International Journal of Industrial Electronics, Control and Optimization © 2018 IECO
Vol. 1, No. 2, pp. 133-142, June (2018)

Designing an Optimal Linear Bid Function in a Pay-as-Bid Electricity Market

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ABSTRACT

In this paper, bidding problem in electricity markets is formulated from the viewpoint of a generation company. With focus on Iran's electricity market structure, the objective is to design an optimal linear bid function considering pay-as-bid pricing mechanism. The market clearing price is considered as a stochastic variable. The bidding problem is explained as a nonlinear optimization problem from the viewpoint of a price-taker generation company. Then, a numerical study is performed to show the effect of stochastic characteristics of market price on the optimal values of bidding parameters. In order to have more expected profit, a mathematical problem is designed and solved to partition the generation capacity. An example is designed to calculate a linear bid function using the proposed technique. Also a comparison between step-wise and linear bidding is presented.

Article Info

Keywords:
Electricity market, Pay-as-bid pricing
Strategic bidding problem

Article History:
Received 2018-03-14
Accepted 2018-06-30

Nomenclature

$C(P)$	GenCo's cost function
P	GenCo's generated power
A , B	The cost function coefficients
$P(P)$	GenCo's bid function
A ,B	The bid function coefficients
P_{MAX}	GenCo's generation capacity
P_M	MCP
P_E	The dispatched capacity in the electricity market
$\pi_{\alpha,\beta}(\rho_m)$	GenCo's profit
$E\{\Pi\}$	GenCo's expected profit
$F_{P_M}(.)$	The probability density function of the electricity MCP
A^*	The optimal value of α
$F_{P_M}(.)$	The cumulative distribution function of the electricity market clearing price
σ, μ	Standard deviation and mean value of the MCP

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I. INTRODUCTION

A. Motivation

In an electricity pool auction, generation companies (GenCos) offer their generation capacities. Also, in a double sided market, consumers offer their demand. Then an Independent System Operator (ISO) clears the market based on received bid functions considering system requirements. The GenCo's goal is to increase its profit, which can be achieved through strategic bidding in electricity markets [1]. This paper proposes a new method to design supply function from the viewpoint of a GenCo.

B. Literature Review

Bidding strategy problem has been widely explored in the literature. In [1], the different methods are classified into four categories. In the first class, the bidding problem is solved estimating market price. In the second, the behaviour of rivals is estimated in order to design bidding strategy. In the 3rd and 4th classes, respectively, game theory and heuristic methods are used to solve the problem. Similarly, in a recent work, the literatures are classified slightly different in [2]. Single GenCo optimization models, game theory based models, agent-based models, and hybrid models constitute the four proposed categories in [2]. References [3-9] are examples of the stated above categories, respectively.

In [10], a bi-level optimization method is proposed, in which the bidding coefficients of rivals are assumed to follow a joint normal probability distribution function. In a single GenCo optimization point of view, the GenCo simulates the rivals' behavior and thereupon calculates the market results. A similar method is used in [11] and [12] for bidding problem in energy and reserve multi-markets.

In [13] a stochastic method is proposed to calculate a step-wise bid function in an electricity market with uniform pricing mechanism. In this method, it is assumed that the historical information about the behavior of rivals is readily available. It is obvious that this assumption is away from reality.

In [14] a piecewise staircase bid function is optimally calculated using linear programming in two cases, complete and incomplete information. In the first case, it is assumed that the complete information on system conditions and rivals strategies is available. In the latter case, different scenarios are generated based on the probability distribution of historical information of demand levels and strategic policies of the competitors. Low, normal, and high levels for demand, and low (bidding at marginal), normal (bidding at %110 of marginal), and high (bidding at %120 of marginal) levels for the rivals' behavior have been considered. The need to scenario analysis and reduction in different market conditions makes the proposed method to be time consuming and makes the results to be far from the optimal values in practice.

In [4], a classical optimization method is proposed in which the market clearing price (MCP) is assumed to be stochastic variable followed a probability density function (PDF). The method is used to find a closed form for optimal bidding prices in a step-wise bidding problem. An extension of the method proposed in [4] for bidding problem in only-energy markets, is utilized by the authors for strategic bidding in joint energy and reserve markets [15,16].

In [17] and [7], Q-learning (QL) approaches are suggested to solve the optimal bidding strategy problem, respectively, in only-energy markets and joint energy and reserve markets. In the mentioned papers, to verify the performance of the QL method, an extension of the method proposed in [4], is utilized.

A min-max regret approach, in a uniform pricing environment, is presented in [18] for a bidding and scheduling problem, based on the confidence intervals of price forecasts. To design a bidding curve, the electricity price interval is partitioned into several subintervals. Then, for each subinterval, the proposed min-max regret model is solved to obtain an optimal generation capacity.

In [19] and [20] four different available parameterization methods are investigated for the construction of the optimal linear supply function bids with varied slope and/or the intercept of the marginal cost functions. In fact, in this approach the impact of the choice of the parameterization method on the market equilibrium solution is examined. Finally, the study concludes that the solutions for all the parameterization methods are very similar for no transmission congestion and no strict voltage limits. But, in large systems, no pattern is concluded and each parameterization method results different number of congested lines and different network operational conditions.

C. Contributions

In this paper, an extension of the method proposed in [4] is presented for the linear bidding problem. According to the categories defined in [1] and [2], the method is a single optimization model based on market price estimation. In [4], the bidding problem is formulated assuming a linear cost function for a GenCo, and optimal bidding prices for a step-wise bid function are calculated. The generation capacity is not considered as an optimization variable, in [4].

In this paper, the market price is assumed to follow a probability density function. Then, independent of the type of the PDF, the expected profit of the GenCo is formulated, considering a linear bid function. The optimal values of the intercept and slope of the bid function can be calculated through optimizing the objective function.

Moreover, it will be shown that the maximum expected profit can be greatly obtained by bidding less than the total capacity. Consequently, due to the prohibition of physical withholding in electricity markets, the GenCo should split its generation capacity and find the optimal bid function for each section. In this case, the expected profit increases. Therefore, our main contribution is to use the extended method, which considers the generation capacity as an optimization variable, to find the optimal linear bid function in a pay-as-bid (PAB) wholesale electricity market. It should be noted that the bid function will cover the total generation capacity and physical withholding will not occur.

D. Paper Organization

In the following, the market structure is explained in section 2 and the strategic bidding problem is formulated in section 3. The capacity partitioning procedure is comprehensively described in section 4, and an example is presented.

II. Structure of The Considered Market

The wholesale hour-ahead single-sided electricity market, which is considered in this paper, is run on PAB pricing mechanism, similar to the Iran's electricity market structure. The main agents are GenCos and the independent system operator. The GenCos bid their piece-wise linear bid functions for energy as a commodity, to the electricity market. After aggregating the all submitted bid functions from all market participants, the ISO clears the energy market and publishes the highest accepted bid, called in this paper as the market clearing price, and informs GenCos from their accepted bids. It is assumed in this paper that the MCP is

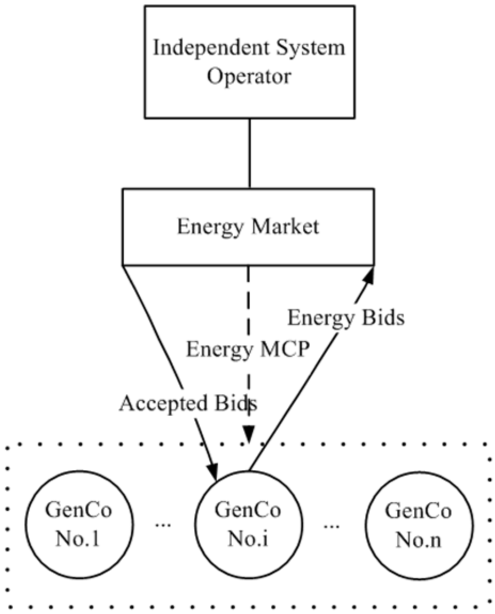

Fig. 1. The considered market structure

publicly available or GenCos estimate it, based on their historical lost and won bidding prices. The above explanations have been summarized in Fig.1.

It should be noted that the proposed method can be easily utilized in a double sided auction and/or in a multi-unit bidding problem. Also, it is assumed that the GenCo is a price-taker that cannot affect the marginal market price.

According to the assumption of an hour-ahead bidding problem and in order to simplify the problem, the physical constraints, for example, ramp rate, have not been considered. It can be assumed that, for the next hour, the GenCo's maximum available capacity is known as $p_{\max}$.

III. Problem Description

Market clearing price and GenCos' bidding strategy can be affected by several factors such as market structure, energy demand, rivals' behavior, and power system constraints like transmission and generation outages. Therefore, using the MCP, the market behavior can be modeled and analyzed. Additionally, the MCP is uncertain because of uncertainties in power system and in participants' actions, and therefore it can be assumed as a random variable [17]. In [21] it is proposed, in each load level, to use a probability density function in order to model and to summarize electricity market price behaviors.

In this paper, the strategic bidding problem is designed and formulated based on the approximated PDF of electricity market prices from the viewpoint of a price-taker GenCo. The PDF of MCP can be approximated based on the historical available information of market clearing prices. In some electricity markets, that the MCP is not available, at least, the weighted average prices are available and the stated above

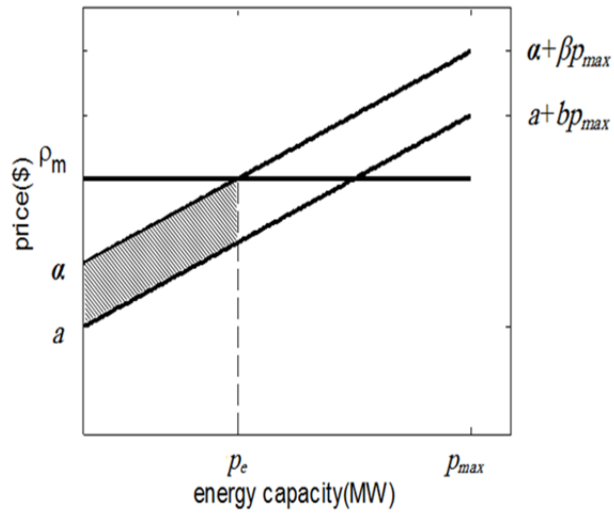

Fig. 2. The marginal cost and the linear bid functions

PDF can be constructed using this available data. The proposed method is also applicable in this case and it is obvious that bidding risk will be lower in this situation.

In the following, at first, the basic problem from the viewpoint of a GenCo is defined. Then the proposed method for constructing an optimal piecewise linear electricity supply function, is formulated and described.

A. GenCo's profit

GenCo's cost function is assumed to be quadratic, $cost(p)=fixed\ cost+ ap+\frac{1}{2}bp^2$. Therefore, the marginal cost, $MC(p)$ is $a+bp$, where p is the generated power. GenCo's linear bid function, $\rho(p)$, is assumed at first, a one-piece linear as $\alpha+\beta p$. The MCP, ρ_m, which does not depend on price-takers' behavior, the marginal cost function and the linear bid function are presented in Fig.2.

If the maximum of bid function is lower than the MCP, the GenCo should deliver p_e MW, in accordance with the MCP, in the specified hour to the electricity market. It should be noted that due to the assumption of a price-taker GenCo, the interaction of bid function and market price, which is shown in Fig.2, may occurs when the competition level is high.

Obviously, the shaded area in Fig.2 shows GenCo's profit. The basic problem is to calculate the optimal values of bid function coefficients. Also, as will be explained in the final section, the bidding capacity, p_e, can be considered as a variable. Thus, the GenCo can optimally partition its available generation capacity, $p_{\max}$, to construct a complete piecewise linear bid function on its total generation capacity.

It should be noted that the method for modelling the basic problem is originally proposed by [4] in which a linear (not quadratic) cost function is considered and a one-step (not linear) bidding problem is designed.

Furthermore, with a new point of view, to split the generation capacity and increase the expected profit, the introduced method in [4] is extended by considering the bidding capacity as a decision variable. The proposed method can be used in different conventional types of strategic bidding formulation, such as fixed slope, fixed intercept, marginal cost multiplication, and so forth.

B. Expected Profit of the GenCo

As stated before, the bid function is assumed as $\rho(p)=\alpha+\beta p,\ 0\le p\le p_e\le p_{max}$. Therefore, the profit of the GenCo is the shaded area in Fig.2 and is calculated by

$$\pi=(\alpha-a)p_e+\frac{1}{2}(\beta-b)p_e^2. \tag{1}$$

Obviously, the bidding price acceptance depends on MCP which is a stochastic variable. Thus, GenCo's profit is a piecewise function of ρ_m:

$$\pi_{\alpha,\beta}(\rho_m)=\begin{cases}0, & \rho_m<\alpha,\\ (\alpha-a)p_e+\frac{1}{2}(\beta-b)p_e^2, & \alpha\le\rho_m<\alpha+\beta p_{max},\\ (\alpha-a)p_{max}+\frac{1}{2}(\beta-b)p_{max}^2, & \alpha+\beta p_{max}\le\rho_m,\end{cases} \tag{2}$$

where, ρ_m is the MCP, p_{max} is the GenCo's total capacity, and $p_e=\frac{\rho_m-\alpha}{\beta}$ is the dispatched capacity in the electricity market.

In fact, the definition of $\pi_{\alpha,\beta}(\rho_m)$ in Equ. (2) states that for bidding prices not greater than MCP, the GenCo wins the competition and sells its generated power to the market.

The profit expectation of the GenCo is calculated as:

$$E\{\pi\}=\int_{-\infty}^{+\infty}\pi_{\alpha,\beta}(\rho_m)f_{\rho_m}(\rho_m)d\rho_m, \tag{3}$$

where $f_{\rho_m}(.)$ is the probability density function of the electricity MCP, ρ_m.

Considering Equs. (2) and (3), one can see that GenCo's expected profit is:

$$\begin{aligned}E\{\pi\}&=A(\alpha,\beta)\cdot\int_{\alpha}^{\alpha+\beta p_{max}}\rho_m^2\cdot f_{\rho_m}(\rho_m)d\rho_m\\&+B(\alpha,\beta)\cdot\int_{\alpha}^{\alpha+\beta p_{max}}\rho_m\cdot f_{\rho_m}(\rho_m)d\rho_m\\&+C(\alpha,\beta)\cdot\int_{\alpha}^{\alpha+\beta p_{max}}f_{\rho_m}(\rho_m)d\rho_m\\&+D(\alpha,\beta)\cdot\int_{\alpha+\beta p_{max}}^{+\infty}f_{\rho_m}(\rho_m)d\rho_m,\end{aligned} \tag{4}$$

in which

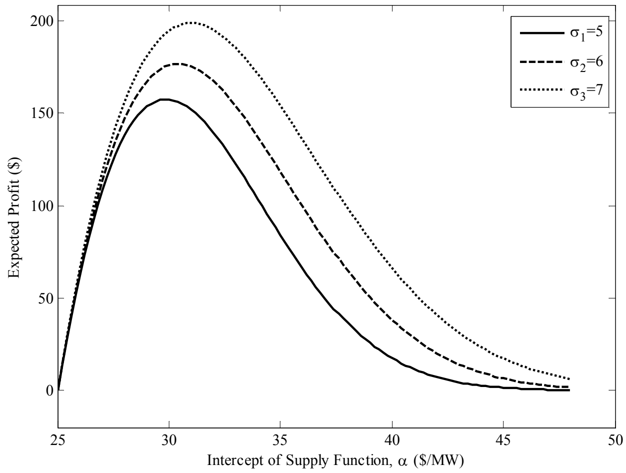

Fig. 3. Expected profit versus α (a=25,b=0.1, p_{max}=200) for normally distributed market prices with σ=6 and different mean values

$$\begin{aligned}A(\alpha,\beta)&=\frac{1}{2}\frac{(\beta-b)}{\beta^2}\\B(\alpha,\beta)&=\frac{b\alpha-a\beta}{\beta^2}\\C(\alpha,\beta)&=\frac{-\alpha^2(\beta+b)}{2\beta^2}+\frac{a\alpha}{\beta}\\D(\alpha,\beta)&=(\alpha-a)p_{max}+\frac{1}{2}(\beta-b)p_{max}^2.\end{aligned} \tag{5}$$

In a specific case, if β=b=0, then the expected profit of the resulting step-wise bid function is:

$$E\{\pi\}=(\alpha-a)p_{max}[1-F_{\rho_m}(\alpha)], \tag{6}$$

and the optimal value of α is

$$\alpha^*-a=\frac{1-F_{\rho_m}(\alpha^*)}{f_{\rho_m}(\alpha^*)}, \tag{7}$$

where $F_{\rho_m}(.)$ is the cumulative distribution function (CDF) of the electricity MCP, ρ_m, and 1-$F_{\rho_m}(\alpha)$, is the probability of acceptance of α or the probability of acceptance of the total bidding capacity. Equs. (6) and (7) previously presented in [4].

In general, it is assumed that $\beta=b\neq0$. So, GenCo's expected profit is:

$$\begin{aligned}E\{\pi\}&=\frac{\alpha-a}{b}\int_{\alpha}^{\alpha+bp_{max}}\rho_m\cdot f(\rho_m)d\rho_m\\&-\alpha\frac{(\alpha-a)}{b}[F(\alpha+bp_{max})-F(\alpha)]\\&+(\alpha-a)p_{max}\cdot[1-F(\alpha+bp_{max})].\end{aligned} \tag{8}$$

In [22], it is shown that electricity market prices usually follow normal distribution in medium and low load levels. The graphical illustrations of the expected profit versus α, for different values of the mean and the standard deviation of

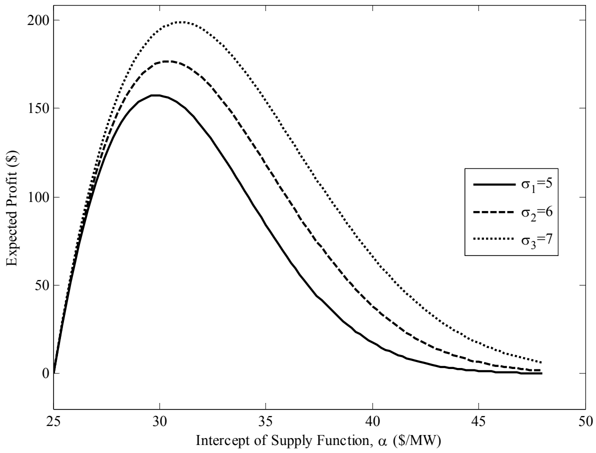

Fig. 4. Expected profit versus α (a=25, b=0.1, p_{max}=200) for normally distributed market prices with μ=32 and different values of standard deviation.

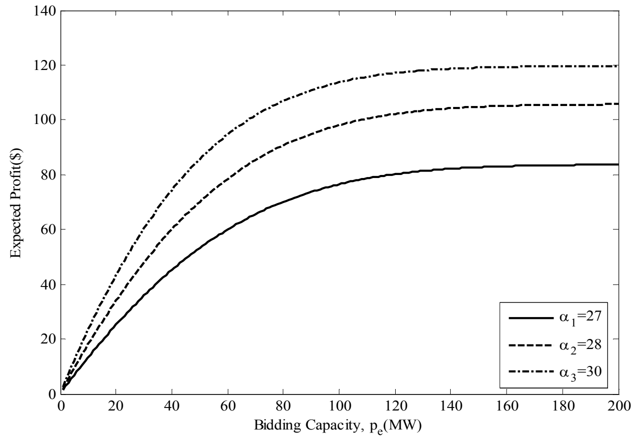

Fig. 5. Expected profit versus p_e (a=25, b=0.1, p_{max}=200, $\rho_m \approx N(30,6)$)

market price, are given in Fig.3 and Fig.4, respectively, when $\beta = b$ and MCP follows a normal distribution. The statistical parameters of market price are selected according to [22].

It can be seen in Fig.3, and Fig.4 that the expectation of the profit is sensitive to statistical characteristics of the electricity market price.

C. GenCo's Objective Function

According to the above explanations, the GenCo should find the optimal values of α and β in order to maximize the expected profit. Therefore the objective of the GenCo is to

$$\begin{aligned} &\underset{\alpha,\beta}{\text{maximize}}\, E\{\pi\} \\ &\text{subject to} \\ &\alpha+\beta p_{max} \leq Price\ Cap. \end{aligned} \tag{9}$$

The GenCo calculates the optimal values of bid function parameters, α^*andβ^*. The expected profit in Equ. (9) is calculated according to Equs. (4) or (8). The problem (9) can be solved numerically using nonlinear optimization methods.

IV. Application in Designing Optimal Bid Function

In this section, we use GenCo's objective function (9) to split the generation capacity in order to increase the expected profit. First, we explain the necessity of the capacity splitting, and then, present the corresponding optimization problem. It should be emphasized that in the modeling method of this paper, the type of PDF of market price is not important and not effective. But, in the rest of the paper, it is assumed mostly, that the market price follows a normal probability density function.

We plot the expected profit versus bidding capacity, p_e, for different values of α, when β=b, and MCP is a normal, Fig.5, and lognormal, Fig.6, distributed random variable. It seems that, independent of the MCP model, almost maximum expected profit is attained by bidding less than the total capacity, p_{max}. Since the physical withholding is forbidden in electricity markets, we will show how to use the above conclusion to split the generation capacity in order to design an optimal bid function.

In what follows, a method is proposed to optimal capacity splitting and numerical results are presented.

A. Capacity splitting

According to Figures 5 and 6, it can be concluded that splitting the generation capacity can increase the expected profit. Here, finding the optimal splitting is formulated as a nonlinear optimization problem.

To maximize the expected profit, we partition the production capacity space, $[0, p_{max}]$, into n intervals $I_0 = [p_0, p_1]$, $I_1 = [p_1, p_2]$, $\cdots$, $I_{n-1} = [p_{n-1}, p_n]$, where $p_0 = 0$, $p_n = p_{max}$ and n is a prespecified integer parameter. Note that, in optimal partition, the number of subintervals may be less than n; that is, $p_k = p_{k+1}$ for some k.

For $k = 0, \cdots, n-1$, let E_{I_k} be the restriction of the expected profit function (9) on interval I_k that is,

$$\begin{aligned} E_{I_k}\{\pi_{\alpha,\beta}\} &= \frac{\alpha - a_k}{b}\int_{\alpha}^{\alpha+bp_{k+1}} \rho_m f(\rho_m) d\rho_m \\ &-\frac{\alpha(\alpha-a_k)}{b}\big(F(\alpha+bp_{k+1}) - F(\alpha)\big) \\ &+(\alpha-a_k)p_{k+1}\big(1-F(\alpha+bp_{k+1})\big), \end{aligned} \tag{10}$$

where $a_k = a_{k-1} + bp_k$, $k = 1, \cdots, n-1$, and $a_0 = a$.

Then, the restriction of maximum expected profit problem on interval $I_k = [p_k, p_{k+1}]$ is

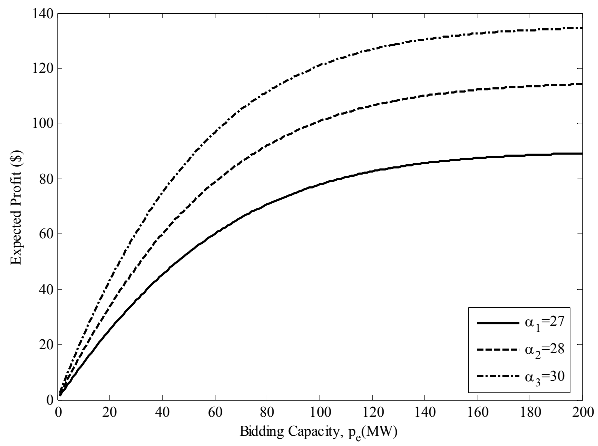

Fig. 6. Expected profit versus p_e (a=25, b=0.1, p_{max}=200, $\rho_m \approx logN(3.4,0.2)$)

$$\begin{aligned} &\max_{\alpha} E_{I_k}\{\pi_{\alpha,b}\} \\ &a_k \le \alpha \le \text{Price Cap.} \end{aligned} \tag{11}$$

Given a vector $p = (p_0, p_1, p_2, \cdots, p_{n-1}, p_n)$ of intermediate points, the total expected profit of GenCo could be determined by solving the following optimization problem:

$$\begin{aligned} z(p) = &\max_{\alpha=(\alpha_0,\cdots,\alpha_{n-1})} \sum_{k=0}^{n-1} E_{I_k}\{\pi_{\alpha_k,b}\} \\ &a_k \le \alpha_k \le \text{Price Cap.} \qquad k = 0, \cdots, n \end{aligned} \tag{12}$$

Now, the objective is to maximize $z(p)$ subject to

$$\begin{aligned} &p_0 = 0, \\ &p_1 \le p_2 \le \cdots \le p_{n-1}, \\ &p_n = p_{max}. \end{aligned} \tag{13}$$

In summary, the whole optimal bidding problem, including capacity partitioning and corresponding prices, is to

$$\underset{p}{\text{maximize}}\, z(p) = \max_{\alpha} \sum_{k=0}^{n-1} E_{I_k}\{\pi_{\alpha_k,b}\}$$

subject to: (14)

$$\begin{aligned} &a_k \le \alpha_k \le \text{Price Cap} \quad k = 0, \cdots, n \\ &p_0 = 0, \\ &p_1 \le p_2 \le \cdots \le p_{n-1}, \\ &p_n = p_{max}. \end{aligned}$$

The above optimization problem could be used to determine the optimal bidding strategy. Given partition p of size $n+1$, problem (12) could be handled by solving n single optimization problem of type (11). In what follows, we show that optimization problem (11) could be solved efficiently when MCP random variable follows normal PDF, $N(\sigma, \mu)$. We begin by showing that the derivative of objective function of expected profit, as a function of α, has a zero in $[a, \infty)$. Note that, in these lemmas, we perform symbolic computations by Wolfram Mathematica [23].

Lemma 1. We have

$$\frac{dE}{d\alpha} = D\, Z, \qquad \frac{d^2E}{d\alpha^2} = \frac{D}{\sigma}\left(2C_1\sigma + C_3(a-\alpha)\right),$$

Lemma 2. $dE/d\alpha$ has a zero in $[a, \infty)$.

where

$$Z = (C_1\mu + C_1 a - 2C_1\alpha + C_2 p + C_3\sigma),$$

$$D = -\frac{1}{2b\sqrt{\pi}} \exp\left(-\frac{(\mu-\alpha)^2 + (\mu - bp - \alpha)^2}{2\sigma^2}\right),$$

$$C_1 = T\left(F(\alpha) - F(\alpha + bp)\right),$$

$$C_2 = -bT\left(1 - F(\alpha + bp)\right),$$

$$C_3 = \frac{1}{\sqrt{\pi}\sigma}\left(\frac{1}{f(\alpha)} - \frac{1}{f(\alpha+bp)}\right),$$

$$T = 2\sqrt{\pi} \exp\left(\frac{(\mu-\alpha)^2 + (\mu - bp - \alpha)^2}{2\sigma^2}\right).$$

Proof.

For $\alpha = k\, a,\ k \in \mathbb{N}$, the value of Z is

$$Z = e^{x^2+(x+y)^2}\sqrt{2}\sqrt{\pi}\sigma h(x,y) + a e^{x^2+(x+y)^2}(-1+k)\sqrt{\pi}(\operatorname{erf} x - \operatorname{erf}(x+y)),$$

where

$$h(x,y) = x \operatorname{erf} x - (x+y)\operatorname{erf}(x+y) + y - \frac{1}{\sqrt{\pi}} \frac{e^{x^2} - e^{(x+y)^2}}{e^{x^2+(x+y)^2}},$$

and

$$x = \frac{k\,a - \mu}{\sqrt{2}\,\sigma}, \quad y = \frac{bp}{\sqrt{2}\sigma}.$$

If $k = 1$; that is, $\alpha = a$, we have

$$Z = \left(e^{x^2+(x+y)^2}\sqrt{2}\sqrt{\pi}\,\sigma\, h(x,y)\right).$$

We show $h(x,y) \ge 0$ if $y \ge 0$. Indeed, given x, $h(x,y)$ has the derivative $1 - \operatorname{erf}(x+y) > 0$. Thus, $h(x,y)$ is increasing as a function of y. Thus $h(x,y) \ge h(x,0) = 0$.

Therefore, for $\alpha = a$, we have $Z > 0$.

We show for sufficiently large k, the value of Z is negative.

One can write Z as

$$Z = \sqrt{2}e^{\frac{(ak+bp-\mu)^2}{2\sigma^2}}\sigma + e^{\frac{(-ak+\mu)^2}{2\sigma^2}}\left(be^{\frac{(ak+bp-\mu)^2}{2\sigma^2}}p\sqrt{\pi} - \sqrt{2}\sigma\right) + e^{\frac{(ak+bp-\mu)^2+(-ak+\mu)^2}{2\sigma^2}}\sqrt{\pi}\left((a-2ak+\mu)\operatorname{erf}\left(\frac{-ak+\mu}{\sqrt{2}\sigma}\right) + (a(-1+2k)+bp-\mu)\operatorname{erf}\left(\frac{-ak-bp+\mu}{\sqrt{2}\sigma}\right)\right).$$

Substituting $x\,\mathrm{erf}\,x$ by x, and $\mathrm{erf}\,x$ by 1, for large value of x, we have

$$Z=\sqrt{2}\,\sigma\left(e^{\frac{(ak+bp-\mu)^2}{2\sigma^2}}-e^{\frac{(-ak+\mu)^2}{2\sigma^2}}\right)<0.$$

Thus $dE/d\alpha$ has a zero in $[a,\infty)$

We use bisection method to find the root α^* of Z. Then, using Lemma 1, we can check the value of $d^2E/d\alpha^2$ at α^* to determine whether α^* is a maximizer or not.

Table I
COMPARISON BETWEEN ONE-PIECE AND OPTIMAL BIDDING (for n=5)

Cost parameters		Expected Profit		Monte Carlo simulation		
a	*b*	Linear One-piece bidding	Optimal bidding	Expectation of Profit	Chance of loss in Optimal bidding (%)	Chance of loss in One-piece bidding (%)
10	0.05	1615.39	1673.33	1443.62	0.60	0.60
10	0.1	1002.55	1108.30	915.25	0.65	0.65
10	0.15	675.25	761.06	604.58	0.62	0.62
10	0.2	506.46	571.16	451.07	0.61	0.61
15	0.05	1007.44	1041.76	993.64	10.62	10.62
15	0.1	586.84	635.21	585.66	10.40	10.40
15	0.15	392.80	428.10	386.31	10.51	10.51
15	0.2	294.60	321.10	289.22	10.44	10.44
20	0.05	535.98	551.47	547.79	39.42	49.75
20	0.1	294.81	311.22	295.08	38.67	49.90
20	0.15	196.78	207.73	194.21	37.70	50.04
20	0.2	147.59	156.07	146.65	38.35	49.92
25	0.05	227.47	232.27	232.64	56.88	89.25
25	0.1	119.73	123.65	120.19	56.43	89.34
25	0.15	79.84	82.48	80.43	56.31	89.23
25	0.2	59.88	61.86	59.68	56.45	89.57

V. NUMERICAL RESULTS: DESIGNING OPTIMAL BID FUNCTION

In the considered decision making problem, as presented before, the generation capacity is divided into n parts and the optimal intercept parameter is calculated for each part through maximizing expected profit, Equs. (9) or (14). The optimal parameters, calculated in each part of capacity split, are used by GenCo to create a temporary supply function. Then, GenCo aggregates the MWs with equal prices in the temporary supply function, to derive the strategic bid function. The strategic bidding problem can be solved by a GenCo in practice as stated above. The process is demonstrated in the following, by a numerical example.

To solve problem (14), for a specific value of n, Particle Swarm Optimization (PSO) method with mutation has been used. Each particle is defined to be a partition of generation capacity to n parts. For each part, α is optimized using Equ. (11). Expected profit $z(p)$, which is the summation over the parts, is set as the fitness function. 40 utilized particles are initialized. The PSO algorithm converges in less than 200 iterations to the optimal partition.

Moreover, for a complete comparison, the expectation of profit is calculated using a Monte Carlo simulation. Also, a comparison between one-piece bidding and the proposed optimal bidding is performed with both classic and Monte Carlo methods. It can be seen in Table I, that the expected profit of optimal bidding is 2-12% more than one-piece bidding. Also, the chance of loss; that is, the percent number of times in which minimum bidding price is more than the market price, for one-piece bidding is more than optimal bidding and increases significantly with the operating cost.

Example. A simple example is presented here to demonstrate the proposed procedure for a GenCo with p_{max}=200MW, a=25\$/MWh, and b=0.1 \$/MWh^2. For simplicity let $\beta = b$. The market price is assumed to be a Gaussian distributed variable with μ=30 and σ=6.

Fig.7 shows the temporary supply function, which is calculated from Equ. (9) with n=5 using PSO algorithm. The numerical values, corresponding to Fig.7, are presented in Table 2. Fig.8 shows the convergence curve for the PSO algorithm.

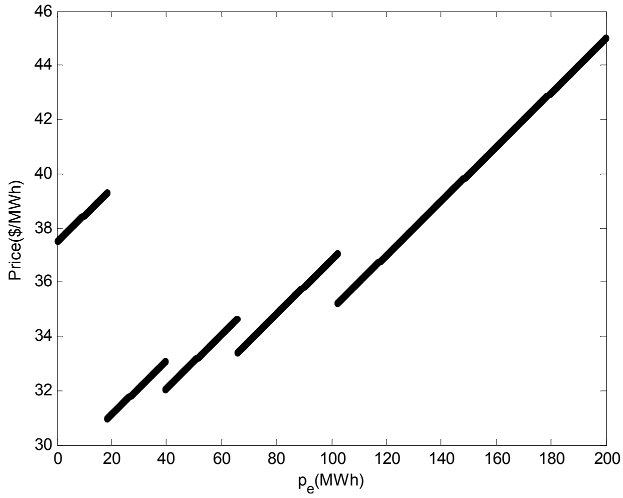

Fig. 7. Temporary supply function: result of the calculation of optimal values

Table II

OPTIMAL VALUES OF OPTIMIZATION PROBLEM (14) (a=25, b=0.1, p_{max}=200, $\rho_m \approx N(30,6)$)

Section No. (i)	p_i(MW)	α^*, \$
1	18.25	37.50
2	21.18	30.98
3	26.11	32.06
4	36.51	33.41
5	97.95	35.24

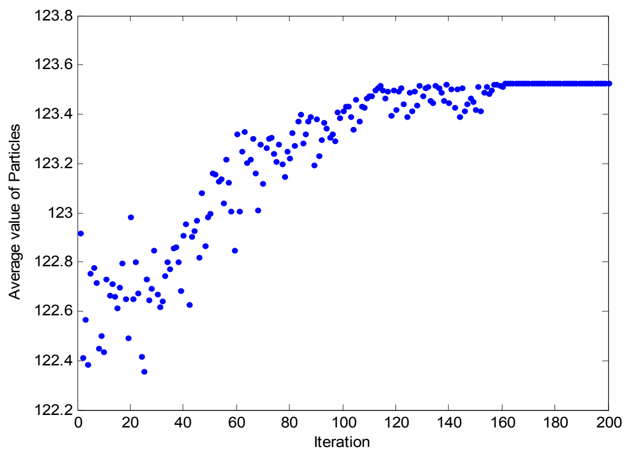

Fig. 8 Average value of particles of PSO algorithm.

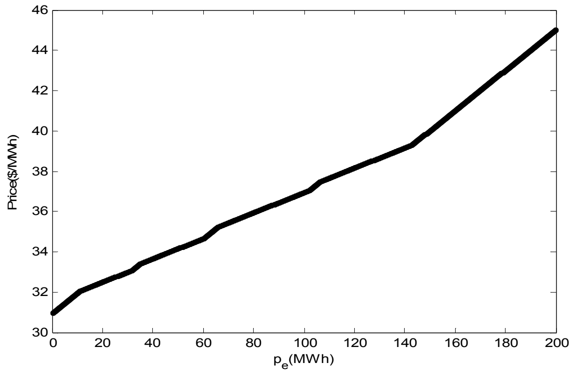

Fig. 9. Optimal bid function

Fig.9 shows the optimal bid function obtained by aggregating results which are shown in Fig.7.

VI. CONCLUSION AND FUTURE WORKS

In this paper a method is proposed for optimal calculation of the parameters of a piece-wise linear bid function in a PAB auction. It is assumed that the market price is a random variable which follows a probability density function. The bidding problem is classically formulated to find the optimal values of supply function parameters: intercept and slope. The method is extended in order to optimally partition the generation capacity and to construct the optimal bidding strategy from the viewpoint of a GenCo. Employing the proposed technique, a price-taker GenCo can partition its total generation capacity and optimize the parameters of the bid function, in the same time.

The linear bidding problem, based on the presented method, can be reconstructed from the view point of buyers, for example, distribution companies, retailers, large consumers and so forth, which will be considered in our future works. Moreover, the extension of method to a day-ahead bidding problem, considering the constraints of unit commitment problem is, assigned to the future works.

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