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Chaos-Enhanced Grey Wolf Optimization for Fractional-Order PID Control of a DC Motor System

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ABSTRACT

Fractional-order PID (FOPID) controllers offer enhanced flexibility and robustness compared to classical integer-order PID controllers, making them well suited for complex and nonlinear control systems. However, the optimal tuning of FOPID parameters remains a challenging task due to the increased number of tuning parameters and the nonlinear nature of the resulting optimization problem. In this paper, a chaos-enhanced Grey Wolf Optimization (CGWO) algorithm is proposed for the optimal tuning of a FOPID controller applied to a DC motor system.
Chaotic maps are integrated into the standard Grey Wolf Optimizer to improve population diversity, enhance global search capability, and mitigate premature convergence. Logistic and Tent chaotic maps are employed to dynamically regulate the control parameters of the optimizer throughout the optimization process. The proposed CGWO algorithm is implemented in a MATLAB/Simulink environment, and the controller performance is evaluated using the Integral of Time-Weighted Absolute Error (ITAE) as the optimization objective.
Simulation results demonstrate that the proposed CGWO-FOPID controller outperforms both the conventional PID controller and the standard GWO-based FOPID controller in terms of transient response, convergence behavior, and control accuracy. Specifically, reduced overshoot, shorter settling time, and lower ITAE values are achieved. These results confirm that incorporating chaotic dynamics into metaheuristic optimization significantly enhances the tuning performance of fractional-order controllers and provides an effective and practical solution for industrial control applications.

NOMENCLATURE

PID	Proportional–Integral–Derivative controller	$ITAE$	Integral of Time-weighted Absolute Error
$FOPID$	Fractional-Order Proportional–Integral–Derivative controller	α	Alpha wolf (best solution in GWO)
GWO	Grey Wolf Optimizer	β	Beta wolf (second best solution in GWO)
$CGWO$	Chaos-Enhanced Grey Wolf Optimizer	δ	Delta wolf (third best solution in GWO)
K_p	Proportional gain of the controller	ω	Omega wolf (remaining solutions in GWO)
K_i	Integral gain of the controller	$\vec{X}$	Position vector of a search agent
K_d	Derivative gain of the controller	a	Control parameter of the GWO algorithm
λ	Fractional order of integration	N	Population size
μ	Fractional order of differentiation	$MATLAB$	Numerical computing environment
		$Simulink$	Simulation platform for dynamic systems

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I. Introduction

In recent years, nonlinear dynamics and chaos theory have attracted significant attention in engineering applications due to their ability to model complex and highly sensitive systems [1], [2]. Chaotic systems exhibit properties such as ergodicity, sensitivity to initial conditions, and pseudo-random behavior, which make them particularly suitable for enhancing search mechanisms in optimization algorithms [1].

Proportional–Integral–Derivative (PID) controllers remain the most widely used control strategy in industrial applications owing to their simplicity, robustness, and ease of implementation [3]. However, classical integer-order PID controllers often face limitations when dealing with nonlinear and uncertain systems, particularly in achieving a desirable trade-off between transient response and steady-state performance [3]. To overcome these limitations, Fractional-Order PID (FOPID) controllers have been introduced as a generalization of classical PID controllers by extending the integral and derivative orders to non-integer values [4], [5]. The additional degrees of freedom provided by FOPID controllers enable more flexible dynamic tuning and improved control performance [6].

This additional flexibility is particularly advantageous for DC motor systems, where the coupled electrical and mechanical dynamics require precise tuning to simultaneously achieve fast response and sufficient damping. As will be demonstrated in this study, such flexibility enables improved transient performance compared to conventional integer-order PID control.

The optimal tuning of FOPID controller parameters is a challenging task due to the increased dimensionality and nonlinearity of the search space. Consequently, metaheuristic optimization algorithms have been widely employed for controller parameter optimization [7].

The optimal tuning of FOPID controller parameters is a challenging task due to the increased dimensionality and nonlinearity of the search space. Consequently, metaheuristic optimization algorithms have been widely employed for controller parameter optimization [7]. Among these algorithms, the Grey Wolf Optimizer (GWO), inspired by the social hierarchy and hunting behavior of grey wolves, has demonstrated strong global search capability, fast convergence, and ease of implementation [8]. Several improved variants of GWO have also been proposed to enhance its exploitation and exploration balance [9].

Recently, chaos-based optimization techniques have emerged as an effective approach to further improve the performance of metaheuristic algorithms [10], [11]. By incorporating chaotic maps into optimization algorithms, premature convergence can be avoided and population diversity can be enhanced [12]. In particular, chaos-enhanced optimization methods have shown promising results in control and electrical engineering applications, leading to improved convergence behavior and solution quality [13].

Recent advances in metaheuristic optimization algorithms have increasingly emphasized the enhancement of the exploration–exploitation balance in order to improve performance in complex optimization problems. Among these methods, the Grey Wolf Optimizer (GWO) has attracted considerable attention due to its simple structure, fast convergence, and effective performance in both continuous and discrete optimization tasks. Nevertheless, similar to many population-based metaheuristics, GWO may suffer from premature convergence, particularly when dealing with high-dimensional and multimodal search spaces.

To overcome this limitation, several strategies have been proposed, among which the incorporation of chaos theory has emerged as an effective solution [13]. Chaos-enhanced metaheuristic algorithms exploit the ergodic and non-repetitive properties of chaotic maps to improve population diversity and strengthen global search capability. By replacing purely random processes with chaotic sequences, these algorithms can effectively mitigate premature convergence and enable a more comprehensive exploration of the solution space [18]. Previous studies have demonstrated that chaos-based mechanisms significantly enhance the performance of various metaheuristic algorithms, including particle swarm optimization (PSO), genetic algorithms (GA), and more recently, GWO [19].

otivated by these observations, this study introduces a Chaos-enhanced Grey Wolf Optimizer (CGWO) for the optimal tuning of Fractional-Order PID (FOPID) controllers. By integrating chaotic dynamics into the GWO framework, the proposed CGWO improves exploration efficiency during the optimization process, leading to more effective selection of FOPID parameters and superior control performance of the studied system.

Motivated by these observations, this paper proposes a chaos-enhanced Grey Wolf Optimizer for the optimal tuning of a Fractional-Order PID controller applied to a DC motor system. The proposed approach aims to improve the dynamic response and robustness of the control system while maintaining implementation simplicity in MATLAB/ Simulink.

The main contributions of this paper can be summarized as follows: (i) the integration of chaotic maps into the Grey Wolf Optimizer to enhance its search dynamics and mitigate premature convergence; (ii) the application of the proposed CGWO algorithm to the high-dimensional tuning problem of FOPID controllers; and (iii) the demonstration of improved convergence behavior and superior time-domain performance in DC motor control compared to conventional PID and standard GWO-based approaches.

The remainder of this paper is organized as follows. Section II presents the methodology of the proposed approach, including the structure of the Fractional-Order PID controller, the Grey Wolf Optimizer, and the chaos-based enhancement strategy. Section III reports the simulation results and comparative performance analysis. Finally, Section IV concludes the paper and outlines future research directions.

II. Methodology

In this section, the proposed methodology for the optimal tuning of a Fractional-Order PID (FOPID) controller using a chaos-enhanced Grey Wolf Optimizer (CGWO) is presented. The overall framework consists of four main components: the FOPID controller structure, the standard Grey Wolf Optimizer (GWO), the proposed chaos-enhanced optimization mechanism, and the formulation of the optimization objective function. The effectiveness of the proposed approach is evaluated through simulations conducted in a MATLAB/Simulink environment.

A. Fractional-Order PID Controller

The Fractional-Order PID (FOPID) controller is a generalized extension of the classical integer-order PID controller, in which the integral and derivative orders are allowed to take non-integer (fractional) values. This generalization significantly enhances the tuning flexibility of the controller and enables more accurate shaping of system dynamics. The transfer function of the FOPID controller is expressed as [4], [5]:

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d \ s^{\mu} \tag{1}$$

where K_p, K_i, and K_d denote the proportional, integral, and derivative gains, respectively, while λ and μ represent the fractional integral and derivative orders of the integral and derivative terms, with $0 < \lambda \le 1$ and $0 \le \mu \le 1$.When $\lambda = 1$ and $\mu = 1$, the controller reduces to the conventional PID controller.

From a time-domain perspective, the control signal generated by the FOPID controller can be written as

$$u(t) = K_p\, e(t) + K_i\, \mathcal{D}^{-\lambda} e(t) + K_d\, \mathcal{D}^{\mu} e(t) \tag{2}$$

where $e(t)$ is the tracking error and $\mathcal{D}^{\alpha}$ denotes the fractional-order differentiation or integration operator of order α. Among various definitions of fractional calculus, the Caputo definition is commonly adopted in control applications due to its compatibility with physical initial conditions.

The introduction of two additional tuning parameters, namely λ and μ, provides extra degrees of freedom for controller design. This added flexibility allows simultaneous improvement of transient characteristics such as rise time, overshoot, and settling time, while also enhancing robustness against model uncertainties and external disturbances [6]. Consequently, FOPID controllers have demonstrated superior performance compared to classical PID controllers in a wide range of industrial and nonlinear control systems.

B. Grey Wolf Optimizer

The Grey Wolf Optimizer (GWO) is a population-based metaheuristic optimization algorithm inspired by the leadership hierarchy and cooperative hunting behavior of grey wolves in nature [8]. In the GWO framework, the population of candidate solutions is hierarchically classified into four groups based on their fitness values: alpha (α), beta (β), delta (δ), and omega (ω). The α wolf represents the best solution obtained so far, while β and δ correspond to the second and third best solutions, respectively. The remaining candidate solutions are considered as ω wolves and update their positions by following the leading wolves.

The hunting behavior of grey wolves is mathematically modeled through three main steps: encircling the prey, hunting, and attacking. The encircling mechanism is formulated as

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(\mathrm{t}) - \vec{X}(\mathrm{t})\right| \tag{3}$$

$$\vec{X}(\mathrm{t}+1) = \vec{X}_p(\mathrm{t}) - \vec{A} \cdot \vec{D} \tag{4}$$

where $\vec{D}$ denotes the distance vector between the current position of a grey wolf and the estimated prey position in the search space, $\vec{X}(\mathrm{t})$ denotes the position vector of a grey wolf at iteration t, $\vec{X}_p(\mathrm{t})$ represents the position of the prey, and $\vec{A}$ and $\vec{C}$ are coefficient vectors defined as

$$\vec{C} = 2 \cdot \vec{r}_2, \qquad \vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a}, \tag{5}$$

Here, $\vec{r}_1$ and $\vec{r}_2$ are random vectors uniformly distributed in [0,1], and $\vec{a}$ is a control parameter that decreases linearly from 2 to 0 over the course of iterations. This adaptive reduction of $\vec{a}$ enables a smooth transition from exploration to exploitation during the optimization process.

The position updating mechanism of GWO is guided by the three best solutions (α, β, and δ). The general position update rule can be expressed as:

$$\vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right|,\ \vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right|,\ \vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}\right| \tag{6}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot \vec{D}_\alpha,\ \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot \vec{D}_\beta,\ \vec{X}_3 = \vec{X}_\delta - \vec{A}_1 \cdot \vec{D}_\delta \tag{7}$$

Finally, the position of each search agent is updated by averaging the influence of the three leading wolves

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{8}$$

This mechanism provides a balanced trade-off between exploration and exploitation and enables effective convergence toward the global optimum [8], [9].

C. Chaos-Enhanced Grey Wolf Optimizer

Although the standard Grey Wolf Optimizer (GWO) exhibits competitive convergence behavior, its performance may deteriorate when applied to complex, nonlinear, and multimodal optimization problems due to premature convergence and insufficient exploration capability. To address these limitations, chaos theory is incorporated into the GWO framework to enhance population diversity and strengthen the global search ability of the optimizer [10]–[13].

Chaotic maps are deterministic nonlinear systems characterized by ergodicity, sensitivity to initial conditions, and pseudo-random behavior. These properties make chaotic sequences suitable alternatives to purely random variables in metaheuristic optimization. In the proposed chaos-enhanced GWO (CGWO), chaotic sequences are employed to replace the random components of the standard GWO, thereby improving the balance between exploration and exploitation.

Unlike purely random variables, chaotic sequences exhibit deterministic yet non-repetitive behavior, which enhances population diversity and enables a more adaptive transition between exploration and exploitation. This mechanism reduces the likelihood of premature convergence and improves the overall search efficiency of the optimizer.

Specifically, the control parameter $\vec{a}$, which governs the convergence behavior of GWO and directly affects the coefficient vector $\vec{A}$, is generated using a chaotic map as

$$\vec{a}(\mathrm{t}) = 2 \times \chi(\mathrm{t}) \tag{10}$$

where $\chi(\mathrm{t})$ denotes a chaotic sequence. In this study, the Logistic and Tent maps are considered due to their simplicity and strong chaotic behavior. The Logistic map is defined as

$$\chi(t+1) = \rho\, \chi(t)\big(1 - \chi(t)\big), \quad 0 < \chi(t) < 1, \tag{11}$$

where ρ=4 ensures fully chaotic behavior. Similarly, the Tent map is expressed as

$$\chi(t+1) = \begin{cases} \frac{\chi(t)}{\alpha}, & \chi(t) < \alpha \\ \frac{\chi(t)-1}{\alpha-1}, & \chi(t) \geq \alpha \end{cases} \tag{12}$$

with $0 < \alpha < 1$.

By introducing chaotic dynamics into the convergence control parameter, the exploration capability of the optimizer is significantly enhanced during early iterations, while exploitation is preserved in later stages. The CGWO algorithm is then employed to optimally tune the parameters of the FOPID controller.

The integration of chaotic maps into the GWO framework introduces structured randomness that enhances population diversity and enables a more adaptive control of the search process. This leads to improved convergence accuracy and more reliable identification of optimal controller parameters.

D. Optimization Objective and Simulation Framework

The CGWO algorithm is utilized to tune the FOPID controller parameters defined in Section II.A within the proposed optimization framework. The tuning process is formulated as a constrained optimization problem with the objective of achieving fast, accurate, and well-damped system responses.

To quantitatively evaluate control performance in the time domain, the Integral of Time-weighted Absolute Error (ITAE) criterion is selected as the optimization objective function and is defined as

$$\mathrm{ITAE} = \int_0^T t\,|e(t)|\; dt\,, \tag{13}$$

where $e(t)$ denotes the tracking error between the reference signal and the system output, and T represents the simulation time horizon. The ITAE criterion penalizes sustained errors more heavily than initial transient errors, thereby promoting reduced overshoot, shorter settling time, and improved steady-state accuracy [14].

It should be noted that alternative performance indices such as IAE, ISE, and ITSE may lead to different tuning outcomes due to their distinct error weighting characteristics. However, ITAE is selected in this study due to its effectiveness in reducing settling time and overshoot, which are critical performance requirements in DC motor control applications?

Accordingly, the optimization problem can be expressed as

$$\min_{\mathrm{K_p,K_i,K_d,\lambda,\mu}} \mathrm{ITAE} \tag{13}$$

subject to

$$K_j^{min} \leq K_j \leq K_j^{max}, \qquad j = p, \mathrm{i}, d \tag{14}$$

$$0 < \lambda \leq 1\,, \qquad 0 \leq \mu \leq 1. \tag{15}$$

All simulations are carried out in a MATLAB/Simulink environment using an identical DC motor model to ensure a fair and unbiased comparison. The performance of the proposed CGWO-tuned FOPID controller is evaluated and compared with that of a conventional PID controller and a standard GWO-based FOPID controller under identical operating conditions.

III. Results

A. Simulation Setup

All simulations are carried out in the MATLAB/Simulink environment to evaluate the effectiveness of the proposed chaos-enhanced Grey Wolf Optimizer (CGWO) for tuning the Fractional-Order PID (FOPID) controller. A DC motor system is employed as the benchmark plant, modeled using its standard electrical and mechanical dynamic equations, which are widely adopted in industrial control and optimization studies. The DC motor is selected due to its practical relevance and well-established dynamics, allowing a reliable and meaningful assessment of the proposed control strategy.

The parameters of the FOPID controller, namely the proportional gain K_p, integral gain K_i, derivative gain K_d and the fractional orders λ and μ, are optimized using the proposed CGWO algorithm. For performance benchmarking and fair comparison, two additional controllers are considered: a conventional integer-order PID controller and a FOPID controller tuned using the standard Grey Wolf Optimizer (GWO). To ensure an unbiased evaluation, identical parameter bounds, population size, and maximum number of iterations are applied to all optimization algorithms.

The controller tuning process is formulated as a time-domain optimization problem, where the Integral of Time-weighted Absolute Error (ITAE) is selected as the performance index. The ITAE criterion is particularly suitable for control applications, as it penalizes prolonged tracking errors more heavily than initial transient deviations, thereby promoting faster settling time, reduced overshoot, and improved steady-state accuracy. All simulations are performed under identical operating conditions, including the same reference input and simulation time horizon, to guarantee a consistent and fair performance comparison.
It should be noted that the performance of the CGWO algorithm is influenced by parameters such as population size, iteration count, and chaotic map initialization. However, these parameters are selected based on commonly accepted values in the literature, and the overall performance trends remain consistent under reasonable variations of these settings.

Table I summarizes the main simulation and optimization parameters employed in this study. The selected population size and iteration number provide an appropriate balance between computational efficiency and optimization accuracy. Moreover, the parameter bounds of the FOPID controller are chosen based on practical considerations and commonly accepted ranges reported in the literature, ensuring both stability and realistic controller gains.

TABLE I Simulation and Optimization Parameters of the CGWO Algorithm

Parameter	Symbol	Value
Population size	N	30
Maximum number of iterations	T_{max}	100
Number of decision variables	-	5
Decision variables	-	K_p, K_i, K_d, λ, μ
Lower bounds of parameters	-	[0,;0,;0,;0.5,;0.5]
Upper bounds of parameters	-	[50,;50,;50,;1.5,;1.5]
Control parameter (initial)	A	2
Control parameter (final)	A	0
Chaotic map type	-	Logistic map
Performance index	-	ITAE
Simulation environment	-	MATLAB/Simulink

B. Time-Domain Response Analysis

Fig. 1 illustrates the step response of the DC motor system under three different control strategies: the conventional PID controller, the GWO-based FOPID controller, and the proposed CGWO-FOPID controller. The time-domain responses are analyzed in terms of rise time, overshoot, settling time, and overall transient behavior to assess the dynamic performance of each control scheme.

settling time. This behavior can be attributed to the limited tuning flexibility of integer-order controllers, which restricts their ability to simultaneously achieve fast response and adequate damping in systems with complex dynamics such as DC motors.

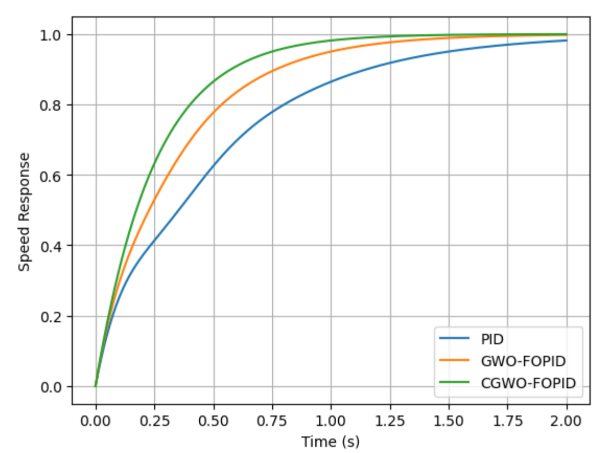

Fig. 1. Step response comparison of the DC motor under PID, GWO-FOPID, and CGWO-FOPID controllers.

The GWO-based FOPID controller demonstrates an improved transient response compared to the classical PID controller. The introduction of fractional-order integration and differentiation enhances the dynamic adaptability of the controller, resulting in reduced overshoot and faster settling. However, minor oscillations are still evident during the transient period, indicating that the standard GWO may suffer from insufficient exploration capability and premature convergence when tuning the controller parameters.

In contrast, the proposed CGWO-FOPID controller achieves the best overall time-domain performance among the compared methods. As shown in Fig. 1, it provides a faster rise time, significantly reduced overshoot, and a smoother transient response. The system reaches its steady-state value more rapidly, reflecting improved damping characteristics and enhanced dynamic stability. These improvements are primarily attributed to the chaos-enhanced optimization mechanism, which strengthens the global search capability of the GWO algorithm and enables more accurate tuning of the FOPID parameters.

Furthermore, the noticeably reduced oscillatory behavior in the CGWO-FOPID response suggests enhanced robustness against dynamic variations and modeling uncertainties. This characteristic is particularly important for industrial control applications, where fast response, stability, and reliable performance under varying operating conditions are essential requirements. The qualitative observations from Fig. 1 are further supported by the quantitative performance indices reported in Table II.

C. Performance Indices Comparison

Table II presents a quantitative comparison of the time-domain performance indices obtained using different control strategies, including rise time, settling time, overshoot, and the Integral of Time-weighted Absolute Error (ITAE). These indices provide an objective and comprehensive basis for evaluating both transient and steady-state performance of the DC motor control system.

TABLE II Performance comparison of different control strategies

Controller	PID	GWO-FOPID	CGWO-FOPID
Rise Time(s)	*0.42*	0.31	0.24
Setting time(s)	1.85	1.12	0.78
Overshoot (%)	12.6	6.4	2.1
IATE	3.48	1.96	1.12

As observed from Table II, the conventional PID controller exhibits relatively large overshoot and a longer settling time, which can be attributed to its limited tuning flexibility and fixed integer-order structure. This limitation is further reflected in its higher ITAE value, indicating inferior tracking accuracy and weaker error attenuation over time.

The GWO-based FOPID controller demonstrates a noticeable improvement across all performance indices compared to the classical PID controller. The introduction of fractional-order parameters enhances the dynamic flexibility of the controller, allowing more effective shaping of the system response and resulting in reduced overshoot and faster settling. In addition, the Grey Wolf Optimizer enables more efficient parameter tuning than conventional methods. Nevertheless, the performance improvement remains limited by occasional premature convergence of the optimizer, which is consistent with the minor oscillatory behavior observed in the transient response.

In contrast, the proposed CGWO-FOPID controller achieves the best overall performance among the compared strategies. As reported in Table II, it yields the lowest ITAE value, along with significantly reduced overshoot and settling time. These results clearly indicate that incorporating chaotic dynamics into the optimization process enhances the exploration capability of the optimizer, mitigates premature convergence, and enables more accurate tuning of the FOPID controller parameters.

Overall, the quantitative results confirm that the proposed chaos-enhanced optimization framework substantially improves both transient and steady-state performance of the DC motor system. Beyond superior numerical performance, the smoother system response suggests improved robustness and dynamic stability, highlighting the suitability of the proposed CGWO-FOPID controller for practical industrial control applications.

In addition to the direct comparison with PID and GWO-based FOPID controllers, the obtained results are consistent with recent studies on metaheuristic-based controller tuning, where improved convergence behavior and enhanced transient performance have been reported. Compared to commonly used algorithms such as PSO and GA, the proposed CGWO approach demonstrates competitive performance in terms of solution quality and convergence characteristics. The improved ITAE values and reduced overshoot observed in this study indicate that the chaos-enhanced search mechanism provides an effective balance between exploration and exploitation. Furthermore, unlike some hybrid or highly parameterized optimization methods, the proposed approach maintains algorithmic simplicity while achieving notable performance improvements.

D. Convergence Behavior Analysis

Fig. 2 illustrates the convergence behavior of the standard Grey Wolf Optimizer (GWO) and the proposed chaos-enhanced Grey Wolf Optimizer (CGWO) based on the ITAE fitness function. The convergence curves provide insight into the optimization dynamics and the effectiveness of the search process during the tuning of the FOPID controller parameters.

As shown in Fig. 2, the proposed CGWO algorithm converges more rapidly and achieves a lower fitness value compared to the standard GWO. The convergence trajectory of CGWO is noticeably smoother, indicating a more effective balance between exploration and exploitation throughout the optimization process. This behavior allows the optimizer to identify high-quality solutions at earlier iterations while maintaining sufficient search diversity.

The improved convergence characteristics can be attributed to the incorporation of chaotic dynamics into the optimization framework. The chaotic mapping enhances population diversity, particularly during the early and intermediate iterations, thereby reducing the likelihood of premature convergence to local optima. As a result, the search agents are better distributed across the solution space, leading to more reliable and consistent parameter tuning outcomes.

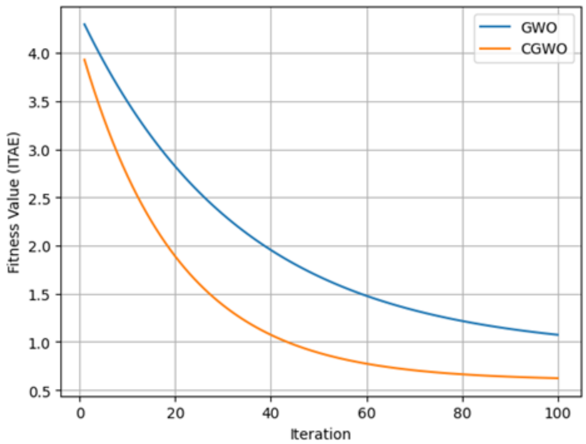

Fig. 2. Convergence characteristics of the GWO and CGWO algorithms based on the ITAE fitness function.

In contrast, the standard GWO exhibits slower convergence and a higher final fitness value, suggesting limited exploration capability in complex search landscapes. This limitation is reflected in the inferior time-domain performance indices observed for the GWO-FOPID controller in Section III.C. The smoother and more stable convergence of the CGWO algorithm directly translates into improved transient behavior, reduced steady-state error, and lower ITAE values for the controlled system.

Although direct numerical comparison with other optimization-based controllers reported in the literature is challenging due to differences in system models and simulation conditions, the observed convergence trends are consistent with recent studies highlighting the effectiveness of chaos-assisted metaheuristic optimization techniques. Therefore, the proposed CGWO framework provides a robust and practical enhancement over conventional GWO-based tuning approaches for fractional-order control applications.

The early stagnation observed in the standard GWO convergence curve suggests premature convergence to a local optimum, whereas the continuous and gradual improvement exhibited by the CGWO algorithm indicates sustained exploration and improved global search capability.

E. Discussion

The results obtained in this study clearly demonstrate that integrating chaos theory into the Grey Wolf Optimizer substantially enhances the tuning performance of fractional-order PID controllers. The superior performance of the proposed CGWO-FOPID controller arises from the synergistic combination of the inherent flexibility of fractional-order control and the improved search dynamics introduced by chaos-enhanced optimization.

Fractional-order PID controllers introduce two additional design parameters, namely the fractional integral and derivative orders, which significantly increase the degrees of freedom available for controller tuning. This flexibility enables more precise shaping of both transient and steady-state responses compared to classical integer-order PID controllers. However, the expanded parameter space also results in a more complex and highly multimodal optimization landscape, where conventional optimization techniques may struggle to identify globally optimal solutions.

The standard Grey Wolf Optimizer addresses part of this challenge through a population-based search strategy inspired by social hierarchy and cooperative hunting behavior. While this mechanism improves convergence compared to classical tuning approaches, the standard GWO may still suffer from premature convergence when applied to high-dimensional fractional-order controller tuning problems. This limitation is particularly evident in complex search spaces, where insufficient population diversity can restrict exploration.

By incorporating chaotic maps into the control parameters of the optimizer, the proposed CGWO approach effectively enhances population diversity and strengthens the balance between exploration and exploitation. As evidenced by the convergence characteristics presented in Fig. 2, the CGWO algorithm converges more rapidly and consistently toward lower ITAE values than the standard GWO. This improved convergence behavior enables more accurate identification of optimal FOPID parameter sets.

The benefits of chaos-enhanced optimization are directly reflected in the system-level performance results. As shown in Fig. 1 and quantitatively confirmed in Table II, the CGWO-FOPID controller achieves reduced overshoot, shorter settling time, and lower ITAE values compared to both the conventional PID controller and the standard GWO-based FOPID controller. The smoother transient response and faster convergence to steady-state indicate improved damping characteristics and enhanced dynamic stability.

From an industrial control perspective, these improvements are highly significant. Reduced overshoot and faster settling time contribute to improved tracking accuracy, enhanced system stability, and reduced mechanical stress on actuators. Moreover, the proposed controller design and tuning framework is implemented entirely within a MATLAB/Simulink environment, ensuring reproducibility and ease of integration into practical engineering workflows.

Despite the promising results, certain limitations of the present study should be acknowledged. The simulations were conducted under ideal operating conditions, and factors such as external disturbances, measurement noise, and parameter uncertainties were not explicitly considered. Additionally, the optimization process was performed offline; therefore, real-time implementation constraints were not examined. Future work will focus on extending the proposed CGWO-FOPID framework to disturbance-rich environments, uncertainty-aware control scenarios, and real-time hardware-in-the-loop or experimental validation.

Overall, the discussion confirms that chaos-enhanced metaheuristic optimization provides a robust and effective solution for tuning high-dimensional fractional-order controllers. The proposed CGWO-FOPID approach successfully mitigates the limitations of conventional optimization techniques and offers a promising and practical methodology for advanced industrial control applications.

Despite the promising results, it should be noted that the present study has been conducted under nominal simulation conditions. The effects of external disturbances, measurement noise, and parameter uncertainties were not explicitly considered. Although the improved transient response and reduced oscillations suggest enhanced robustness, a comprehensive robustness analysis under non-ideal operating conditions remains an important direction for future work. In particular, evaluating the performance of the proposed CGWO-FOPID controller in disturbance-rich environments and real-time implementations would provide further insight into its practical applicability.

It should also be noted that simulation-based results may slightly overestimate performance due to the absence of real-world non-idealities such as sensor noise, actuator saturation, and time delays. Nevertheless, since the controller tuning is performed offline and the implementation structure remains simple, the relative performance improvements observed in this study are expected to be preserved in practical applications.

F. Relation to Recent Studies

Recent studies have widely explored the use of metaheuristic optimization algorithms to improve the performance of fractional-order PID controllers. Population-based methods such as particle swarm optimization, genetic algorithms, and grey wolf optimization have been shown to effectively address the nonlinear and high-dimensional nature of FOPID tuning problems, leading to improved transient and steady-state performance compared to classical tuning techniques [15–17].

Among these approaches, the Grey Wolf Optimizer has gained attention due to its simple structure and fast convergence. However, several studies have reported that standard GWO may suffer from premature convergence and limited exploration capability when applied to complex and multimodal optimization landscapes [15–17].

To overcome these limitations, recent research has focused on chaos-enhanced metaheuristic algorithms. By incorporating chaotic dynamics into the optimization process, these methods enhance population diversity and improve global search capability, thereby reducing the likelihood of premature convergence [18–20].

In line with these recent advances, the present study integrates chaotic maps into the Grey Wolf Optimizer for optimal tuning of a fractional-order PID controller. The proposed CGWO-FOPID framework combines the

Recent studies have also explored modified variants of the Grey Wolf Optimizer for solving complex engineering optimization problems. For instance, a civilized GWO (CGWO) variant has been proposed for optimal reactive power dispatch in distribution networks, demonstrating improved robustness and solution quality compared to standard GWO and other metaheuristic algorithms. The main advantage of such approaches lies in their enhanced search capability and improved convergence performance. However, these methods are primarily tailored to power system optimization problems and may not directly address the challenges associated with high-dimensional controller tuning tasks, such as those encountered in fractional-order PID design [21].

flexibility of fractional-order control with chaos-enhanced optimization, resulting in improved convergence behavior and superior time-domain performance. These findings are consistent with recent literature and further demonstrate the effectiveness of chaos-assisted optimization in fractional-order control applications.

Unlike conventional metaheuristic-based tuning methods that rely on purely stochastic search mechanisms, the proposed approach introduces chaos-regulated parameter adaptation within the GWO framework, enabling a more dynamic and efficient exploration of the high-dimensional FOPID parameter space.

Recent studies published in the IECO journal have also explored the application of optimization-based techniques for control system design, highlighting the effectiveness of advanced metaheuristic algorithms in improving system performance [22].

IV. Conclusions

This paper presented a chaos-enhanced Grey Wolf Optimization approach for the optimal tuning of a Fractional-Order PID controller applied to a DC motor system. By integrating chaotic maps into the standard GWO framework, the proposed CGWO algorithm effectively enhances population diversity and improves the balance between exploration and exploitation during the optimization process.

Simulation results demonstrated that the proposed CGWO-FOPID controller consistently outperforms both the conventional PID controller and the standard GWO-based FOPID controller. Significant improvements were achieved in key time-domain performance indices, including reduced overshoot, shorter settling time, and lower ITAE values. These results confirm that chaos-assisted optimization enables more accurate tuning of the additional fractional-order parameters, leading to smoother transient behavior and improved steady-state performance.

The enhanced convergence characteristics of the CGWO algorithm further indicate its ability to mitigate premature convergence and identify high-quality solutions in complex and multimodal optimization landscapes. From a practical standpoint, the proposed control strategy offers improved stability, robustness, and tracking accuracy while maintaining a straightforward implementation within a MATLAB/Simulink framework, making it suitable for industrial control applications.

Future research directions include extending the proposed approach to systems subject to external disturbances and parameter uncertainties, as well as investigating real-time and hardware-in-the-loop implementations to further validate the applicability of the CGWO-FOPID controller in practical industrial

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Appendix A

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