International Journal of Industrial Electronics, Control and Optimization © 2019 IECO
Vol. 2, No. 3, pp. 239-246, July (2019)

Stability Analysis of Discrete-time Switched Linear Systems with Parametric Uncertainties

Nasrollah Azam Baleghi[1], and Mohammad Hossein Shafiei[2,†]

[1,2] Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran

ABSTRACT

This paper considers the stability problem of discrete-time switched linear systems in the presence of parametric uncertainties. This type of uncertainty is sometimes called structured uncertainty because of its known structure. However, some of the parameters in the system are uncertain. From a practical viewpoint, it is important to guarantee the robust stability of uncertain switched systems. Therefore, based on the structure of the uncertainty matrix and the common Lyapunov function for the nominal switched system, sufficient conditions for robust exponential stability of the discrete-time uncertain switched system (under any switching signal) are derived. These sufficient conditions are formulated in terms of matrix inequalities and using fixed values for some parameters, they will be solved via LMI techniques and based on numerical methods. Moreover, a procedure is proposed to determine the maximum admissible bounds of the uncertain parameters to guarantee the exponential stability of the uncertain switched system. Finally, numerical examples are provided to verify the proposed theoretical results.

Article Info

Keywords:
Discrete-time switched systems, Exponential stability, LMI, Parametric uncertainties

Article History:
Received 2018-11-28
Accepted 2019-02-25

I. INTRODUCTION

Switched systems are an important class of hybrid systems, which consist of a family of continuous-time or discrete-time subsystems and a switching condition determining the active subsystem at any time instant. The motivation for the study of switched systems comes from the fact that these systems arise in many engineering applications that cannot be described by continuous or discrete models, such as electrical and chemical engineering, automotive industry, aerospace control, networked control systems and many other fields [1]-[5].

There has been an increasing interest in the stability analysis and control design of switched systems during the last decades. There are three basic problems regarding the stability of switched systems [6]: (1) Finding conditions to guarantee the stability or controller design of switched systems under an arbitrary switching signal; (2) Identifying stabilizing switching signals; (3) Designing the stabilizing switching signal for the switched system. The first problem has been of particular interest due to its direct applications to various engineering problems. To solve this problem, many effective methods have been developed. In the case of stability analysis under arbitrary switching signals, all the subsystems should be asymptotically stable. However, the stability of all subsystems is not sufficient to guarantee the stability property under any arbitrary switching signal, except for some cases [7], [8]. On the other hand, if there exists a common Lyapunov function for all the subsystems, the stability of the switched system is guaranteed under arbitrary switching [1]. Therefore, some attempts have been made to construct a common Lyapunov function [9]-[11]. This approach provides us with a way to solve the uncertainty problem in a switched system.

One of the basic issues in the analysis and control of dynamic systems is the effect of uncertainties on the stability

†Corresponding Author: shafiei@sutech.ac.ir
Tel: +98-7137354500, Fax: +98-7137353502, Shiraz University of Technology
Faculty of Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran

of systems. Due to inaccuracy in a system's parameters, their uncertainties are often included in the modeling. Parametric uncertainty is sometimes called structured uncertainty because the structure of the model is known. However, some of the parameters are uncertain. For switched systems with uncertainties, there are many available results. Two kinds of uncertainties have been considered in the literature for switched systems such as polytopic and norm-bounded uncertainties [12]-[18]. The polytopic uncertain systems are less conservative than systems with norm-bounded uncertainties. It should be noted that parametric uncertainty can also be considered polytopic uncertainty. Compared with the existing results of polytopic uncertainties, parametric uncertainties analysis can provide a simple method to study the stability of a switched system and to obtain the bounds of uncertainty. Although extensive research has focused on stability analysis of continuous-time switched systems with polytopic or norm-bounded uncertainties, to the best of our knowledge, little results have been reported in the literature on the stability of discrete-time switched systems with parametric uncertainties. In [19] and [20], stability analysis has been presented for continuous-time switched systems with parametric uncertainties. In contrast, [21]-[24] have presented stability analysis and stabilization for discrete-time nonlinear switched systems with time-delay and affine parametric uncertainties. When the dynamics of a system contains time-delay, the stability analysis is more complicated and more conservative than non-delay cases. Therefore, an analysis method that has been developed for non-delay cases may reduce conservatism.

To achieve a less conservative analysis, this paper considers the exponential stability analysis for a class of discrete-time switched systems with parametric uncertainties under an arbitrary switching signal. It is assumed that a common quadratic Lyapunov function exists for the nominal discrete-time switched system that guarantees the stability of the nominal system. Finding a common quadratic Lyapunov function is computationally feasible because it can be obtained by solving linear matrix inequalities (LMIs) for a nominal switched system with a finite number of subsystems.

The contributions of the present paper mainly include two aspects. First, sufficient conditions that are based on the common Lyapunov function for the nominal system are derived for robust exponential stability of discrete-time switched linear systems under arbitrary switching signal. Second, the maximum admissible bounds of the uncertain parameters are determined. Furthermore, the proposed approach can be further used as a constructive solution to the problem of feedback stabilization.

The paper is organized as follows. In Section 2, the preliminaries and a useful lemma are given for later use. Section 3 includes theorems on the stability analysis of a switched linear system. Section 4 proposes a procedure for the stability analysis. In Section 5, numerical examples are presented to investigate the main results. Finally, the paper is concluded in Section 6.

II. Preliminaries and Problem Formulation

Consider a discrete-time switched linear system described by the following model:

$$S_{\sigma(k)}:\ x(k+1) = A_{\sigma(k)}(q)\, x(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $A_{\sigma(k)}(q) \in \mathbb{R}^{n\times n}$ is the system's matrix and $\sigma(k): \mathbb{Z} \to M = \{1, \dots, m\}$ is the switching signal, m is a finite integer, $\mathbb{Z}$ is the set of positive integers, and $q = [q_1, q_2, \dots q_r]$ is a vector of uncertain parameters. Assume that all subsystems are uncertain systems. Therefore, the switched system is composed of m subsystems with the same dimensions, which are expressed as follows:

$$S_i:\ x(k+1) = A_i(q)\, x(k) \qquad i \in \{1, \dots, m\} \tag{2}$$

where $A_i(q)\,, i = 1, \dots, m,$ is a parametric uncertain matrix of subsystems with r uncertain parameters also defined as

$$A_i(q) = A_i^0 + \sum_{j=1}^{r} \delta q_j\, E_j^i \tag{3}$$

where A_i^0 is the constant nominal matrix, r is a positive integer value that depends on the number of uncertain parameters and δq_j is the perturbation around the nominal value of the jth uncertain parameter. E_j^i is the uncertainty structure matrix with known real parameters that describe how A_i depends on the uncertain parameter δq_j. If the uncertain parameter does not enter in the i-th subsystem, then E_j^i is a zero matrix. The present paper tries to find the conditions of robust stability of the uncertain switched system (1) under any switching signal and the maximum admissible bounds of the uncertain parameters. Now, let to introduce the following Assumption and Lemma:

Assumption 1: It is assumed that a common quadratic Lyapunov function $V(x(k)) = x^T(k)Px(k)$ exists for the nominal discrete-time switched system $x(k+1) = A_i^0\, x(k)$. Namely, there exists a positive definite matrix $P \in \mathbb{R}^{n\times n}$, such that

$$(A_i^0)^T P A_i^0 - P < 0 \qquad \forall i \in M \tag{4}$$

Also, it is assumed that the eigenvalues of the matrix $(A_i^0)^T A_i^0$ for each subsystem are located in the open unit circle. This assumption is necessary for feasibility problem of the proposed LMI conditions.

Remark 1: Finding a Lyapunov function for discrete-time switched systems is much more difficult than for continuous-time cases because the Lyapunov equation

$A^T PA - P = -Q$ for a discrete linear system is not linear with respect to A.

Lemma 1 [25]: For a given positive definite symmetric matrix $Q \in \mathbb{R}^{n\times n}$, and for any $x \in \mathbb{R}^n$, we have

$$\underline{\sigma}(Q)\|x\|_2^2 \le x^T Qx \le \overline{\sigma}(Q)\|x\|_2^2 \tag{5}$$

where $\overline{\sigma}(Q)$ and $\underline{\sigma}(Q)$ are the largest and smallest singular values of the matrix Q, respectively. Moreover, for any symmetric matrix $Q \in \mathbb{R}^{n\times n}$ and any $x \in \mathbb{R}^n$, we have

$$\lambda_{\min}(Q)x^T x \le x^T Qx \le \lambda_{\max}(Q)x^T x \tag{6}$$

where $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ are the largest and smallest eigenvalues of matrix Q, respectively.

Property 1 [26]**:** Suppose A and B are square matrices. According to the norm consistency condition, $\|AB\| \le \|A\|\|B\|$, and the following property is obvious:

$$\overline{\sigma}(AB) \le \overline{\sigma}(A)\overline{\sigma}(B) \tag{7}$$

III. Main Results

This section proposes sufficient conditions for robust exponential stability analysis of the uncertain switched system (1). Then, the maximum admissible bounds of the uncertain parameters are determined to guarantee robust exponential stability.

Theorem 1: Consider the discrete-time switched linear system (1) with the parametric uncertainties $\delta q_j \in [-c_j,\ c_j]$. The switched system under arbitrary switching signal is robustly stable if a positive definite matrix $P \ge I$ and positive scalars α_i exist such that

$$(A_i^0)^T PA_i^0 - P \le -\alpha_i P \qquad \forall i \in M \tag{8}$$

$$\beta_i{}^2 + 2\|A_i^0\|\beta_i < \frac{\alpha_i}{\|P\|} \qquad \forall i \in M \tag{9}$$

where $\beta_i = \sum_{j=1}^{r} c_j\left\|E_j^i\right\|$.

Proof: Existence of a common Lyapunov function is a sufficient condition for exponential stability of a discrete-time switched linear system [27]. Therefore, if common quadratic Lyapunov function $V(x(k)) = x^T(k)Px(k)$ stays decreasing along the trajectories of each subsystem for all uncertainties of parameters, the switched system (1) with parametric uncertain matrices (3) is robustly exponentially stable.

Consider the Lyapunov function $V(x(k)) = x^T(k)Px(k)$ for the uncertain system (1). The forward difference of $V(x)$ along with the solution of the system by considering Assumption 1 is given by

$$\Delta V\big(x(k)\big) = V\big(x(k+1)\big) - V\big(x(k)\big) \tag{10}$$

$$= \left[\left(A_i^0 + \sum_{j=1}^{r}\delta q_j\, E_j^i\right)x(k)\right]^T P\left[\left(A_i^0 + \sum_{j=1}^{r}\delta q_j\, E_j^i\right)x(k)\right] - x^T(k)Px(k)$$

$$= x^T(k)\left[(A_i^0)^T PA_i^0 - P + \left(\sum_{j=1}^{r}\delta q_j\, E_j^i\right)^T PA_i^0 + (A_i^0)^T P\left(\sum_{j=1}^{r}\delta q_j\, E_j^i\right) + \left(\sum_{j=1}^{r}\delta q_j\, E_j^i\right)^T P\left(\sum_{j=1}^{r}\delta q_j\, E_j^i\right)\right]x(k)$$

Assumption (8) and considered intervals $[-c_j,\ c_j]$ of the parameter uncertainties, leads to

$$\Delta V\big(x(k)\big) \le -\alpha_i x^T(k)Px(k) + \left[2\sum_{j=1}^{r} c_j\| E_j^i\|\|P\|\|A_i^0\| + \left(\sum_{j=1}^{r} c_j\| E_j^i\|\right)^2 \|P\|\right]x^T(k)x(k) \tag{11}$$

According to Lemma 1, we have

$$\Delta V\big(x(k)\big) \le \left[-\alpha_i\underline{\sigma}(P) + 2\sum_{j=1}^{r} c_j\| E_j^i\|\|P\|\|A_i^0\| + \left(\sum_{j=1}^{r} c_j\| E_j^i\|\right)^2 \|P\|\right]\|x\|_2^2 \tag{12}$$

Since $P \ge I$, and $\underline{\sigma}(P) \ge 1$ and given the inequality (9), we have $\Delta V\big(x(k)\big) < 0$, which implies that the Lyapunov function stays decreasing along with the trajectories of each subsystem for the parametric uncertainties. Therefore, the uncertain switched system (1) with parametric uncertain matrices (3) is robustly stable and the proof is completed.

Now, the admissible bounds of the uncertain parameters are determined. Theorem 2 determines how parameters of a switched linear system can vary from their nominal values so that the robust exponential stability of the uncertain arbitrary switched system is guaranteed.

Theorem 2: Consider the discrete-time switched system (1) with the parametric uncertainties $\delta q_j \in [-\gamma w_j,\ \gamma w_j]$, where γ is a positive constant and w_j, $j = 1, \ldots r$ are the weight of uncertain parameters. Assume that a positive definite matrix

$P \geq I$ and positive scalars α_i exist such that:

$$(A_i^0)^T PA_i^0 - P \leq -\alpha_i P \qquad \forall i \in M \tag{13}$$

The switched system under arbitrary switching signal is robustly stable for all δq_j, if:

$$0 < \gamma < \min_{i\in M}\left(\frac{-b_i + \sqrt{b_i^2 + a_i c_i}}{a_i}\right) \tag{14}$$

where

$$a_i = \left(\sum_{j=1}^{r} w_j \|E_j^i\|\right)^2,\ b_i = \|A_i^0\| \sum_{n=1}^{r} w_j \|E_j^i\| \quad \text{and} \quad c_i = \frac{\alpha_i}{\|P\|} \tag{15}$$

Proof: Similar to the proof of Theorem 1, if the common quadratic Lyapunov function stays decreasing along the trajectories of each subsystem for all uncertain parameters, the switched system (1) with parametric uncertain matrices (3) is robustly stable. Thus, (11) can be rewritten as

$$\Delta V(x(k)) \leq x(k)^T \left[-\alpha_i + 2\gamma\|P\| \sum_{j=1}^{r} w_j \| E_j^i\| \|A_i^0\| + \gamma^2 \left(\sum_{j=1}^{r} w_j \| E_j^i\|\right)^2 \|P\|\right] x(k) \tag{16}$$

Therefore, if

$$-\alpha_i + 2\gamma\|P\| \sum_{j=1}^{r} w_j \| E_j^i\| \|A_i^0\| + \gamma^2 \left(\sum_{j=1}^{r} w_j \| E_j^i\|\right)^2 \|P\| < 0, \tag{17}$$

the common quadratic Lyapunov function Vstays decreasing $(\Delta V(x(k)) < 0)$ and the switched system (1) with parametric uncertain matrices (3) is robustly stable. Using (15), the inequality (17) can be written as

$$a_i\gamma^2 + 2b_i\gamma - c_i < 0 \tag{18}$$

Thus, the proof is completed.

IV. Stability Analysis Procedures in Steps

In order to analyze the robust exponential stability of the uncertain discrete-time switched system (1), the complete procedure is given in the following steps:

1) Check Assumption 1. In order to proceed with the stability analysis of the uncertain switched system (1), the nominal switched system should be stable. The problem of finding a common Lyapunov function for the nominal switched system is analogous to finding a symmetric matrix P satisfying (4).

2) Select arbitrary positive scalars $0 < \alpha_i < 1$, $i = 1, 2, \dots, m$ and find a positive definite matrix $P \geq I$ satisfying linear matrix inequalities (8). For all given uncertain parameters $\delta q_j \in [-c_j,\ c_j]$, if the inequality (9) holds, the uncertain switched system (1) is robustly stable. However, if the condition (9) for the given uncertain parameters does not hold, we can use Theorem 2 and proceed to the next step to calculate the maximum admissible interval of the uncertain parameters.

3) Consider the uncertain parameters $\delta q_j \in q_j^0[-\gamma,\ \gamma]$. Find the maximum value of $\frac{\alpha_i}{\|P\|}$ with respect to different values of α_i such that the linear matrix inequalities (13) with a positive definite matrix $P \geq I$ are feasible. Calculate a_i, b_i, c_i from (15). The resulted γ from (14) shows the tolerance bound of the uncertain parameters.

Remark 2: The problem of finding a symmetric matrix P satisfying (4) or (8) can be solved by solving linear matrix inequalities. Efficient solvers for solving such inequalities are available [28]. Also, the Matlab LMI or YALMIP toolboxes are most widely used for solving such linear matrix inequalities.

V. Numerical Examples

In this section, computer simulations are performed to illustrate the effectiveness of the proposed analysis.

Example 1: In this example, the results of the proposed method are compared with the method based on parametric uncertainty presented in [23]. Consider the discrete-time switched linear system (1) with $m = 2$ and the parameters which are given below:

$$A_1 = \begin{bmatrix} 0.8 + \delta q_1 & 0.2 + \delta q_2 \\ 0.1 + \delta q_3 & -0.4 + \delta q_4 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.5 + \delta q_1 & 0.3 + \delta q_2 \\ 0.6 + \delta q_3 & 0.4 + \delta q_4 \end{bmatrix}. \tag{19}$$

where the uncertain parameters are $\delta q_1 \in [-0.01, 0.01]$, $\delta q_2 \in [-0.02\ 0.02]$, $\delta q_3 \in [-0.03, 0.03]$ and $\delta q_4 \in [-0.01, 0.01]$. Therefore, the nominal matrices are

$$A_1^0 = \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & -0.4 \end{bmatrix}, \qquad A_2^0 = \begin{bmatrix} -0.5 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}. \tag{20}$$

It can be verified using the MATLAB LMI toolbox that the nominal system has a common diagonal Lyapunov function. Therefore, the switched system (1) with nominal matrices (20) is globally exponentially stable and Assumption 1 holds. In addition, the structure of uncertainty matrices is as,

$$E_1^1 = E_1^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ E_2^1 = E_2^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ E_3^1 = E_3^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ E_4^1 = E_4^2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{21}$$

Now, Theorem 1 can be used to investigate the robust stability of the considered switched linear system. For the nominal matrices (20) and uncertainty structure matrices (21), there exists a positive definite matrix $P = \begin{bmatrix} 2.2972 & 0.1820 \\ 0.1820 & 1.9593 \end{bmatrix}$ and a positive scalar $\alpha_1 = \alpha_2 = 0.3$ such that inequalities (8), (9) holds. Therefore, the switched system (1) with uncertain matrices (19) is robustly exponentially stable for uncertain parameters belonging to the considered intervals.

Now, we compare the results of the proposed method with

Theorem 1 of [23]. In order to be able to compare, we must consider zero for the time-delay matrices in [23]. Although the results obtained by [23] cannot be provided any feasible solutions for the arbitrary switching signals, the proposed method guarantees the stability for this example.

In the simulation, suppose that $\delta q_1 = 0.01\sin(k)$, $\delta q_2 = 0.02\sin(2k)$, $\delta q_3 = 0.03\sin(3k)$, and $\delta q_4 = 0.01\sin(4k)$. Also, the subsystems of the switched system (2) with uncertain matrices (19) are activated according to Figure 1. The trajectories of the system states are shown in Figure 2. It is observed that the uncertain switched system is robustly stable in the presence of parameter uncertainties.

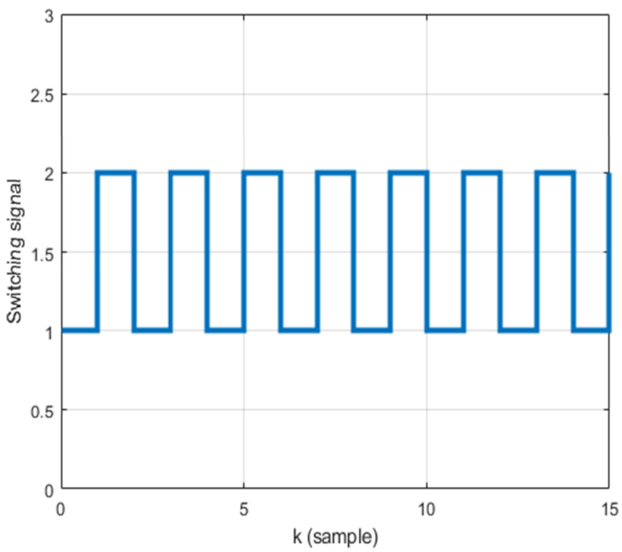

Fig. 1. Time history of the switching signal.

Remark 3: The considered system in [23] involves time-delay, nonlinear terms, and affine parametric uncertainties. Therefore, the stability analysis presented with more conservatism. In this paper, to obtain less conservative results, a simple method is presented.

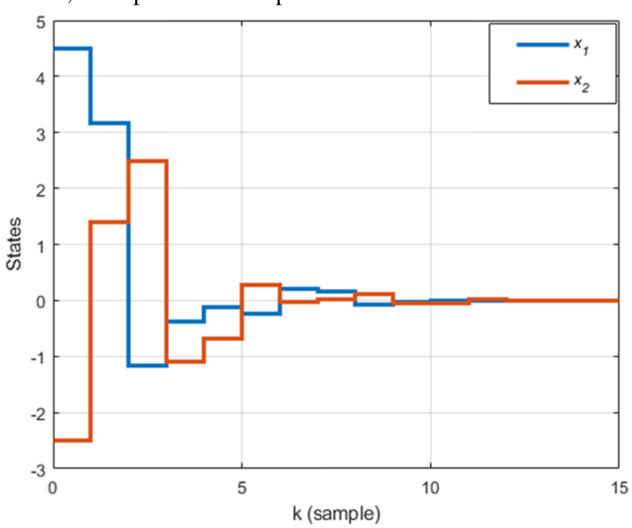

Fig. 2. Time history of states in Example 1.

Example 2: Consider the following discrete-time switched linear system with the parametric uncertainties and $m = 2$:

$$A_1 = \begin{bmatrix} 0.42+0.2\delta q_1 & -0.02 & -0.12 \\ 0.03 & 0.6+3\delta q_2 & 0.01+0.1\delta q_1 \\ 0.18 & 0.02+0.02\delta q_3 & 0.77 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.7+5\delta q_1+\delta q_2 & 0.16 & \delta q_1+0.2 \\ -0.06 & 0.5+\delta q_3 & -0.1 \\ -0.02 & -0.1-\delta q_1 & 0.5-5\delta q_1+\delta q_3 \end{bmatrix} \tag{22}$$

where the uncertain parameters are $\delta q_1 \in [-0.001, 0.001]$, $\delta q_2 \in [-0.002, 0.002]$, and $\delta q_3 \in [-0.001, 0.001]$. Therefore, the nominal matrices are

$$A_1^0 = \begin{bmatrix} 0.42 & -0.02 & -0.12 \\ 0.03 & 0.6 & 0.01 \\ 0.18 & 0.02 & 0.77 \end{bmatrix},\quad A_2^0 = \begin{bmatrix} 0.7 & 0.16 & 0.2 \\ -0.06 & 0.5 & -0.1 \\ -0.02 & -0.1 & 0.5 \end{bmatrix} \tag{23}$$

It can be verified using the LMI toolbox that the nominal system has a common Lyapunov function. Therefore, the switched system (1) with nominal matrices (23) is globally exponentially stable and Assumption 1 holds. In addition, the structure of uncertainty matrices is

$$E_1^1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.1 \\ 0 & 0 & 0 \end{bmatrix},\ E_2^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ E_3^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.02 & 0 \end{bmatrix}, E_1^2 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & -5 \end{bmatrix},\ E_2^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{24}$$
$$E_3^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

According to the parameter uncertainty intervals, Theorem 1 can be used to investigate the robust stability of the considered switched linear system. For the nominal matrices (23) and uncertainty structure matrices (24), there exists a positive definite matrix $P = \begin{bmatrix} 2.83 & 0.18 & 0.52 \\ 0.18 & 2.78 & -0.004 \\ 0.52 & -0.004 & 3.12 \end{bmatrix}$ and a positive scalar $\alpha_1 = \alpha_2 = \alpha_3 = 0.2$ such that inequalities (8) and (9) holds. Therefore, the switched system (1) with uncertain matrices (22) is robustly exponentially stable for uncertain parameters belonging to the considered intervals.

Now, based on Theorem 2, the maximum interval of uncertain parameters can be calculated. For this purpose, weights of uncertainties are chosen as $w_j = 1$. Using the LMI toolbox, the maximum value of $\frac{\alpha_i}{\|P\|}$ can be calculated. Figure 3 shows this maximum for $\alpha_1 = \alpha_2 = \alpha_3$ such that inequality (14) holds for $\gamma = 0.014$. Therefore, the switched system (1) with uncertain matrices (22) is robustly exponentially stable for $\delta q_j \in [-0.014, 0.014], j = 1,2,3$. It can be seen that this interval has a higher bound compared to the uncertain parameters mentioned in (22).

For computer simulation, $\delta q_1 = 0.001\sin(k)$, $\delta q_2 = 0.002\sin(2k)$, $\delta q_3 = 0.001\sin(3k)$ are considered to show uncertainties in the system's parameters. Also, the subsystems of the switched system (2) with uncertain matrices (22) are activated according to Figure 1. The simulation result is shown in Figure 4 in which the trajectories of the system states are depicted. It can be observed that the uncertain switched system is robustly stable under parameter uncertainties.

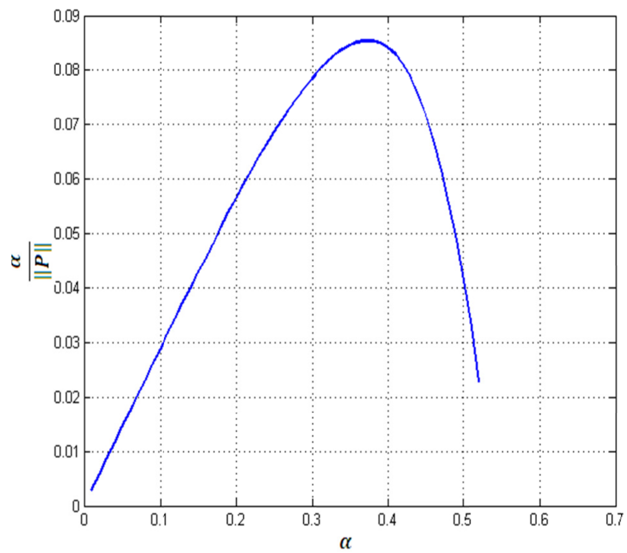

Fig. 3. Graph of $\frac{\alpha}{\|P\|}$ with respect to different values of $\boldsymbol{\alpha}$ in Example 2.

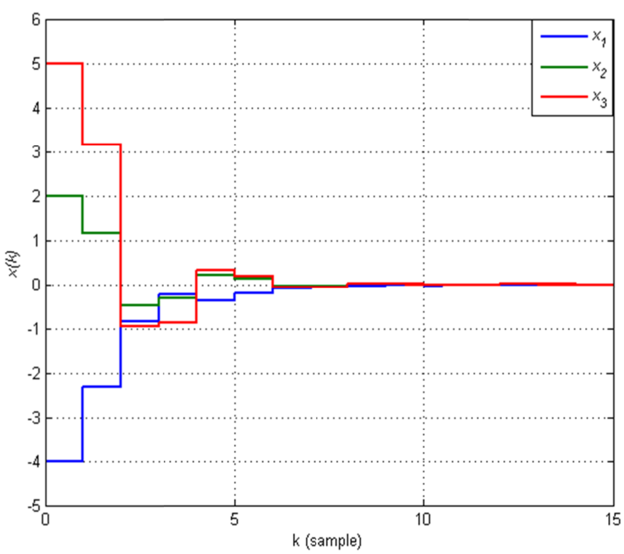

Fig. 4. Time history of states in Example 2.

VI. Conclusions

This paper considered the problem of robust stability for linear discrete-time switched systems. First, sufficient conditions for the stability of uncertain discrete-time switched linear systems were proposed. The derived conditions guaranteed the stability of switched linear systems with parametric uncertainties under any arbitrary switching signal. Secondly, in the proposed procedure for stability analysis, the admissible bounds of uncertain parameters were determined. Finally, numerical examples were presented to verify the theoretical results.

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Nasrollah Azam Baleghi was born in Mashhad, Iran. He received his B.Sc. degree in Electronics Engineering from Shahid Bahonar University, Kerman, Iran, in 2002, and his Ph.D. degree in Control Engineering from the Shiraz University of Technology, Shiraz, Iran, in 2018. His current research interests include discrete-time systems, nonlinear control, robust control, and switching systems.

Mohammad Hossein Shafiei was born in Shiraz, Iran, in 1979. He received his B.Sc. and M.Sc. degrees in Electronics Engineering from Shiraz University, Shiraz, Iran, in 2002 and 2005, respectively, and his Ph.D. degree in Control Engineering from the University of Tehran, Tehran, Iran, in 2010. He is now the Associate Professor of the Faculty of Electrical Engineering, Shiraz University of Technology, Shiraz, Iran. His current research interests include nonlinear control, robust and optimal methods, and switching systems.

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