Innovative Hybrid Backward Input Estimation and Data Fusion for High Maneuvering Target Tracking

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ABSTRACT

A hybrid unknown input estimation based on a new two-sample backward model and data fusion for high maneuvering target tracking is proposed. This new approach is based on consideration of more than one state and input components from the current single observation. These extracted state and input components would be augmented in a single vector, and the final estimation for unknown target acceleration will be determined. Using a combination of the new backward modeling and traditional modified input estimation (MIE) technique, more information will be extracted. This new hybrid scheme which using more input information can better estimate the target maneuvering structure. Despite the traditional methods, the proposed algorithm introduces two different strategies to state the input estimation including online and delayed estimation scenarios. Also, this paper suggests several different data fusion methods through these strategies. The results are compared with a typical MIE method to evaluate the performance of the proposed hybrid scheme especially for problems in high maneuvering target tracking. The results show that the backward algorithm makes advantages such as reduction of the transient state error and more stability for the estimation by an appropriate combination of the MIE estimator.

I. INTRODUCTION

Target tracking problem referees to the determination of the position, velocity and acceleration of a maneuvering target from its noisy observations. Standard Kalman filter (SKF) as an optimal least square estimation (LSE) algorithm is widely used in the field of non-maneuvering target tracking [1-4]. Since that the acceleration considered as a deterministic and unknown value for a maneuvering target, SKF loses its optimality in tracking especially for high maneuvering targets [4]. Various methodologies have been suggested different solutions to overcome the unknown input estimation (UIE) difficulty. Some basic approaches, such as input estimation (IE) methods and interacting multiple model (IMM) algorithms have been widely used to handle the case of unknown target maneuvers [5]. A typical IE algorithm is based on maneuver magnitude recognition and compensate the non-maneuver state vector. One of the early algorithm based on IE approach was showed by Chan et al., where the magnitude of the unknown input is determined by the LSE when a target maneuver is detected [3]. And the estimated acceleration compensates the target position and velocity using a SKF. In addition, during the time intervals which there is no maneuver detection, the SKF well be used individually. Although almost all IE methods are attractive in the field of typical maneuvering target tracking, their performances would be degraded for high maneuver targets due to a constant input assumption [6]. Modified input estimation (MIE) [7] and enhanced input estimation (IEI) [8] were among the most important algorithms have been introduced to improve the IE algorithm with some special assumptions. Moreover MIE methods have been shown the better performance especially to overcome some drawbacks in IE algorithms [9]. These drawbacks in IE approach come from ignoring the dependence between dynamic and observer noises especially in those algorithms based on augmentation approaches [4].
An innovative algorithm based on a new generalized dynamic modelling has been introduced to overcome some of these kind of problems called optimal two-stage Kalman estimation algorithm [10]. Combined Kalman filter and fuzzy logic [11], tracking algorithm based on unknown input and multiplicative noises [12-13], dual Kalman filter [14], intelligent Kalman filter [15] and repetitive optimization [16-17] are some of new papers in the area of input estimation and detection approach.

In order to avoid a maneuver detection algorithm, IMM methodologies as a main category of target tracking methods have been proposed [18-20]. The IMM basic idea is to assume a set of maneuver modes from a possible true candidates at the time. The mode changes are modelled by a hidden Markov process, run a bank of basic filters, each based on a unique model in the set; and finally the overall estimates are generated. Several types of IMM based approach such as fuzzy logic based IMM algorithm [21], IMM combined with square-root cubature Kalman filter with correlated noises (IMM-SCKF-CN) in maneuvering emitter tracking [22], combined IMM with particle filter [23] have been used to improve the performance of IMM basic algorithm. In the recent years, studies have significantly focused on multisensory fusion for both military and non-military purposes [24-26].

A new hybrid algorithm is proposed in this paper combines some traditional methods such as MIE and data fusion [27]. Almost all of the traditional algorithms in target tracking implement the current observed data and don’t use the benefits of the estimated states and inputs over the previous times. The obtained estimation from most of the available algorithms contains a transient state which affects the final tracking during the forward time after the input applied. This problem in control methods doesn’t make a significant error, however this slight difference in transient period causes divergence in target tracking problem. In some control methods such as model predictive control (MPC), the main objective of design is the influence analysis of applied input in the future time and matching the output with desired value in a certain time period [28].

Although in traditional tracking algorithms, the goal is matching the estimated states and input vector with the real values counterparts in current time, the new proposed algorithm is focused on matching estimated and real values in a time interval between the current and previous period times. Moreover this algorithm is based on extracting more input estimated components using data fusion techniques. Some of the most important ones of the data fusion techniques will be studied. Therefore, with the availability of more information and by fusion of them, the more appropriate input estimator would be achieved. Finally, target states are calculated by a uniform framework estimator. To extracting more input components, target model is changed such that instead of the model dependence to the current input value, it is modified by a vector of input in different times. The proposed model in this way called backward model. The final estimation is followed by determining the final amount of extracted input and state estimation. The last part of the proposed hybrid algorithm is the correction of the states.

II. THE DYNAMIC MODEL OF THE TARGET AND THE OBSERVER

In this section, dynamic motion of target is assumed to be two-dimensional as follows,
\[
X(n+1) = FX(n) + Cu(n) + Gw(n)
\]
\[
Z(n) = HX(n) + v(n)
\]

Where \(X(n+1)\) is the state vector, \(Z(n+1)\) is the observation vector; \(u(n)\) is the unknown input or target acceleration vector. \(w(n)\) and \(v(n)\) are considered as two independent white random processes in dynamic and observation, respectively. The error covariance matrices of these noises are,
\[
E[v(n)u^T(n)] = \begin{bmatrix} R(n_1) = r_{12x2} & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{bmatrix}
\]
\[
E[w(n_1)w^T(n_2)] = \begin{bmatrix} Q(n_1) = q_{12x2} & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{bmatrix}
\]
(2)
\[
E[v(n_1)v^T(n_2)] = 0
\]

And the target state vector includes both target position and velocity is defined as,
\[
X(n) = [x(n) \ v_x(n) \ y(n) \ v_y(n)]^T
\]

Where \(F, C, G\) and \(H\) are presented as state transition matrix, input and plant noise matrices and measurement matrix, respectively. All the previous matrices are considered as the known functions of time interval \(T\) (\(T\) is the time interval between two consecutive measurements) [4] as follows,
\[
F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix}^T
\]
(3)

III. PROPOSED HYBRID BACKWARD INPUT ESTIMATION ALGORITHM

In order to applying the benefits of MIE methods, SKF and data fusion, the new hybrid backward input estimation algorithm is proposed. Therefore, this proposed algorithm is the result of an intelligent combination of MIE algorithm, data fusion and SKF. Basis of the proposed algorithm is the modification of traditional target model into a new augmented model with multiple input components. On the other hand, the new augmented input vector contains current input value and some of its backward samples. It should be noted that if the new augmented vector includes many number of backward input components then the vector and some related matrices dimension would be increased and makes the calculations heavy. Therefore this new algorithm proposes a model with two backward input components or two-sample backward (TSB) model. The overall operation of the hybrid TSB algorithm is illustrated by four stages according to Fig. 1.

First stage: The dynamic model of the target motion is changed into a novel model to extract more than one input
component from one observation in each sample time. Proposed model is based on a modified target dynamic using more different samples over the last time. The new model will be introduced as the backward model (BM).

Second stage: In this section, the input vector estimation based on MIE approach is performed. MIE as a classical approach is the best estimator which can provide simple estimates compared with other methods [4].

Third stage: The final value of input acceleration which correspond to the real target acceleration is obtained. In addition, two strategies are presented and some data fusion approaches are also proposed where each has its own advantages and disadvantages.

Fourth stage: In this stage, the target position and velocity are estimated by the estimated input vector from the Third stage. A SKF with known input (estimated input) vector is used for estimation in this stage.

The proposed algorithm would help to smooth out the target parameters such as position, velocity and acceleration while preventing the fluctuations from the transient response. In the following, there are more details about the new hybrid backward input estimation and data fusion algorithm for high maneuvering target tracking.

A- First stage: Proposed Two-sample backward (TSB) model

From (1), the target dynamic model can be calculated over the m previous sample times as:

\[ X(n + 1) = FX(n) + Cu(n) + Gw(n) \]

\[ X(n) = FX(n - 1) + Cu(n - 1) + Gw(n - 1) \] (4)

\[ X(n + 1 - m) = FX(n - m) + Cu(n - m) + Gw(n - m) \]

New dynamic motion is achieved by substituting (4) in (1):

\[ X(n + 1) = F^{m+1}X(n - m) + F^mCu(n - m) + \ldots + FCu(n - 1) + Cu(n) + F^mGw(n - m) + \ldots + Gw(n - 1) + Gw(n) \] (5)

After rearrangement in the matrix framework:

\[
\begin{bmatrix}
X(n + 1) \\
X(n) \\
X(n - 1) \\
\vdots \\
X(n + 1 - m)
\end{bmatrix}
= 
\begin{bmatrix}
0 & \ldots & \ldots & 0 & F^{m+1} \\
I & 0 & \ldots & \ldots & \ldots \\
0 & I & 0 & \ldots & \ldots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & I & 0
\end{bmatrix}
\begin{bmatrix}
X(n) \\
X(n - 1) \\
X(n - 2) \\
\vdots \\
X(n - m)
\end{bmatrix}
+ 
\begin{bmatrix}
C \\
FC \\
F^2C \\
\vdots \\
F^mC
\end{bmatrix}
\begin{bmatrix}
u(n) \\
u(n - 1) \\
u(n - 2) \\
\vdots \\
u(n - m)
\end{bmatrix}
\] (6)

\[
\begin{bmatrix}
G \\
FG \\
F^2G \\
\vdots \\
F^mG
\end{bmatrix}
\begin{bmatrix}
w(n) \\
w(n - 1) \\
w(n - 2) \\
\vdots \\
w(n - m)
\end{bmatrix}
\]

Based on the new state vector augmentation, the observation equation would be changed as follows:

\[ Z(n) = [H 0 \ldots 0] \begin{bmatrix} X(n) \\ X(n - 1) \\ X(n - 2) \\ \vdots \\ X(n - m) \end{bmatrix} + \nu(n) \] (7)

Where in (6) and (7), \( I \) and 0 denote the 4 x 4 identity and zero matrices, respectively (4 in the dimension of state vector \( X(n) \)). Consequently, the process and observation noise covariance matrices could be obtained as follows:

\[
Q_{BM} = E \begin{bmatrix}
w(n) \\
w(n - 1) \\
w(n - 2) \\
\vdots \\
w(n - m)
\end{bmatrix}^T \begin{bmatrix}
w(n) \\
w(n - 1) \\
w(n - 2) \\
\vdots \\
w(n - m)
\end{bmatrix} = qI_{2(m+1)x2(m+1)}
\]

\[
R_{BM} = E[\nu(n)\nu^T(n)] = rI_{2x2}
\] (8)

The new augmented model is introduced as the backward model (BM). The backward horizon \( m \) could be selected as a
trade-off between the estimation improvement and the computational cost. By using \( m = 2 \) the BM is called the proposed TSB model. In following, the equations are represented for proposed TSB model, but equations and structures could be generally investigated for \( m > 2 \) as well.

The model of target dynamic and radar observation for TSB models are as follow:

\[
X_{TSB}(n+1) = F_{TSB}X_{TSB}(n) + C_{TSB}u_{TSB}(n) + G_{TSB}w_{TSB}(n),
\]

\[
Z(n) = H_{TSB}X_{TSB}(n) + v(n) \tag{9}
\]

Where \( X_{TSB} \) and \( u_{TSB} \) are state and input vectors of TSB model, respectively and \( w_{TSB} \) is uncertainty vector of this model,

\[
X_{TSB}(n) = \begin{bmatrix} X(n) \\ X(n-1) \\ X(n-2) \end{bmatrix}, \quad u_{TSB}(n) = \begin{bmatrix} u(n) \\ u(n-1) \\ u(n-2) \end{bmatrix}
\]

\[
w_{TSB}(n) = \begin{bmatrix} w(n) \\ w(n-1) \\ w(n-2) \end{bmatrix} \tag{10}
\]

Relationship of matrices \( F_{TSB}, \ G_{TSB}, \ C_{TSB} \) and \( H_{TSB} \) with matrices \( F, \ G, \ C \) and \( H \) are expressed as follows:

\[
F_{TSB} = \begin{bmatrix} 0 & F^3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C_{TSB} = C_{TSB} = \begin{bmatrix} C & FC & F^2C \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
H_{TSB} = [H \quad 0 \quad 0] \tag{11}
\]

Consequently the covariance matrices of the TSB model are given by

\[
Q_{TSB} = \begin{bmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} = qI_{6x6}, \quad R_{TSB} = rI_{2x2} \tag{12}
\]

B- Second stage: Modified Input Estimation Method [4]

The MIE technique which has been proposed in [4], is an algorithm where the input vector as an unknown term augmented to the state vector. This technique is used in the second part of our hybrid TSB algorithm. Thus, the new acceleration vector \((u_{TSB})\) and the new state vector \((X_{TSB})\) would be augmented as follows:

\[
\begin{bmatrix} X_{TSB}(n+1) \\ u_{TSB}(n+1) \end{bmatrix} = \begin{bmatrix} F_{TSB} & C_{TSB} \\ 0 & I \end{bmatrix} \begin{bmatrix} X_{TSB}(n) \\ u_{TSB}(n) \end{bmatrix} + \begin{bmatrix} G_{TSB} \\ 0 \end{bmatrix} w_{TSB}(n)
\]

\[
Z(n) = [H_{TSB} F_{TSB} \quad H_{TSB} C_{TSB}] \begin{bmatrix} X_{TSB}(n) \\ u_{TSB}(n) \end{bmatrix} + w_{TSB}(n)
\]

\[
H_{TSB} F_{TSB} w_{TSB}(n) + v(n) \tag{13}
\]

Now, a standard non-maneuvering augmented state model is obtained as follows:

\[
X_{Aug}(n+1) = F_{Aug}X_{Aug}(n) + G_{Aug}w_{Aug}(n)
\]

\[
Z(n) = H_{Aug}X_{Aug}(n) + v(n) \tag{14}
\]

where \( X_{Aug}(n), \ F_{Aug}, \ G_{Aug}, \ H_{Aug}, \ v_{Aug}(n) \) and \( w_{Aug}(n) \) are calculated as,

\[
X_{Aug}(n) = \begin{bmatrix} X_{TSB}(n) \\ u_{TSB}(n) \end{bmatrix}, \quad F_{Aug} = \begin{bmatrix} F_{TSB} \\ 0 \end{bmatrix}, \quad C_{Aug} = \begin{bmatrix} C_{TSB} \\ 0 \end{bmatrix}
\]

\[
H_{Aug} = [H_{TSB} F_{TSB} \quad H_{TSB} C_{TSB}] \tag{15}
\]

\[
v_{Aug}(n) = H_{TSB} F_{TSB} w_{TSB}(n) + v(n), \quad w_{Aug}(n) = w_{TSB}(n)
\]

The covariance matrices of this new model are expressed as,

\[
Q_{Aug}(n) = E\{w_{Aug}(n)w_{Aug}^T(n)\} = Q_{TSB}
\]

\[
R_{Aug}(n) = E\{v_{Aug}(n)v_{Aug}^T(n)\} = H_{Aug} Q_{Aug} H_{Aug}^T + R_{TSB} \tag{16}
\]

\[
T_{Aug}(n) = E\{w_{Aug}(n)v_{Aug}^T(n)\} = Q_{Aug} G_{Aug}^T H_{Aug}^T \tag{17}
\]

\[
K_{Aug}(n+1) = \begin{bmatrix} P_{Aug}(n+1|n)H_{Aug}^T \quad + G_{Aug} T_{Aug}(n) \end{bmatrix} \times R_{Aug}^{-1} P_{Aug}(n+1|n+1)
\]

\[
P_{Aug}(n+1|n) = F_{Aug} P_{Aug}(n|n) F_{Aug}^T + G_{Aug} Q_{Aug} G_{Aug}^T \tag{18}
\]

As shown in Fig. 2, in the augmented TSB method, for a step acceleration as input, estimated inputs are calculated as a vector for each observation data. For example, using the observation at sample time \( n + 1 \) i.e. \( x(n + 1) \) and the inputs at sample times \( n + 1 \), \( n \), \( n \) \( n - 1 \) i.e. \( u(n + 1) \), \( u(n) \), \( u(n - 1) \) could be used (also see Section 2 of Fig.1) and so on the inputs at sample times \( n - 1 \), \( n - 2 \), \( n - 3 \) would be used based on the observation at time \( n - 1 \), i.e. \( x(n - 1) \). Therefore, the number of input estimations at each sample times \( n + 1 \), \( n \), \( n - 1 \) are equal to 1, 2 and 3 respectively (i.e. one \( u(n + 1) \), two \( u(n) \) and three \( u(n - 1) \) could be calculated, see Fig. 3). On the other hand, these estimations are the extracted information come from inputs.

Therefore, by considering the TSB model, the maximum number of estimated inputs is equal to six or \( 1 + 2 + 3 = 6 \) input data samples are extracted for each of the x and y axes. Generally, for the model of \( m \) backward sample, \( 1 + 2 + 3 + ... + m + 1 = (m + 1) \times (m + 2)/2 \) input data samples are used to determine the final input values in \( m + 1 \) different sample times.
C- Third stage: Data Fusion.

Different data fusion approaches are used to find a proper amount of troubled information which maybe arisen from different viewpoint or incomplete information. By fusion of these types of information, besides the correction and completion, results with proper improvement and high stability could be achieved [29].

The Data fusion technique which is used in the proposed algorithm in this paper, is unlike to the data fusion related to the multi sensors problems such as [30, 31]. In these kind of papers, the information obtained from several observers are combined in each sample time. While in the proposed scheme, different input estimations obtained in Second stage (i.e. \( \hat{u}(n + 1) \), \( \hat{u}(n) \) and \( \hat{u}(n - 1) \) within the vector \( \hat{X}_{\text{Aug}}(n + 1) \)), would be combined that come from only one observer. At the Third stage of the proposed algorithm, the different data fusion algorithms for extracted data is used to achieve the best final input estimation.

![Real Input vs Input Estimation](image)

**Fig. 3.** Data Extraction in different sample times and input function identification strategy

There are several methodologies in the field of data fusion. In this paper there are two inputs in each x and y axis, so for each axis the proposed method is used separately. Therefore, the notation \( \hat{u}_{x/y}(n) \) is used for the input estimation in the arbitrary x or y axis. The data composition approach in this paper is based on two strategies. The first strategy determines the final estimation value in a specified sample time using the extracted data in that sample time. This strategy is introduced as the specific sample determination (SSD). For example, final estimation of \( \hat{u}(n - 1) \) is performed using the data of \( \hat{u}_{\text{TSB}}(n - 1|n - 1) \), \( \hat{u}_{\text{TSB}}(n - 1|n) \) and \( \hat{u}_{\text{TSB}}(n - 1|n + 1) \) (see Section 2 and 3 of Fig.1). The second strategy is based on determining the final estimation value using all of the extracted data (see Fig. 3) that this strategy is introduced as input function identification (IFI). In this strategy, we often seek to identify and smooth out the estimator response. The final values of input estimation are achieved from the identification function. Some of the most important ones of the data fusion techniques are as follows:

1) **Mean method**

One of the most common methods of data fusion is the so-called mean method. In this methodology, the final estimation value in a specified sample is equal to the average value of the extracted data. This method is a simple solution from the SSD strategy. The final value of input estimation in the samples \( n - 1, n \) and \( n + 1 \) is calculated as follows:

\[
\hat{u}_{x/y}(n - 1) = \frac{1}{2} \sum_{i=1}^{3} \hat{u}_{\text{TSB}}(n - 1|n + i - 2) \\
\hat{u}_{x/y}(n) = \frac{1}{2} \sum_{i=1}^{3} \hat{u}_{\text{TSB}}(n|n + i - 1) \\
\hat{u}_{x/y}(n + 1) = \hat{u}_{\text{TSB}}(n + 1|n + 1)
\]

2) **Min-Max method**

Another common kind of data fusion approaches is the Min-Max method. In this methodology, the final estimation value in a specified sample is determined from maximum/minimum of the extracted data in that sample time. This approach follows the SSD strategy, too. The final value of input estimation for each of the \( x \) and \( y \) axes in samples \( n - 1, n \) and \( n + 1 \) is calculated follows:

\[
\hat{u}_{x/y}(n - 1) = \min/\max_{i=1,2,3} \hat{u}_{\text{TSB}}(n - 1|n + i - 2) \\
\hat{u}_{x/y}(n) = \min/\max_{i=1,2} \hat{u}_{\text{TSB}}(n|n + i - 1) \\
\hat{u}_{x/y}(n + 1) = \hat{u}_{\text{TSB}}(n + 1|n + 1)
\]

3) **Curve Fit method**

In this data fusion methodology, in each of the \( x \) and \( y \) axes an input function is identified and the final estimation is obtained from that function. This method follows the IFI strategy. A common approach to system identification is to assume that the model structure corresponds to an autoregressive with exogenous inputs description, where the unknown parameters are estimated using least square criterion [32-35]. Assume that the input movement on the linear regression equation be as follows:

\[
\hat{u}_{x/y}(t) = P^T \theta + e
\]

For each sample time \( t = nT \) where \( \theta \) is unknown parameters vector of the model, \( P \) is the system data vector, and \( e \) is the error rate. The vectors \( \theta \) and \( P \) are defined as

\[
P = [1 \ t \ t^2 \ \ldots \ t^N]^T \\
\theta = [a_0 \ a_1 \ a_2 \ \ldots \ a_N]^T
\]

Where, \( N \) is the degree of the polynomial function. Using the best least unbiased error (BLUE) method, the optimum value of the vector \( \theta \) is calculated as follows:

\[
\theta_{\text{op}_{x/y}} = (P^T P)^{-1} P^T \hat{u}_{x/y}(nT)
\]

According to the equation (27), the optimum function is obtained as follow:

\[
\hat{u}_{\text{op}_{x/y}}(nT) = P^T \theta_{\text{op}_{x/y}}
\]

The final value of input estimation is obtained by replacing times aligned to the samples \( n - 1, n \) and \( n + 1 \) in equation (27). In order to reduce the computational cost of the methodology, it is recommended that polynomial functions of
degree 1 \((N = 1)\) or 2 \((N = 2)\) to be used:
\[
\begin{align*}
\hat{u}_{x/y}(t) &= a_0 + a_1 t \\
\hat{u}_{x/y}(t) &= a_0 + a_1 t + a_2 t^2
\end{align*}
\]  \(\text{(29)}\)
\(\text{(30)}\)

4) The innovative fast curve method

One of the major advantages of the backward algorithm over the other traditional methodologies is that the backward algorithm could have the fast response by using a novel data fusion technique. This technique is similar to the curve fit method, but instead of the optimal function, a function with faster response is used for each sample point (see Fig. 4).

Despite of the curve fit method; a new function is identified for each data sample. The optimum parameters obtained from the curve fit method will be used in this approach. The difference between the functions recognized in this method and previous method is its quick slope. If we consider the optimal polynomial functions of degree 1 or 2 as follows:
\[
\begin{align*}
\hat{u}_{x/y}(t) &= a_0 + a_1 t \\
\hat{u}_{x/y}(t) &= a_0 + a_1 t + a_2 t^2
\end{align*}
\]  \(\text{(31)}\)
\(\text{(32)}\)

By transforming the equation \(\text{(31)}\) and \(\text{(32)}\) about the sample time \((n - 2)T\),
\[
\begin{align*}
\hat{u}_{x/y}(n - 2)T) &= a_0 + a_1(n - 2)T \\
\hat{u}_{x/y}(n - 2)T) &= a_0 + a_1(n - 2)T + a_2((n - 2)T)^2
\end{align*}
\]  \(\text{(33)}\)
\(\text{(34)}\)

And by subtracting equation \(\text{(31)}\) from \(\text{(33)}\) and subtracting equation \(\text{(32)}\) from \(\text{(34)}\),
\[
\begin{align*}
\hat{u}_{x/y}(n) &= a_1(t - (n - 2)T) + \hat{u}_{x/y}((n - 2)T) \\
\hat{u}_{x/y}(n) &= a_1(t - (n - 2)T) + a_2t^2 + a_2((n - 2)T)^2 + \\
\hat{u}_{x/y}(n) &= \hat{u}_{x/y}((n - 2)T)
\end{align*}
\]  \(\text{(35)}\)
\(\text{(36)}\)

On the other hand, the equations \(\text{(35)}\) and \(\text{(36)}\) are considered based on the polynomial functions of degree 1 and 2, between two sample points \(t\) and \((n - 2)T\). In order to make the response faster, the derivative of the proposed functions should be increased. The derivative of the \(\text{(35)}\) and \(\text{(36)}\) are as follows:
\[
\begin{align*}
\frac{d}{dt} \hat{u}_{x/y}(t) &= a_1 \\
\frac{d}{dt} \hat{u}_{x/y}(t) &= a_1 + 2a_2 t
\end{align*}
\]  \(\text{(37)}\)
\(\text{(38)}\)

By multiplying an accelerating factor \(k_{fast}\) to the equations \(\text{(37)}\) and \(\text{(38)}\) and then integrating them around sample time \((n - 2)T\),
\[
\begin{align*}
\hat{u}_{fastx/y}(t) &= k_{fast}a_1(t - (n - 2)T) + \hat{u}_{x/y}((n - 2)T) \\
\hat{u}_{fastx/y}(t) &= k_{fast}a_1(t - (n - 2)T) + k_{fast}a_2t^2 - a_2((n - 2)T)^2 + \hat{u}_{x/y}((n - 2)T)
\end{align*}
\]  \(\text{(39)}\)
\(\text{(40)}\)

The final value of the input estimation is calculated by replacing the times \(n + 1\), \(n\) and \(n - 1\) in equations \(\text{(39)}\) and \(\text{(40)}\). The value of acceleration factor \(k_{fast}\) for each sample time could be different. By considering \(k^{n-1}_{fast}\) as the accelerating factor for the function at the sample time \(n - 1\), \(k^n_{fast}\) for the function at the sample time \(n\) and \(k^{n+1}_{fast}\) for the function at the sample time \(n + 1\), the relation between these factors is considered as follows (see Fig. 5):
\[
k^n_{fast} > k^n_{fast} > k^{n+1}_{fast} \equiv 1
\]  \(\text{(41)}\)

The numerical results of all data fusion methods will be investigated in the result section.

D- Fourth stage: SKF with estimated input (known input) assumption

The final value of the input estimation from the previous section is used in this section as the known input. According to the Fig. 1, the Kalman filter is used for the state estimation. The SKF is the most optimum state estimator for the linear non-maneuvering models with known input vector. The number of required filters for the model of \(m\) backward samples are \(m\). So, two standard Kalman filters are used for the TSB model (see Fig. 5). The proposed algorithm would use the SKF frame work as follows:

First of all, the state estimation process, \(\hat{X}(n + 1)\) could be calculated using \(\hat{X}(n - 1)\), \(\hat{u}(n - 1)\) and \(Z(n)\) with the SKF 1 (see Fig. 5). Consequently, \(\hat{X}(n)\) would be calculated using the previous \(\hat{X}(n)\) and \(\hat{u}(n)\) with SKF 2.

Finally, two estimation scenarios including online estimate and delayed estimation are expressible for the backward algorithm.

- The online estimation:

The online estimation is achieved by the present observation, for example, the estimation of states and input in the sample time \(n + 1\) is obtained by the present observation, i.e. \(Z(n + 1)\).

- The delayed estimation:

The delayed estimate is the estimation that is performed using \(m\) samples observation after the present, for example, the estimation of the states and input in the sample \(n + 1\) is achieved by the observation \(Z(n + m + 1)\), (i.e. \(m\) samples after
On the other hand, the online scenario estimation is done similar to the traditional algorithms, but the delayed estimation scenario is performed after some delayed samples. Due to using several observations, the delayed estimation scenario will have better results compared with the online estimation scenario.

### IV. SIMULATION RESULTS

The performance of the new hybrid scheme is compared with the traditional MIE method in two different examples. It is assumed that the target moves in a two-dimensional plane. Since that the mean and max/min data fusion methods have the simplest structure and they have not shown favorable results the simulation of the proposed algorithm is done only with the curve fit and fast curve data fusion methods. All the initial conditions and the other alignment parameters using in the following examples are the same in the proposed methods including curve fit and fast curve and the MIE approach.

**Example 1:** As the first example, a target with initial condition

\[
X(0) = [3 \text{ km} \ 0 \text{ m/s} \ 2 \text{ km} \ 20 \text{ m/s}^2]^T
\]

and the acceleration \( u(t) = [0 \ 0]^T (\text{ms}^{-2}) \) for time interval \( 0 < t < 100 \) (Sec) is considered. In this simulation, the target begins to high maneuver as \( u(t) = [30 \ 15]^T (\text{ms}^{-2}) \) for time interval \( 100 < t < 300 \) (Sec). The sample time is \( T = 2 \) (s) and the elements of the covariance matrices in (12) are selected as \( q = 1 \) and \( r = 10000 \) (m²). As mentioned, the simulation of the proposed algorithm is done only with the curve fit and fast curve data both with degree 2. Constant gains in fast curve with degree 2 in (40) are assumed \( k_{\text{fast}}^0 = 6, \ k_{\text{fast}}^1 = 2 \) and \( k_{\text{fast}}^2 = 1 \), respectively. Due to use the MIE method, proposed by Khaloozadeh and Karsaz in [4] as the traditional augmentation approach or modified input estimation (MIE), for data extraction in the proposed method, also a single MIE approach is simulated to compare the results.

Fig. 6 shows the actual values and the estimations of \( u_x(t) \) and \( u_y(t) \) and also their corresponding errors by the proposed method and the method of MIE, respectively. Fig. 7 shows the actual values and the estimations of \( v_x(t) \) and \( v_y(t) \) and also their corresponding errors. Fig. 8 illustrates the actual values and the estimations of target positions also their corresponding errors in the X and Y directions for the proposed method and the method of MIE, respectively.
Fig. 7. The actual and estimation values of target velocity and also their corresponding errors by the proposed method and the method of MIE

Fig. 8. The actual and estimation values of target position and also their corresponding errors by the proposed method and the method of MIE

To proof the robustness of our obtained results over the MIE approach a Monte-Carlo simulation approach is used. So, Table
I, shows the results obtained by the new proposed algorithm with different approaches and their corresponding errors with 100 Monte-Carlo runs in maneuvering target. Although, the time consumption of the proposed scheme is comparable with the MIE method, the RMSE of proposed algorithm are reduced by almost half in target position estimations.

TABLE I

<table>
<thead>
<tr>
<th>Method</th>
<th>( x ) (km)</th>
<th>( y ) (km)</th>
<th>( v_x ) (m/s)</th>
<th>( v_y ) (m/s)</th>
<th>( u_x ) (m/s²)</th>
<th>( u_y ) (m/s²)</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIE</td>
<td>8.0762</td>
<td>4.0399</td>
<td>99.3511</td>
<td>50.2273</td>
<td>8.2113</td>
<td>4.3595</td>
<td>0.6324</td>
</tr>
<tr>
<td>Proposed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curve Fit (poly 2)</td>
<td>4.02919</td>
<td>2.0207</td>
<td>79.0956</td>
<td>36.6177</td>
<td>7.5806</td>
<td>4.0897</td>
<td>0.6436</td>
</tr>
<tr>
<td>Fast Curve (poly 2)</td>
<td>3.92989</td>
<td>1.9237</td>
<td>78.0456</td>
<td>33.0077</td>
<td>6.5863</td>
<td>4.0097</td>
<td>0.6754</td>
</tr>
</tbody>
</table>

Example 2: For this example target initial condition is supposed with state \( X(0) = [3 \text{ km} \ 0 \text{ ms}^{-1} \ 2 \text{ km} \ 20 \text{ ms}^{-1}]^T \) and the acceleration \( u(t) = [0 \ 0]^T(\text{ms}^{-2}) \) for time intervals \( 0 \leq t < 200 \text{ s} \) and \( 400 \leq t < 600 \text{ s} \). In this simulation, the target begins to high maneuver as \( u(t) = [5 \ 5]^T(\text{ms}^{-2}) \) for \( 200s \leq t < 400s \). The sample time and the elements of the covariance matrices are selected as those selected in Example 1. The proposed algorithm is simulated using methods include curve fit with degrees 1 and 2 and fast curve with degrees 1 and 2. Constant gains in fast curve with degree 1 and 2 method in (39) and (40) are assumed \( k_{fast}^{-1} = 7 \), \( k_{fast} = 3 \) and \( k_{fast}^{+1} = 1 \), respectively.

Results of the proposed method with a variety of data fusion methods which have been stated in Third stage of the proposed algorithm, are shown in Fig 9-11. For each method is used both online and delay estimation separately as stated in (III-D). In delay estimation, input will be estimated after two sample time or after \( 2T = 4 \text{ (sec)} \) of the present observation. Fig. 9 shows the results of input estimation and Fig. 10 and Fig. 11 show the results of velocity and position estimations, respectively.

![Fig. 9](image_url) The actual values and the estimation of X and Y accelerations and their corresponding errors
Performance evaluation of above mentioned approaches is calculated by the mean value of error (ME), the mean of absolute error (MAE) and the root mean square error (RMSE) indices. ME and MAE measures demonstrate the level of estimation matching with real value, and RMSE measured represents the rate of estimate distribution with real value. To proof the robustness of our obtained results over the MIE approach a Monte-Carlo simulation approach is used. So, Table II, shows the results obtained by the new proposed algorithm with different approaches and their corresponding errors with 100 Monte-Carlo runs in maneuvering target. According to Figs. 9-11 and Table II, the results for each data fusion method are as follows.

**Curve Fit:** For acceleration estimation it is clear that the results are approximately improved compared to the MIE estimation. In this method the performance of the second order polynomial
is relatively better than first order.

Fast Curve: This is the only approach that could reduce the transient effect of approximation. The estimation results indicate a significant improvement in acceleration detection compared to the curve fit and MIE approach. Due to the better performance and fast convergence ability the estimation of target position has been improved as well as velocity. In this method the first order of polynomial has effective performance compared to second of order polynomial. Also delay estimation mode has suitable improvement compared to online estimation mode. According to above mentioned issues, all proposed methods represent appropriate results especially the fast curve with first order of polynomial approach (see Table II, green rows). Nevertheless, several proper methods could be selected for each criterion according to application of each criterion in different scenarios. The fast curve method which is a heuristic one in data fusion field is introduced as high risk method with very well results.

V. CONCLUSION

This paper deals with a new hybrid modelling of unknown input estimation problem for tracking high maneuvering targets and proposed the two-sample backward (TSB) model. The new hybrid algorithm for high maneuvering target tracking provides an online performance as well as the other methodologies besides the advantages for modification of previous states.

Simulation results are provided to confirm the theoretical and experimental developments of the proposed scheme in two different examples. The results are compared with the work of Khaloozadeh and Karsaz in [4] as the traditional augmentation approach or modified input estimation (MIE). The results showed that the backward algorithm makes advantages such as reduction of the transient state error and more stability for the estimation by an appropriate combination of the traditional estimators. The innovative Fast curve approach in data fusion section which is the main stage of the proposed algorithm could reduce the transient effect of approximation. The estimation results indicate a significant improvement in acceleration, velocity and position estimations compared to the MIE approach.

<table>
<thead>
<tr>
<th>MIE method</th>
<th>x (km)</th>
<th>y (km)</th>
<th>vx (m/s)</th>
<th>vy (m/s)</th>
<th>ux (m/s²)</th>
<th>uy (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curve Fit (poly 1)</td>
<td>online</td>
<td>-1.2051</td>
<td>-1.2347</td>
<td>-3.7714</td>
<td>-4.0138</td>
<td>0.0298</td>
</tr>
<tr>
<td></td>
<td>delay</td>
<td>-1.1850</td>
<td>-1.2212</td>
<td>-3.2461</td>
<td>-3.3606</td>
<td>0.0330</td>
</tr>
<tr>
<td>Curve Fit (poly 2)</td>
<td>online</td>
<td>-1.2038</td>
<td>-1.2350</td>
<td>-3.8658</td>
<td>-4.3752</td>
<td>0.0359</td>
</tr>
<tr>
<td></td>
<td>delay</td>
<td>-1.1832</td>
<td>-1.2200</td>
<td>-3.3452</td>
<td>-3.7143</td>
<td>0.0388</td>
</tr>
<tr>
<td>Fast Curve (poly 1)</td>
<td>online</td>
<td>-1.2049</td>
<td>-1.2346</td>
<td>-3.7981</td>
<td>-4.0720</td>
<td>0.0265</td>
</tr>
<tr>
<td></td>
<td>delay</td>
<td>-1.1849</td>
<td>-1.2210</td>
<td>-3.2837</td>
<td>-3.4081</td>
<td>0.0246</td>
</tr>
<tr>
<td>Fast Curve (poly 2)</td>
<td>online</td>
<td>-1.2039</td>
<td>-1.2421</td>
<td>-3.6606</td>
<td>-3.8789</td>
<td>0.0277</td>
</tr>
<tr>
<td></td>
<td>delay</td>
<td>-1.1843</td>
<td>-1.2291</td>
<td>-3.1351</td>
<td>-3.2268</td>
<td>0.0269</td>
</tr>
</tbody>
</table>

| MAE | | | | | | |
| MIE method | x (km) | y (km) | vx (m/s) | vy (m/s) | ux (m/s²) | uy (m/s²) |
| Curve Fit (poly 1) | online | 1.2111 | 1.2479 | 16.9567 | 16.9898 | 0.8050 | 0.8024 |
| | delay | 1.2062 | 1.2394 | 16.4678 | 16.3726 | 0.7927 | 0.7894 |
| Curve Fit (poly 2) | online | 1.2189 | 1.2514 | 16.8709 | 17.1735 | 0.8062 | 0.8011 |
| | delay | 1.2038 | 1.2430 | 16.3839 | 16.5551 | 0.7951 | 0.7886 |
| Fast Curve (poly 1) | online | 1.2180 | 1.2471 | 15.1828 | 15.2454 | 0.7692 | 0.7613 |
| | delay | 1.2028 | 1.2384 | 14.6100 | 14.5343 | 0.6888 | 0.6824 |
| Fast Curve (poly 2) | online | 1.2184 | 1.2548 | 16.2846 | 16.3384 | 0.7966 | 0.7901 |
| | delay | 1.2038 | 1.2466 | 15.6490 | 15.5800 | 0.8944 | 0.8858 |

| RMSE | | | | | | |
| MIE method | x (km) | y (km) | vx (m/s) | vy (m/s) | ux (m/s²) | uy (m/s²) |
| Curve Fit (poly 1) | online | 0.8808 | 0.8703 | 25.2791 | 25.6931 | 1.5329 | 1.5263 |
| | delay | 0.8763 | 0.8763 | 24.4856 | 24.4557 | 1.5100 | 1.5032 |
| Curve Fit (poly 2) | online | 0.8808 | 0.8744 | 24.8849 | 25.9554 | 1.5296 | 1.5270 |
| | delay | 0.8764 | 0.8824 | 24.0798 | 24.7313 | 1.5094 | 1.5057 |
| Fast Curve (poly 1) | online | 0.8872 | 0.8779 | 22.3762 | 23.0249 | 1.4695 | 1.4657 |
| | delay | 0.8821 | 0.8836 | 21.3433 | 21.4891 | 1.3027 | 1.2992 |
| Fast Curve (poly 2) | online | 0.8820 | 0.8723 | 24.1274 | 24.3471 | 1.5150 | 1.5021 |
| | delay | 0.8768 | 0.8778 | 23.0864 | 22.8211 | 1.4744 | 1.4631 |

TABLE II
Monte-Carlo simulation results for 100 runs in Example 2
VI. REFERENCES


Ali Karsaz received his B.S. degree in Electrical Engineering from the Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, in 1999. He received his M.Sc. and Ph.D. degrees in Control Engineering both from Ferdowsi University of Mashhad, Iran, in 2004 and 2008, respectively. Since 2008, he has been Assistant Professor of control and biomedical engineering at Khorasan Institute of Higher Education and he was Chair of the Division of Control Department from 2012 until now. He has consulted for Iranian Diabetes Society (IDS) in glucose-insulin modeling and control system design and he is developing an algorithm for automated plasma glucose control using optimal-based robust approach. He has also worked as a research scientist at the National Center of Medical Image Processing within the School of Medicine, Mashhad University of Medical Sciences form 2015 until now. In 2017, he also served as a consulting faculty at the Hashemi-Nezhad Refinary (Khangiran Gas Refinery) in some related research fields. He has received several awards including: Best Researcher Award, National Student Science Organization of Electrical Engineering (NSSOEE 2006), K. N. Toosi University’s (KNTU) Research Grant, 2006, FUM-ADO Award from the School of Medicine, 2015, The Outstanding Faculty of the Year Ph.D. Student Researcher Award, 2006, Outstanding Graduate Student Award in 2003. His current research interests include the development of mathematical models for analysis and control of biological systems, system biology mathematical modeling, medical image processing and video target tracking, deep learning, convolutional neural networks, time series analysis and prediction, inertial navigation systems, multi-sensory multi-target tracking. He has published over 140 peer-reviewed articles in related research fields.

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