$

where α_i, $i \in \mathbb{M}$, is the bounding parameter on the perturbation function $h_i(k,x)$ and $x = [x_1^T, \cdots, x_N^T]^T \in \mathbb{R}^n$ is the state vector of overall interconnected system. H_i, $i \in \mathbb{M}$, is a constant $l_i \times n$ matrix and k is time instance.

The dynamics of the overall interconnected system can be represented as

$$x(k+1) = A_D x(k) + B_D u(k) + \tilde{h}\big(k, x(k)\big), \tag{4}$$

where $u = [u_1^T, \dots, u_M^T]^\mathrm{T} \in \mathbb{R}^m$ is the control input of overall interconnected system and $m = \sum_{i=1}^{M} m_i$. The definition of the overall vectors x and u means that the subsystems are disjoint. $A_D = \mathrm{diag}\{A_1,\dots,A_M\}$ and $B_D = \mathrm{diag}\{B_1,\dots,B_M\}$ are constant matrices of appropriate dimensions. $\tilde{h} = [h_1^T,\dots,h_M^T]^T$ is the overall interconnection function. By obeying the decentralized information structure constraints, it is assumed that the control of each subsystem is performed by using only its locally state vector x_i. According to assumption 2, $\tilde{h}$ satisfies the following quadratic inequality bound for all $(k,x) \in \mathbb{Z}^+ \times \mathbb{R}^n$,

$$\tilde{h}^T(k,x(k))\tilde{h}(k,x(k)) \leq x^T(k)(\sum_{i=1}^{M} \alpha_i^2 H_i^T H_i)x(k). \quad (5)$$

For given matrices H_i, Ineq. (5) defines a class of vector functions as

$$\mathbf{H}_{\boldsymbol{\alpha}} = \{\tilde{h}: \mathbb{R}^{n+1} \to \mathbb{R}^n | \tilde{h}^T\tilde{h} \leq x^T(k)(\sum_{i=1}^{M} \alpha_i^2 H_i^T H_i)x(k), \text{ for all } k \in \mathbb{Z}^+\}, \quad (6)$$

where $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_N]^T$. This class includes functions $\tilde{h}(k,x)$ that satisfy $\tilde{h}(k,0) = 0$, and so $x = 0$ is an equilibrium point for the overall interconnected system (4).

Definition 1. The interconnected system (4) is robustly connectively stable with degree $\boldsymbol{\alpha}$ if the equilibrium point $x = 0$ is globally stable for all $\tilde{h}(k,x) \in \boldsymbol{H_\alpha}$.

Assumption 3. The pairs (A_i, B_i), $i \in \mathbb{M}$, are robustly stabilizable, i.e., there exists a linear state-feedback control law of the form $\bar{k}_f^i(x_i) = K_i x_i$, where $K_i \in \mathbb{R}^{m_i \times n_i}$, $\forall i \in \mathbb{M}$, such that the closed-loop system $x_i(k+1) = (A_i + B_i K_i)x_i(k) + h_i(k,x)$, $i \in \mathbb{M}$, is robustly connectively stable with degree α_i.

It should be noticed that if the pairs (A_i, B_i), $i \in \mathbb{M}$, are robustly stabilizable, then the pair (A_D, B_D) is robustly stabilizable, and therefore there exists a decentralized state-feedback control law $\bar{k}_f(x) = K_D x$, where $K_D = diag(K_i) \in \mathbb{R}^{m \times n}$, such that the closed-loop system $x(k+1) = (A_D + B_D K_D)x(k) + \tilde{h}(x)$ is robustly connectively stable with degree $\boldsymbol{\alpha}$. By connective stability, the objective is that the interconnected system (4) remains stable whether the subsystems are connected or disconnected.

Definition 2. By neglecting the interconnections between subsystems, the model of the isolated subsystem can be represented as

$$x_i^{IS}(k+1) = A_i x_i^{IS}(k) + B_i u_i^{IS}(k), i \in \mathbb{M}, \quad (7)$$

where $x_i^{IS} \in \mathbb{R}^{n_i}$ is state vector of i^{th} isolated subsystem. Therefore, the overall isolated system is presented as

$$x^{IS}(k+1) = A_D x^{IS}(k) + B_D u^{IS}(k), \quad (8)$$

where $x^{IS} = [x_1^{IS^T}, \cdots, x_M^{IS^T}]^T$ and $u^{IS} = [u_1^{IS^T}, \cdots, u_M^{IS^T}]^T$ are the state vector and the control input of the overall isolated system.

In following, first, in section 3, the stability problem of overall isolated system is formulated. Then, in section 4, the results obtained in section 3 are used to establish the robust connective stability of the interconnected system.

III. NOMINAL MPC FOR OVERALL ISOLATED SYSTEM

A. Nominal MPC

The discrete-time dual-mode nominal MPC law [12] for regulation of each isolated subsystem $i \in \mathbb{M}$, denoted by $u_i^{IS} = k_{MPC}(x_i^{IS})$, is defined through a finite-horizon optimal control problem $\mathbb{P}_N^i(x_i^{IS})$, which is solved online at each time sample k:

$$\mathbb{P}_N^i(x_i^{IS}): J_i(x_i^{IS}) = \min_{u_i} \sum_{t=0}^{N-1} l_i(x_i^{IS}(t), u_i^{IS}(t)) + V_f^i(x_i^{IS}(N)) \quad (9a)$$

$$\text{s.t., } x_i^{IS}(0) = x_i^{IS}(k) \quad (9b)$$

$$x_i^{IS}(t+1) = A_i x_i^{IS}(t) + B_i u_i^{IS}(t) \quad \forall t \in \{0, \dots, N-1\}, \quad (9c)$$

$$\left(x_i^{IS}(t), u_i^{IS}(t)\right) \in \mathcal{X}^i \times \mathrm{U}^i, \ \forall t \in \{0, \dots, N-1\}, \quad (9d)$$

$$x_i^{IS}(N) \in \mathcal{X}_f^i, \quad (9e)$$

where N is the prediction horizon and, $\forall i \in \mathbb{M}$, $\mathcal{X}_f^i$ is terminal constrained set and both the terminal penalty cost $V_f^i(x_i^{IS}(N))$ and the stage cost $l_i(x_i^{IS}(k), u_i^{IS}(k))$ are positive definite convex functions. The optimal solution of i^{th} optimal control problem is denoted by $u_i^{IS^*} = \{u_{i,0}^{IS^*}, \dots, u_{i,N-1}^{IS^*}\}$. Using Receding Horizon strategy, the closed loop control law for i^{th} isolated subsystem is defined by

$$k_{MPC}^i = u_{i,0}^{IS^*}, \ i \in \mathbb{M}. \quad (10)$$

Assumption 4. For i^{th} isolated subsystem ($i \in \mathbb{M}$), a locally linear state feedback control law, $k_f^i(x_i^{IS})$, terminal set $\mathcal{X}_f^i$ and terminal penalty cost $V_f^i(x_i^{IS}(N))$ are chosen such that:

1. $\mathcal{X}_f^i \subset \mathcal{X}^i$, $\mathcal{X}_f^i$ is closed, $0 \in \mathcal{X}_f^i$,
2. $k_f^i(x_i^{IS}) \in \mathrm{U}^i$, $\forall x_i^{IS} \in \mathcal{X}_f^i$,
3. $\mathcal{X}_f^i$ is a controlled invariant set for $x_i^{IS}(k+1) = A_i x_i^{IS}(k) + B_i k_f^i(x_i^{IS})$,
4. $\beta_1(\|x_i^{IS}\|_2) \leq V_f^i(x_i^{IS}) \leq \beta_2(\|x_i^{IS}\|_2)$.
5. $\Delta V_f^i(x_i^{IS}(k)) \leq -l_i\left(x_i^{IS}(k), k_f^i(x_i^{IS})\right)$, $\forall x_i^{IS} \in \mathcal{X}_f^i$,

where $\Delta V_f^i\left(x_i^{IS}(k)\right) = V_f^i\left(x_i^{IS}(k+1)\right) - V_f^i\left(x_i^{IS}(k)\right)$.

Theorem 1 [12]. Let $\beta_1(\cdot)$ and $\beta_2(\cdot)$ be K_∞ class functions. If assumption 4 holds, then the i^{th} closed-loop isolated subsystem $x_i^{IS}(k+1) = A_i x_i^{IS}(k) + B_i k_{MPC}^i(k), i \in \mathbb{M}$, is asymptotically stable with the domain of attraction $\mathbb{X}_N^i$. $\mathbb{X}_N^i \subset \mathbb{R}^{n_i}$ denotes the set of all initial state vectors of isolated subsystem i for which problem (9) is feasible.

Lemma 1 [35]. For the i^{th} isolated subsystem (7) with perfect state measurement and no disturbances, the feasibility of the open-loop optimal control problem $\mathbb{P}_N^i(x_i^{IS})$ at time sample $k = 0$ implies its feasibility for all $k > 0$.

B. Nominal MPC for Isolated System

Since it is assumed that the subsystems are disjoint, the M optimal problems $\mathbb{P}_N^i(x_i^{IS}), i \in \mathbb{M}$, which are defined above, can be augmented in one optimal problem as

$$\mathbb{P}_N(x^{IS}): J(x^{IS}) = \min_{u} \sum_{t=0}^{N-1} l(x^{IS}(t), u^{IS}(t)) + V_f(x^{IS}(N)) \quad (11a)$$

$$\text{s.t., } x^{IS}(t+1) = A_D x^{IS}(t) + B_D u^{IS}(t), \ \forall t \in \{0, \dots, N-1\}, \quad (11b)$$

$$x^{IS}(0) = x^{IS}(k), \quad (11c)$$

$$\left(x^{IS}(t), u^{IS}(t)\right) \in \mathcal{X} \times \mathrm{U}, \ \forall \mathrm{t} \in \{0, \dots, N-1\} \quad (11d)$$

$$x^{IS}(N) \in \mathcal{X}_f, \quad (11e)$$

where $l(x^{IS}(t), u^{IS}(t)) \triangleq \sum_{i=1}^{M} l_i(x_i^{IS}(t), u_i(t))$ and $V_f(x^{IS}(N)) \triangleq \sum_{i=1}^{M} V_f^i\left(x_i^{IS}(N)\right)$. Also, $\mathrm{U} \triangleq U_1 \times \cdots \times U_M$, $\mathcal{X} \triangleq \mathcal{X}_1 \times \cdots \times \mathcal{X}_M$ and $\mathcal{X}_f \triangleq \mathcal{X}_f^1 \times \cdots \times \mathcal{X}_f^M$.

Therefore, the optimal solution to the MPC problem for overall isolated system $\mathbb{P}_N(x^{IS})$, which is denoted by $u^{IS^*} = \{u_0^{IS^*}, \dots, u_{N-1}^{IS^*}\}$, includes the optimal solutions of M optimal problems, which are solved separately. The control law for the overall isolated system is as follows,

$$k_{MPC}(x^{IS}) \triangleq u^{IS^*}(\mathrm{k}) = \left[u_{1,0}^{IS^*}, \dots, u_{M,0}^{IS^*}\right]^T. \quad (12)$$

The overall closed loop isolated system is represented as

$$x(k+1) = A_D x(k) + B_D k_{MPC}(x^{IS}). \quad (13)$$

Theorem 2 (Stability of overall isolated system). Consider the nominal isolated subsystem (7). Suppose that state x_i^{IS} is measurable in current time k and (a) assumptions 1, 3 and 4 are satisfied $\forall i \in \mathbb{M}$, (b) the M optimal problems $\mathbb{P}_N^i(x_i^{IS})$, $i \in \mathbb{M}$, are feasible at $k = 0$. Then, the closed loop overall isolated system (13) is asymptotically stable with domain of attraction $\mathbb{X}_N \triangleq \mathbb{X}_N^1 \times \dots \times \mathbb{X}_N^M$. $\mathbb{X}_N \subset \mathbb{R}^n$ denotes the set of all initial state vectors of overall isolated system for which assumption (b) is satisfied.

Proof. According to Lemma 1, assumption (b) guaranties the feasibility of the open loop optimal control problem $\mathbb{P}_N(x^{IS})$ which includes M optimal control problems. Form assumption (a) and Theorem 1, the i^{th} isolated subsystem is asymptotically stable with domain of attraction $\mathbb{X}_N^i$. Therefore, there is a Lyapunov functional $\mathrm{V}_i(x_i^{IS}): \mathbb{R}^{n_i} \to \mathbb{R}$, for i^{th} closed loop isolated subsystem, such that:

$$\mathrm{V}_i(0) = 0, \ \mathrm{V}_i(x_i^{IS}) > 0 \ \text{and} \ \Delta\mathrm{V}_i(x_i^{IS}) < 0 \ \text{for all} \ x_i^{IS} \in \mathbb{X}_N^i \ \text{and} \ x_i^{IS} \neq 0 \quad (14)$$

Now, let the Lyapunov functional $\mathrm{V}(x^{IS}): \mathbb{R}^n \to \mathbb{R}$ is considered for overall isolated system (8) as

$$\mathrm{V}(x^{IS}) = \sum_{i=1}^{M} \beta_i \mathrm{V}_i(x_i^{IS}), \quad (15)$$

where β_i, $i \in \mathbb{M}$, are positive constants. According to (14) and (15),

$$\mathrm{V}(0) = 0, \ \mathrm{V}(x^{IS}) > 0 \ \text{and} \ \Delta\mathrm{V}(x^{IS}) < 0 \ \text{for all} \ x^{IS} \in \mathbb{X}_N \ \text{and} \ x^{IS} \neq 0. \quad (16)$$

So, the closed loop overall isolated system (13) is asymptotically stable with domain of attraction $\mathbb{X}_N$.

Remark 1. According to assumption 4 and Theorem 1, for the overall isolated system (8), (a) There exists a locally linear state feedback control law $k_f(x^{IS})$, terminal set $\mathcal{X}_f$, and terminal penalty cost $V_f(x^{IS}(k))$ such that:

1. $\mathcal{X}_f \subset \mathcal{X}$, $\mathcal{X}_f$ is a closed, $0 \in \mathcal{X}_f$ as an interior point,
2. $k_f(x^{IS}) \in U$, for all $\mathrm{x}^{IS} \in \mathcal{X}_f$,
3. $\mathcal{X}_f$ is a controlled invariant set for $x^{IS}(k+1) = A_D x^{IS}(k) + B_D k_f(x^{IS})$,
4. $\Delta V_f(x^{IS}(k)) \leq -l\left(x^{IS}(k), k_{MPC}(x^{IS}(k))\right)$ for all $x^{IS} \in \mathcal{X}_f$.

(b) The problem (11) is feasible and its optimal solution is Equ. (12).

The part (a) in remark 1 presents sufficient conditions for asymptotic stability of overall isolated system under control law (12).

IV. Robust Decentralized MPC for Interconnected System

Let consider the interconnected system presented by Equ. (4). In order to implement an RDMPC strategy for this system, the term $\tilde{h}(k, x(k))$ in Equ. (4) is considered as an additive state-dependent perturbation. Therefore, the prediction of the state trajectories of the interconnected system is calculated based on the overall isolated system (8). Consequently, the overall RDMPC problem is as follows,

$$\mathbb{P}_N(x) \colon J(x) = \min_u \sum_{t=0}^{N-1} \bar{l}(x(t), u(t)) + \bar{V}_f(x(N)) \quad (17a)$$

$$\text{s.t.,} \ x(0) = x(k),, \quad (17b)$$

$$x(t+1) = A_D x(t) + B_D u(t), \quad t \in \{0, \dots, N-1\}, \quad (17c)$$

$$\left(x(t), u(t)\right) \in \mathcal{X} \times U, \ \forall t \in \{0, \dots, N-1\}, \quad (17d)$$

$$x(N) \in \bar{\mathcal{X}}_f, \quad (17e)$$

where $\bar{l}(x(t), u(t)) \triangleq x^T(t) Q_D x(t) + u^T(t) R_D u(t)$. $Q_D = diag(Q_i)$and $R_D = diag(R_i)$, where $Q_i \in \mathbb{R}^{n_i \times n_i}$ and $R_i \in \mathbb{R}^{m_i \times m_i}$, $\forall i \in \mathbb{M}$, are all positive definite. Also, the overall terminal set $\bar{\mathcal{X}}_f$ and terminal cost $\bar{V}_f(x)$ are defined as $\bar{\mathcal{X}}_f = \bar{\mathcal{X}}_f^1 \times \cdots \times \bar{\mathcal{X}}_f^M$ and $\bar{V}_f(x) = \sum_{i=1}^{M} \bar{V}_f^i(x_i)$, respectively. Since the subsystems are disjoint, problem (17) consists of M optimal control problems denoted by $\mathbb{P}_N(x_i), i \in \mathbb{M}$. Every subsystem i has its own MPC law, which is a function of only its locally available state x_i. Therefore, the overall control law, which is optimal solution of problem (17), represented by $\bar{k}_{MPC}(x)$ is defined by

$$\bar{k}_{MPC}(x) \triangleq \bar{u}^*(\mathrm{k}) = \{\bar{u}_{1,0}^*, \dots, \bar{u}_{M,0}^*\}. \quad (18)$$

To guarantee the robust connective stability of the interconnected system (4) under the MPC law $\bar{k}_{MPC}(x)$, there must exist $\bar{k}_f(x)$, $\bar{\mathcal{X}}_f$, and $\bar{V}_f(x(k))$ such that the conditions in remark 1(a) are satisfied in presence of the perturbation function $\tilde{h}(k, x)$. In other words, $\bar{\mathcal{X}}_f$, $\bar{V}_f(x)$ and $\bar{k}_f$ must be chosen such that they are robust version of $\mathcal{X}_f$, $V_f(x)$ and k_f against $\tilde{h}(x)$, respectively.

A. Constructing $\bar{\mathcal{X}}_f$, $\bar{V}_f(x)$ and $\bar{k}_f$

Let consider a quadratic form for terminal cost function $\bar{V}_{f_i}(x_i) \triangleq x_i^T P_i x_i$ with $P_i = P_i^T$ and $P_i > 0$ for each subsystem. Therefore, the overall terminal cost function $\bar{V}_f(x)$ is as follows

$$\bar{V}_f(x) = \sum_{i=1}^{M} \bar{V}_{f_i}(x_i) = x^T P_D x, \quad (19)$$

where $P_D = diag\{P_i\}$. Having defined the matrices P_i, the terminal set $\bar{\mathcal{X}}_f^i$ can be considered as the largest set $\Omega_i(\varepsilon)$ which is defined by

$$\Omega_i(\varepsilon) = \{x_i : \bar{V}_{f_i}(x_i) = x_i^T P_i x_i \leq \varepsilon, x_i \in X_i, K_i x_i \in U_i\},\ \forall i \in \mathbb{M}. \quad (20)$$

In order to specify the matrix P_D, the fourth condition in remark 1(a) is rewritten for the interconnected system (4) as follows

$$\Delta\bar{V}_f(x) < -x^T Q_D x - u^T R_D u \ \text{ for all } \ x \in \bar{\mathcal{X}}_f. \quad (21)$$

From Equs. (4), (19) and (21), it follows that

$$\Delta V_f(x) = x^T(\bar{A}^T P_D \bar{A} - P_D + Q_D + K_D^T R_D K_D)x + \tilde{h}^T P_D \tilde{h} + x^T \bar{A}^T P_D \tilde{h} + \tilde{h}^T P_D \bar{A} x < 0, \quad (22)$$

where $\bar{A} = A_D + B_D K_D$. The above inequality can be written as the following matrix form:

$$\Delta\bar{V}_f(x) = \begin{bmatrix} x \\ \tilde{h} \end{bmatrix}^T \begin{bmatrix} \bar{A}^T P_D \bar{A} - P_D + Q_D + K_D^T R_D K_D & \bar{A}^T P_D \\ P_D \bar{A} & P_D \end{bmatrix} \begin{bmatrix} x \\ \tilde{h} \end{bmatrix} < 0. \quad (23)$$

The Ineq. (5) is equivalent to the following one

$$\begin{bmatrix} x \\ \tilde{h} \end{bmatrix}^T \begin{bmatrix} -H_D^T \Sigma^{-1} H_D & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ \tilde{h} \end{bmatrix} < 0, \quad (24)$$

where $H_D = [H_1^T, \dots, H_M^T]^T$, $\Sigma = diag\{\gamma_i I_{l_i}, i \in \mathbb{M}\}$ and $\gamma_i = \frac{1}{{\alpha_i}^2}$, $i \in \mathbb{M}$. Using the S-procedure [36] and with respect to (24), Ineq. (23) is satisfied if the following inequality holds:

$$\Delta\bar{V}_f(x) = \begin{bmatrix} x \\ \tilde{h} \end{bmatrix}^T \begin{bmatrix} X & \bar{A}^T P_D \\ P_D \bar{A} & P_D - I \end{bmatrix} \begin{bmatrix} x \\ \tilde{h} \end{bmatrix} < 0, \quad (25)$$

Where $X = \bar{A}^T P_D \bar{A} - P_D + Q_D + K_D^T R_D K_D + H_D^T \Sigma^{-1} H_D$.

Rewriting the last inequality and applying Schur complement [37], one may have

$$\begin{bmatrix} [-P_D + P_D(P_D - I)^{-1} P_D]^{-1} & \bar{A} \\ \bar{A}^T & XX \end{bmatrix} < 0, \quad (26)$$

where $XX = -P_D + Q_D + K_D^T R_D K_D + H_D^T \Sigma^{-1} H_D$.

In order to simplify Ineq. (26), the following lemma is used.

Lemma 2. For any invertible matrix C and matrices D, E, and F, the following holds:

$$(C + DEF)^{-1} = C^{-1} - C^{-1} D (I + EFC^{-1} D)^{-1}. \quad (27)$$

In Equ. (27), let $C = -P_D$, $D = P_D$, $E = (P_D - I)^{-1}$, and $F = P_D$. Then for the element $\{1,1\}$ of (26), it obtains

$$[-P_D + P_D(P_D - I)^{-1} P_D]^{-1} = -{P_D}^{-1} + I. \quad (28)$$

Taking into account Equ. (28), the Ineq. (26) is identical to the following one:

$$\begin{bmatrix} -P_D + Q_D + K_D^T R_D K_D + H_D^T \Sigma^{-1} H_D & \bar{A}^T \\ \bar{A} & -P_D^{-1} + I \end{bmatrix} < 0 \quad (29)$$

Since the Ineq. (29) is a BMI (Bilinear Matrix Inequality), it is not possible to synthesize the Lyapunov matrix P_D and state feedback matrix K_D from it. To deal with this bilinearity, the method introduced by de Oliveira et al. [38] is applied. To do so, Ineq. (29) is multiplied from the right by $diag(G_D, I)$ and from the left by $diag(G_D^T, I)$. Then

$$\begin{bmatrix} XXX & G_D^T \bar{A}^T \\ \bar{A} G & -P_D^{-1} + I \end{bmatrix} < 0, \quad (30)$$

where $XXX = G_D^T(-P_D + Q_D + K_D^T R_D K_D + H_D^T \Sigma^{-1} H_D) G_D$, G_D is a slack block-diagonal matrix variable and $G_D = diag\{G_i\}$. Matrices G_i are non-singular matrices of appropriate dimensions. Now, bounding $-G_D^T P_D G_D$ by $P_D^{-1} - G_D - G_D^T$ and applying Schur complement twice to LMI (30) results in the following LMI

$$\begin{bmatrix} l_1 & [G_D^T \ \ Y_D^T] & G_D^T H_D^T & l_2 & 0 \\ * & \begin{bmatrix} -Q_D & 0 \\ 0 & -R_D \end{bmatrix}^{-1} & 0 & 0 & 0 \\ * & * & -\Sigma & 0 & 0 \\ * & * & * & -V_D & I \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (31)$$

Where $l_1 = V_D - G_D - G_D^T$, $l_2 = G_D^T A_D^T + Y_D^T B_D^T$, $V_D = P_D^{-1}$ and $Y_D = K_D G_D$ is a block-diagonal matrix so that $Y_D = diag\{Y_i\}$ and $Y_i = K_i G_i$. The symbol $*$ induces a symmetric structure. Therefore, the LMI (31) is equivalent to the robust version of the fourth condition in remark 1(a) for interconnected system (4). Having LMI (31) solved, matrices P_D and K_D and consequently matrices P_i and K_i are obtained for each subsystem.

In order to establish robust connective stability of interconnected system (4) in sense of Definition 2 and, at the same time, maximize the bounds on the interconnection functions, the parameters $\gamma_i, i \in \mathbb{M}$ must be minimized subject to LMI (31). In other words, the following optimization problem must be feasible.

Problem 1. Min $\sum_{i=1}^{M} \theta_i \gamma_i$ subject to LMI (30) and

$$\gamma_i - \frac{1}{{\bar{\alpha}_i}^2} < 0,\ \ i \in \mathbb{M}, \quad (32)$$

where θ_i, $i \in \mathbb{M}$, are constant tuning parameters. $\bar{\alpha}_i$ is minimal uncertainty bound on α_i.

Remark 2. By using the constraint (32), a desired robustness degree $\bar{\boldsymbol{\alpha}} = [\bar{\alpha}_1, \dots, \bar{\alpha}_M]$ is guaranteed for the interconnected system (4).

If problem 1 is feasible, the robust version of the fourth condition in remark 1(a) is satisfied. Once the local terminal controllers K_i and the terminal cost functions $\bar{V}_{f_i}(x_i)$ are fixed, one needs to find the terminal sets $\bar{\mathcal{X}}_f^i$, $i \in \mathbb{M}$, such that state and input constraint satisfaction is guaranteed for the terminal closed-loop system. The terminal set $\bar{\mathcal{X}}_f^i$ for each subsystem is achieved by solving a linear programming (LP) optimization problem as follows

Problem 2. $\rho_i = \sup\ x_i^T P_i x_i$,

s.t. $x_i \in X_i, K_i x_i \in U_i$,

and the terminal set for i^{th} subsystem is as follows

$$\bar{\mathcal{X}}_f^i = \{x_i : \bar{V}_{f_i}(x_i) = x_i^T P_i x_i \leq \rho_i\}. \quad (33)$$

According to the contents mentioned above, the robust connective stability of closed loop interconnected system (4) can be established by the following theorem.

Theorem 3 (stability of the interconnected system). Consider the discrete-time interconnected dynamical system (4) and control law (18). Suppose that

(a) assumptions 1-3 are satisfied,

(b) the current state of each subsystem $x_i(k)$ is measurable,

(c) the optimization problem 1 is feasible,

(d) the problem (17) is feasible at $k = 0$, Then, there exists local MPC controllers $\mathbb{P}_N^i\left(x_{\mathcal{N}_i}\right)$ with their stage cost as $l_i(\cdot) = x_i^T Q_i x_i + u_i^T R_i u_i$, terminal cost function as $\bar{V}_{f_i}(x_i) = x_i^T P_i x_i$, and terminal set as $\bar{\mathcal{X}}_f^i = \{x_i : \bar{V}_{f_i}(x_i) = x_i^T P_i x_i \le \rho_i\}$, $i \in \mathbb{M}$, which locally stabilize the overall closed loop interconnected system (4) for all $\tilde{h}(k,x) \in \boldsymbol{H_\alpha}$ with the region of attraction $\bar{\mathbb{X}}_N = \bar{\mathbb{X}}_N^1 \times \ldots \times \bar{\mathbb{X}}_N^M$. $\bar{\mathbb{X}}_N \subseteq \mathbb{R}^n$ denotes the set of all initial overall state vectors for which assumption (d) is satisfied.

The proof of this theorem with a comparison to Theorem 1 is straightforward.

Since the subsystems are disjoint, problem (17) includes M individual sub-problems as $\mathbb{P}_N(x_i), i \in \mathbb{M}$. $\mathbb{P}_N(x_i)$ is the robust version of $\mathbb{P}_N(x_i^{IS})$.

Remark 3. The bounding procedure described by the Ineq. (3) can also incorporate uncertainties in the elements of matrix A_i as well as the perturbation function $h_i(k,x)$. This feature is of high importance since it is often difficult (and sometimes altogether impossible) to specify the model precisely. In order to incorporate the effects of uncertainties in matrix A_i, it is introduced a perturbation $\Delta h_i(k,x)$ into the corresponding component of the function $h_i(k,x)$.

Remark 4. The summary of main steps of the proposed RDMPC synthesis procedure is given in algorithms 1. It shows the procedure for the synthesis of the main components of each MPC optimization problem $\mathbb{P}_N^i(x_i)$, $i \in \mathbb{M}$, i.e., $\bar{V}_{f_i}(x_i)$ and $\bar{\mathcal{X}}_f^i$. Once these stabilizing components are specified offline, at each time sample k, each subsystem $i, i \in \mathbb{M}$, measures its local states $x_i(k)$ and solves its associated optimal control problem $\mathbb{P}_N^i(x_i)$. Finally, each subsystem applies its local control input $\bar{k}_{MPC}^i = \bar{u}_{i,0}^*$.

Algorithm 1. Offline synthesis of $\bar{V}_{f_i}(x_i)$ and $\bar{\mathcal{X}}_f^i$

Step 1. Solve problem 1 to find $\bar{V}_{f_i}(x_i) = x_i^T P_i x_i$ and K_i and, at the same time, maximize the bounding parameter α_i.

Step 2. Solve problem 2 to find the largest feasible level set $\{x_i : \bar{V}_{f_i}(x_i) = x_i^T P_i x_i \le \rho_i\}$, locally at each subsystem i, $i \in \mathbb{M}$, and consider it as the terminal set $\bar{\mathcal{X}}_f^i$.

V. SIMULATION RESULTS

In this section, the proposed RDMPC algorithm is applied to an interconnected unstable system composed of $M = 5$ inverted pendulums, with masses of $m_i = 1kg$, $i \in \mathbb{M}$, and rod lengths of $l = 1m$, coupled by springs as shown in Fig. 1.

It is assumed that each inverted pendulum is an individual subsystem. Every two adjacent pendulums (i.e., m_i and m_{i+1}, $i \in \mathbb{M} - \{1\}$) are coupled by a sliding spring of $k_i = 3N/m$, which its position $a_i(k,x)$ is an uncertain parameter. The nonlinear equation of motion of the i^{th} inverted pendulum is as follows:

$$m_i l^2 \ddot{\theta}_i \cos\theta_i = m_i g l \sin\theta_i + k_i a_i^2(\theta_i - \theta_{i+1}) + u_i, \quad i \in \mathbb{M} \tag{34}$$

in which θ_i is the angle of the i^{th} inverted pendulum and u_i is the torque applied in its pivot. It is assumed that the position of each spring is a piecewise function in both time and state such that it can slide up and down the rods between the support and the high $\bar{a}_i \in [0 \quad l]$. For all $i \in \mathbb{M}$, by linearizing the above equation of motion in the vicinity of $\theta_i = 0$, defining the state vector of each subsystem consisting of its angle θ_i and its angular velocity $\dot{\theta}_i$, i.e., $x_i = [\theta_i \quad \dot{\theta}_i]^T$, and defining a normalized interconnection parameter as $e_i = (a_i/\bar{a}_i)^2 \in [0,1]$ the state-space model of each subsystem in form of (1) will be obtained accordingly as follows:

$$\dot{x}_i(t) = \begin{bmatrix} 0 & 1 \\ \varphi_i & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ \beta_i \end{bmatrix} u_i + h_i(k,x), i \in \mathbb{M}, \tag{35}$$

where $\varphi_i = g/l$, $\beta_i = 1/m_i l^2$, $\gamma_i = \bar{a}_i{}^2 k_i$. The uncertain state-dependent perturbation function for the i^{th} subsystem is defined as

$$h_i(x) = e_{i-1}\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{\gamma_{i-1}}{\beta_i} & 0 & -\frac{\gamma_{i-1}}{\beta_i} & 0 \end{bmatrix}\begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix} + e_i\begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{\gamma_i}{\beta_i} & 0 & \frac{\gamma_i}{\beta_i} & 0 \end{bmatrix}\begin{bmatrix} x_i \\ x_{i+1} \end{bmatrix}, i \in \mathbb{M}, \tag{36}$$

The linearized continuous-time mathematical model of the system is discretized by the Euler method with sampling time $0.1s$. The local state and input constraints are considered as $\|x_i\|_\infty \le 10$ and $\|u_i\|_\infty \le 15$, respectively. The prediction horizon is $N_i = 5$, Q_i and R_i are identity, $\forall i \in \mathbb{M}$. Minimal uncertainty bounds on α_i denoted by $\bar{\alpha}_i$, which is the smallest feasible value of α_i in problem 1, are chosen as $\bar{\alpha}_i = 1$, $\forall i \in \mathbb{M}$. The control goal is to design a decentralized Predictive control strategy in order to keep the pendulums in the upright position via the inputs u_i, $i \in \mathbb{M}$, despite a variation in the parameters a_i. The initial states are chosen as

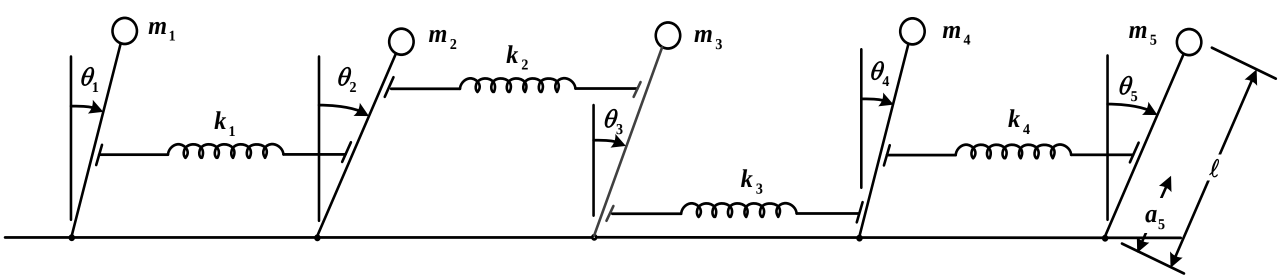

Fig. 1. The schematic of overall system with five subsystems

$x_1 = [1.2, 0]^T$, $x_5 = [-0.2, 0]^T$ and $x_i = [0, 0]^T, i = 2, \ldots, 4$. Solving the optimization problem 1 using the YALMIP MATLAB Toolbox [39], results in $\alpha_1 = 2.89$. If we solve problem 1 in centralized fashion (the selection of matrix H for centralized and overall decentralized systems are quite different), in which the obtained bounds $\tilde{\alpha}_i$ for all subsystems are identical and equal to $\tilde{\alpha}$, the value $\tilde{\alpha}_i = \tilde{\alpha} = 1.73$ is obtained.

In decentralized case, the obtained value for $\alpha_i, i = 1, \ldots, 5$, are shown in Table I. As shown in this table, the tolerable uncertainty bound for each subsystem in decentralized MPC case is larger than the corresponding values for centralized MPC. Therefore, by designing decentralized controllers we can maximize the interconnection bounds the can be tolerated by each subsystem separately. The values of α_i greater than 1 mean that the subsystems can also tolerate uncertainty in other parameters such as masses m_i and springs coefficients k_i.

TABLE I

OBTAINED BOUNDS FOR CENTRALIZED AND DECENTRALIZED STRUCTURES

Subsystems	# 1	# 2	# 3	# 4	# 5
$\tilde{\alpha}_i$ (Centralized)	1.73	1.73	1.73	1.73	1.73
α_i (Decentralized)	2.89	2.64	2.58	2.43	2.59

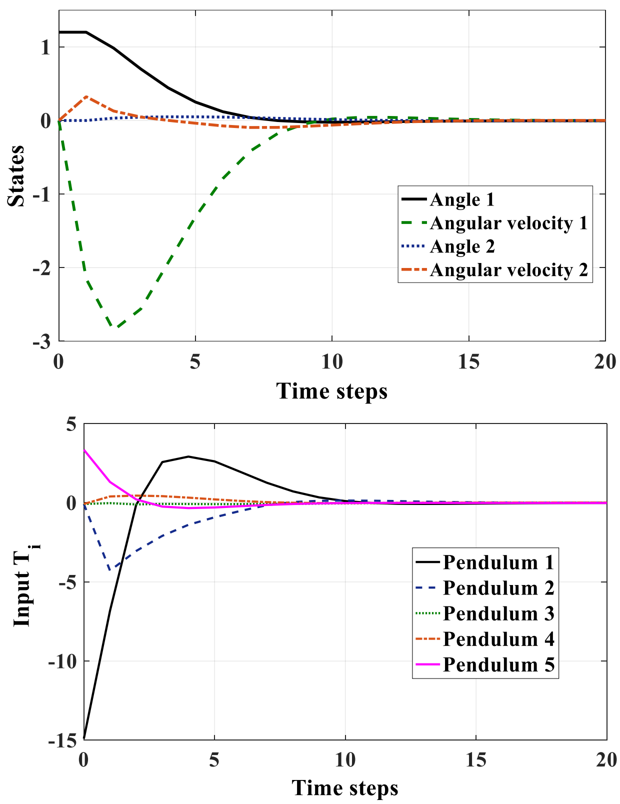

Fig. 2. Trajectories under RDMPC for i=1,2.

In order to show the feasibility and performance of the proposed RDMPC algorithm, two different simulations were done. In first simulation, it is assumed that the parameters m_i and k_i have their nominal values and only the position of first spring is variable according to the following profile.

$$e_1(k) = \begin{cases} 0.8 & 0 \le k < 5 \\ 0.5 & 5 \le k < 10 \\ 1 & 10 \le k \end{cases}$$

Fig. 2 shows the simulated closed-loop trajectories for the control inputs and states of the first two pendulums. In second simulation, despite the uncertain interconnections, the robustness of the proposed control strategy with respect to uncertainty in the model's parameters k_i and m_i is evaluated. During the simulation process, for $i = 1, 5$, these parameters were allowed to vary randomly subject to $0.8k_i^0 \le k_i \le 1.2k_i^0$, and $0.8m_i^0 \le m_i \le 1.2m_i^0$ (k_i^0 and m_i^0 represent nominal values for i^{th} subsystem).

The simulated closed-loop results for the control inputs and angles of all subsystems are shown in Fig. 3. These simulation results show that the proposed RDMPC strategy performs well and stabilizes the state variables of the inverted pendulums system despite the interaction between inverted pendulums and a wide range of parameter variations.

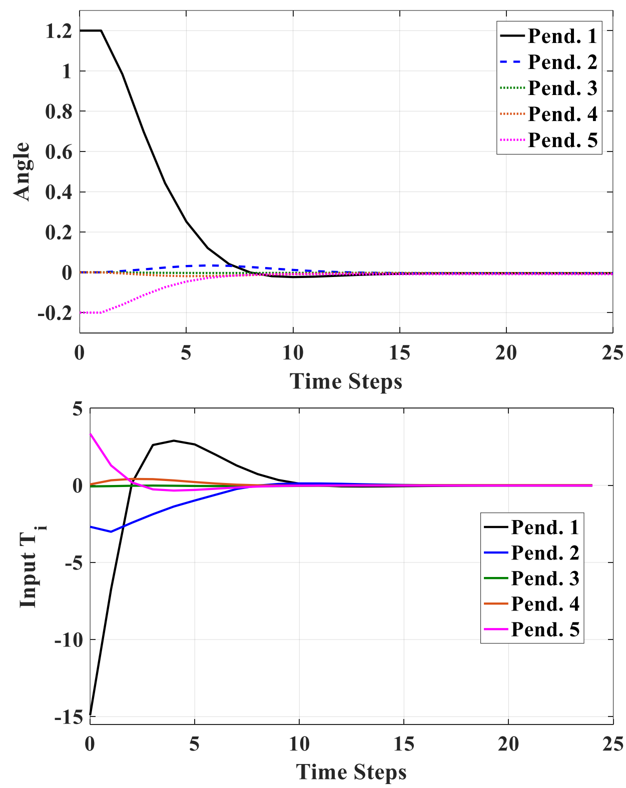

Fig. 3. Closed-loop trajectories under RDMPC in presence of parameter uncertainty for i=1,5.

Now, we evaluate the performance of the proposed RDMPC approach compared to robust decentralized static state feedback (RDSSF) suggested in [40] in Table II. The summation of absolute error $\left(SAE = \sum_0^{t_s} |e(k)|\right)$ [41] performance index is applied to evaluate and compare the performance (in the stabilization problem $e(k) = x(k)$). Fig. 4 shows the simulated closed-loop trajectories of the first two pendulums controlled by RDMPC and RDSSF controllers. As one sees in Fig. 4 and Table II, our proposed RDMPC approach improves the performance of the closed-loop system with compare to RDSSF approach.

TABLE II

COMPARISON OF SAE PERFORMANCE INDICES FOR RDMPC AND SSF STRUCTURES FOR SUBSYSTEMS 1, 2 AND 5

States		θ_1	$\dot{\theta}_1$	θ_2	$\dot{\theta}_2$	θ_5	$\dot{\theta}_5$
SAE	(RDMPC)	5.13	12.42	0.41	1.04	1.09	1.91
	(RDSSF)	10.39	11.72	4.75	5.23	2.76	1.93

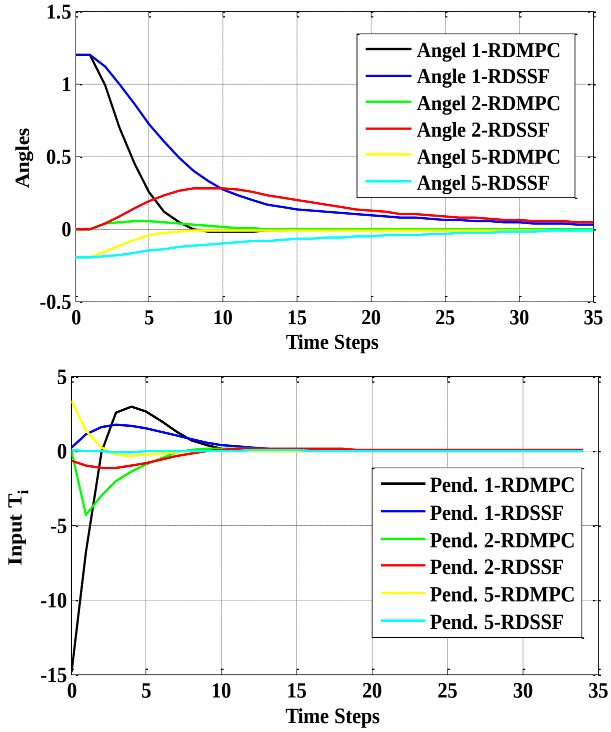

Fig. 4. Closed-loop trajectories under RDMPC and RDSSF for i=1,2,5.

VI. CONCLUSION

In this paper, it is proposed a robust decentralized MPC strategy for a class of discrete-time interconnected systems. Each nominal LTI subsystem is subjected to an additive quadratic ally bounded perturbation, which can be nonlinear in general. The perturbation can be arisen due to interactions between the subsystems in a networked system. To guarantee robust asymptotic stability, the sufficient conditions for designing MPC stabilizing components based on dual-mode MPC stability theorem in the form of an LMI optimization problem was characterized. To illustrate the effectiveness of the proposed approach, it was applied to a system composed of five coupled inverted pendulums, which is a typical interconnected system, in a decentralized fashion. The results confirmed the graceful capability of the proposed RMPC approach. In the future study, we will consider the information of neighbor subsystems in the local cost function and prediction in order to enlarge the permitted bound of uncertainties and enhance the performance of overall system.

REFERENCES

[1] D. D. Siljak, *Decentralized control of complex systems*: Courier Corporation, 2011.

[2] D. Wang, D. Liu, C. Mu, and H. Ma, "Decentralized guaranteed cost control of interconnected systems with uncertainties: a learning-based optimal control strategy," *Neurocomputing*, vol. 214, pp. 297-306, 2016.

[3] S. Ghali, I. S. Aousgi, and S. Elloumi, "Decentralized Optimal Control of Interconnected systems using a direct approach," in *2017 International Conference on Advanced Systems and Electric Technologies (IC_ASET)*, 2017, pp. 79-84.

[4] Y.-X. Li, S. Tong, and G.-H. Yang, "Observer-based adaptive fuzzy decentralized event-triggered control of interconnected nonlinear system," *IEEE transactions on cybernetics*, 2019.

[5] L. Li, R. R. Negenborn, and B. De Schutter, "Distributed model predictive control for cooperative synchromodal freight transport," *Transportation Research Part E: Logistics and Transportation Review*, vol. 105, pp. 240-260, 2017.

[6] V. Puig, C. Ocampo-Martínez, J. Romera, J. Quevedo, R. Negenborn, P. Rodríguez, *et al.*, "Model predictive control of combined irrigation and water supply systems: application to the Guadiana river," in *Proceedings of 2012 9th IEEE International Conference on Networking, Sensing and Control*, 2012, pp. 85-90.

[7] A. Casavola, G. Franzè, D. Menniti, and N. Sorrentino, "Voltage regulation in distribution networks in the presence of distributed generation: A voltage set-point reconfiguration approach," *Electric power systems research*, vol. 81, pp. 25-34, 2011.

[8] F. Tedesco, A. Casavola, and E. Garone, "A distributed parallel command governor strategy for the coordination of multi-agent networked systems," *IFAC Proceedings Volumes*, vol. 45, pp. 478-483, 2012.

[9] F. Tedesco, D. M. Raimondo, and A. Casavola, "Collision avoidance command governor for multi-vehicle unmanned systems," *International Journal of Robust and Nonlinear Control*, vol. 24, pp. 2309-2330, 2014.

[10] M. S. Mahmoud, "Resilient decentralized stabilization for interconnected discrete-time systems," *Journal of optimization theory and applications*, vol. 145, pp. 507-525, 2010.

[11] M. S. Elliott and B. P. Rasmussen, "Model-based predictive control of a multi-evaporator vapor compression cooling cycle," in *American Control Conference, 2008*, 2008, pp. 1463-1468.

[12] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica,* vol. 36, pp. 789-814, 2000.

[13] D. Q. Mayne, "Model predictive control: Recent developments and future promise," *Automatica,* vol. 50, pp. 2967-2986, 2014.

[14] P. W. Sauer, M. A. Pai, and J. H. Chow, *Power System Dynamics and Stability: With Synchrophasor Measurement and Power System Toolbox*: Wiley, 2017.

[15] J. B. Rawlings and M. J. Risbeck, "Model predictive control with discrete actuators: Theory and application," *Automatica,* vol. 78, pp. 258-265, 2017.

[16] J. H. Lee, "Model predictive control: Review of the three decades of development," *International Journal of Control, Automation and Systems,* vol. 9, pp. 415-424, 2011.

[17] P. O. Scokaert and J. B. Rawlings, "Stability of model predictive control under perturbations," in *IFAC Symposium on Nonlinear Control System Design, Tahoe City, California*, 1995, pp. 21-26.

[18] D. L. Marruedo, T. Alamo, and E. Camacho, "Input-to-state stable MPC for constrained discrete-time nonlinear systems with bounded additive uncertainties," in *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, 2002, pp. 4619-4624.

[19] G. Betti, M. Farina, and R. Scattolini, "A robust MPC algorithm for offset-free tracking of constant reference signals," *IEEE Transactions on Automatic Control,* vol. 58, pp. 2394-2400, 2013.

[20] D. Raimondo, L. Magni, and R. Scattolini, "Decentralized MPC of nonlinear systems: An input-to-state stability approach," *International Journal of Robust and Nonlinear Control,* vol. 17, pp. 1651-1667, 2007.

[21] H. K. Khalil and J. W. Grizzle, *Nonlinear systems* vol. 3: Prentice hall Upper Saddle River, NJ, 2002.

[22] S. Riverso, M. Farina, and G. Ferrari-Trecate, "Plug-and-play decentralized model predictive control," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, 2012, pp. 4193-4198.

[23] S. Riverso, M. Farina, and G. Ferrari-Trecate, "Plug-and-play decentralized model predictive control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, pp. 2608-2614, 2013.

[24] M. S. Ghasemi and A. A. Afzalian, "Robust tube-based MPC of constrained piecewise affine systems with bounded additive disturbances," *Nonlinear Analysis: Hybrid Systems,* vol. 26, pp. 86-100, 2017.

[25] A. Alessio, D. Barcelli, and A. Bemporad, "Decentralized model predictive control of dynamically coupled linear systems," *Journal of Process Control,* vol. 21, pp. 705-714, 2011.

[26] D. Barcelli and A. Bemporad, "Decentralized model predictive control of dynamically-coupled linear systems: Tracking under packet loss," *IFAC Proceedings Volumes,* vol. 42, pp. 204-209, 2009.

[27] L. Lu, Y. Zou, and Y. Niu, "Event-triggered decentralized robust model predictive control for constrained large-scale interconnected systems," *Cogent Engineering,* vol. 3, 2016.

[28] T. Keviczky, F. Borrelli, and G. J. Balas, "Decentralized receding horizon control for large scale dynamically decoupled systems," *Automatica,* vol. 42, pp. 2105-2115, 2006.

[29] D. D. Šiljak and D. M. Stipanovic, "Robust stabilization of nonlinear systems: The LMI approach," *Mathematical Problems in Engineering,* vol. 6, 2000.

[30] D. D. Siljak, D. M. Stipanovic, and A. I. Zecevic, "Robust decentralized turbine/governor control using linear matrix inequalities," *IEEE Transactions on Power Systems,* vol. 17, pp. 715-722, 2002.

[31] A. S. Tlili, S. Dhbaibi, and N. B. Braiek, "Robust decentralised observer based guaranteed cost control for nonlinear uncertain interconnected systems. Application to multi-machine power systems," *International Journal of Systems Science,* vol. 43, pp. 1713-1727, 2012.

[32] A. I. Zecevic, G. Neskovic, and D. D. Siljak, "Robust decentralized exciter control with linear feedback," *IEEE Transactions on Power Systems,* vol. 19, pp. 1096-1103, 2004.

[33] A. I. Zečević and D. D. Šiljak, "Estimating the region of attraction for large-scale systems with uncertainties," *Automatica,* vol. 46, pp. 445-451, 2010.

[34] S. V. Naghavi, S. V. Naghavi, A. Safavi, A. Safavi, M. H. Khooban, M. H. Khooban*, et al.*, "A robust control strategy for a class of distributed network with transmission delays: LMI-based model predictive controller," *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering,* vol. 35, pp. 1786-1813, 2016.

[35] H. Chen and F. Allgöwer, "A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability," *Automatica,* vol. 34, pp. 1205-1217, 1998.

[36] V. Yakubovich, "The S-procedure in nonlinear control theory," *Vestnik Leningrad Univ. Math,* vol. 4, pp. 73-93, 1977.

[37] J. G. VanAntwerp and R. D. Braatz, "A tutorial on linear and bilinear matrix inequalities," *Journal of Process Control,* vol. 10, pp. 363-385, 2000.

[38] M. C. de Oliveira, J. Bernussou, and J. C. Geromel, "A new discrete-time robust stability condition," *Systems & control letters,* vol. 37, pp. 261-265, 1999.

[39] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *2004 IEEE international conference on robotics and automation (IEEE Cat. No. 04CH37508)*, 2004, pp. 284-289.

[40] D. Stipanovic and D. Siljak, "Robust stability and stabilization of discrete-time non-linear systems: the LMI approach," *International Journal of Control,* vol. 74, pp. 873-879, 2001.

[41] E. Zakeri, S. A. Moezi, and M. Eghtesad, "Optimal interval type-2 fuzzy fractional order super twisting algorithm: A second order sliding mode controller for fully-actuated and under-actuated nonlinear systems," ISA transactions, vol. 85, pp. 13-32, 2019.

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