Control Law Design with Dead-beat Property in Continuous-Time Dynamical Systems

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A B S T R A C T

In this paper, an open-loop control scheme is developed to design of a dead-beat control effort in the high order continuous-time LTI systems. The dead-beat control is really a finite-time control law. In this method, the input signal is manually selected such that the output signal becomes constant in a finite time. In the LTI systems, having known the step response, a control signal could be exactly selected such a way that the control objective would be met in a finite time. For this goal, the dead-beat control problem is firstly investigated in a standard first order system. Then a similar problem is studied in the second order systems. Finally a general design framework would be developed to obtain a dead-beat control policy in the high order continuous-time systems. In the proposed method, the control design problem is deliberately converted into the solution of a linear matrix equation. Therefore the control signal would be determined via the solution of an algebraic equation. The suggested procedure has been successfully applied in some continuous-time LTI systems. The simulation results show the effectiveness of the proposed control method for designing of a finite-time control law in the continuous-time LTI systems.

I. INTRODUCTION

Theoretically the control synthesis problems have been mainly investigated in closed loop control system. The closed loop control schemes may have some advantages over the corresponded open-loop ones. These briefly includes improvement of the stability property and also robustness with respect to the parameter changes. But some extra costs should be paid due to existence of some measurement sensors and control design [1]. Hence sometimes the open-loop control scheme would be interested especially when there is not any disturbance and or parameter uncertainty. In the control literatures, some open-loop control policies have been proposed in the dynamical systems as well as dead-beat control [2, 3], iterative learning control (ILC) [4] , input shaping control [5] and signal correction control technique [6].

The dead-beat control problem is firstly investigated in the discrete-time SISO systems [7]. In the discrete-time system,

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the relation between input and output is simply described by a difference equation rather than differential one. Hence, in the discrete-time version of the dead-beat control, the input signal would be designed such a way that the input signal (control effort) becomes exactly zero after some sampling times. Then the output signal would become a constant value. This problem is also studied in the multivariable systems [8]. The dead-beat property would be very important in some control applications as well as grid connected systems [9, 10], electric machines [11-13], active power filter [14], electronic convertors [15, 16] and aerospace vehicles [17].

The discrete-time control signal may be converted into a continuous-time signal by adding a zero-order hold (ZOH) element but it may make some reconstruction errors [18-20]. Although the dead-beat control problem is essentially suggested in the discrete-time systems but such a control concept can be also extended to the continuous-time ones [2, 21, 22]. In other word, in the continuous-time dead-beat control, the input signal is manually selected such that the output signal becomes constant in a finite time. It seems that a dead-beat control would be a finite-time control law in the open-loop control structures. But the concept of the

finite-time control system has been mainly studied in the closed loop schemes [23].

Recently several control methods have been suggested in the literatures to realize the finite settling-time control in a continuous-time control system. These mainly includes the Posicast control and feedback control using bang-bang or hysteresis element [23-25]. In the second order systems, the Posicast control policy is designed to obtain dead-beat response with zero initial condition [3, 26, 27]. Then it is extended to nonzero initial condition [28]. Similar problem is also studied in the fractional order systems [29] and sampled-data system [30].

In a typical continuous-time system, sometimes there exists a piecewise continuous-time control signal which provides a dead-beat response. This point motivates the author to study the dead-beat control law in the high order continuous-time systems. For achieving this goal, the dead-beat problem is firstly investigated in the standard first order system. Then it is studied in the second order systems. Finally a general framework would be developed to obtain a dead-beat control policy in the high order continuous-time systems. Hence some benefits of the proposed method can be listed as the following:

1- This method is inherently an open loop structure. Thus it does not need any further sensor.

2- The control system structure is a very simple. Thus the control signal is directly determined.

3- It does not need to run the control system continuously. The controller is active in a finite time.

4- This method can be efficiently used in some control applications like guidance problem.

5- The control system design procedure is very simple for the designer.

6- In the proposed method, the dead-beat control is constructed via using of some pulse-shape signals.

Therefore the main contribution of this paper concentrates on designing of a dead-beat control law in the continuous-time LTI systems. The suggested method would be an open-loop scheme which is realized with combination of some sequences of the pulse signals.

The rest of this paper is organized as follows: In the next section, some mathematical notations and symbols are addressed. Then in the sections 3, the problem formulation and main result of the paper is presented. The application of proposed method is studied in some numerical examples in the section 4. Finally some concluding remarks are placed in the last section.

II. MATHEMATICAL NOTATIONS

Throughout this paper, the symbol $u(t)$ denotes the unit step signal, $s(t)$ is the unit step response in a continuous-time LTI system. The system's input and output denote by $x(t)$ and $y(t)$ respectively. The star symbol $*$ is used for the convolution integral. The s symbol also states the Laplace operator. The mathematical definition of the dead-beat control in the continuous-time form can be expressed as follows:

Definition 1: The response of a continuous-time system would have a dead-beat property, if its output $y(t)$ is a constant value after a finite-time. In a simple word, it can be mathematically written as follows:

$$y(t) = cte \quad \forall \quad t > t_n \tag{1}$$

The switching-time t_n would be exactly the settling-time of the continuous-time system (1). The standard first order system can be described with the following transfer function:

$$G(s) = \frac{k}{1+sT} \tag{2}$$

where k is the system gain and $T > 0$ is the time-constant of the LTI system (2). The standard second order system is written as the following:

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \tag{3}$$

where $0 < \xi < 1$ is the damping ratio and $\omega_n > 0$ is the natural frequency of the second order system (3). A typical high order continuous-time LTI system with all real-poles may be described as follows:

$$G(s) = k\frac{(s+\gamma_1)(s+\gamma_2)\dots(s+\gamma_m)}{(s+\beta_1)(s+\beta_2)\dots(s+\beta_n)} \ , \qquad m < n \tag{4}$$

In the equation (4), the positive parameters β_i indicates poles of the open-loop system (4) and the coefficients γ_i determines the system's zeros. It is assumed that the poles are simple and stable. The system's zeros may be some real or complex numbers. Next, the dead-beat problem is formulated in the continuous-time system. Then some technical results would be proposed.

III. PROBLEM FORMULATION

In some continuous-time control applications, an open-loop control law design may be interested. Sometimes it may be found a piecewise continuous-time control signal which provides a dead-beat response. The input signals in form of the pulse structure would be concerned in this paper. Then a sequence of the pulse signals would be considered as a control signal. The parameters of such pulse signal is determined such that the system output would be constant after a finite time. Therefore, in this section, the dead-beat problem is mainly investigated in the standard first order and second order systems. Then a general framework will be developed to obtain dead-beat control policy in the high order continuous-time systems (4). For this purpose, firstly, a first order system is considered. Such a system is a simple continuous-time one. Then it can be easily analyzed. In the next lemma, a simple dead-beat control law is addressed.

Lemma 1: In the first order system (2), if the switching-time t_s is deliberately selected as $t_s = T.Ln\left(1+\frac{1}{\alpha}\right)$ then a dead-beat response would be provided by the following

signal:

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 + \alpha & 0 < t < t_s \\ 1 & t \geq t_s \end{cases} \quad (5)$$

where α can be any positive constant and the switching-time t_s would be the settling-time of the system (2).

Proof: In the first order system (2), the unit step response $s(t)$ can be found as follows:

$$s(t) = k\left(1 - e^{-\frac{t}{T}}\right)u(t) \quad (6)$$

It is easy check that the response $y(t)$ due to an input signal $x(t) = (1 + \alpha)u(t) - \alpha u(t - t_s)$ would be computed as the following:

$$y(t) = (1 + \alpha)s(t) - \alpha s(t - t_s) \quad (7)$$

By substitution of the switching-time $t_s = T.Ln\left(1 + \frac{1}{\alpha}\right)$, the system response $y(t), t \geq t_s$ would be obtained as follows:

$$y(t) = (1 + \alpha)k\left(1 - e^{-\frac{t}{T}}\right) - \alpha k\left(1 - e^{-\frac{t-t_s}{T}}\right) \quad (8)$$

Then a dead-beat response $y(t) = k$, $t \geq t_s$ would be appeared. It completes the proof.

Remark 1: In the lemma 1, the constant α would be an independent variable. It can be shown that the switching-time t_s could be selected as an independent variable when the constant parameter α is chosen as $\alpha = \left(e^{\frac{t_s}{T}} - 1\right)^{-1}$.

Remark 2: It is seen that in the first order system (2), the dead-beat response would not be unique. Then in order to obtain a dead-bead response via the lemma 1, there are too many choices for selection of the constant α.

Similarly such idea can also be used in a second order system to obtain a dead-beat response. Consider the standard second order system (3). It can be shown that the following control signal will provide a dead-beat response in the standard second order system (3):

$$x(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{1+M_p} & 0 < t < t_p \\ 1 & t \geq t_p \end{cases} \quad (9)$$

In the control literature, the control signal (9) is referred as the Posicast control law [3]. In the input signal $x(t)$ there exist two terms M_p and t_p which are defined as follows:

$$M_p \stackrel{def}{=} e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \text{ and } t_p \stackrel{def}{=} \frac{\pi}{\omega_n}$$

They denote the maximum overshoot and peak-time respectively due to unit step response in the standard second order system (3). Next, the dead-beat problem would be investigated in a general LTI systems.

Assumption 1: In the continuous-time LTI systems (4), assume that the parameters β_i are some real and positive ones and the unit step response $s(t)$ is represented as the following form:

$$s(t) = \left(\alpha_0 + \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} + \cdots + \alpha_n e^{-\beta_n t}\right)u(t) \quad (10)$$

In the equation (10), for the stability purpose, it is assumed that the coefficients β_i must be some positive ones.

Remark 3: The high order LTI system (4) can be mathematically decomposed into some first order subsystems. Then a dead-beat control signal can be designed for each subsystem by using of the Lemma 1. It does not guarantee that a linear combination of the designed control signals, for the first order subsystems, would provide a dead-beat response in the high order system (4).

The remark 3 can be checked with a simple parallel system like $G(s) = G_1(s) + G_2(s)$. For this purpose, assume that $x_1(t)$ and $x_2(t)$ are some dead-beat control signals for the subsystem $G_1(s)$ and $G_2(s)$ respectively. Let select a linear combination of the mentioned signal like $x(t) = ax_1(t) + bx_2(t)$. Then by using of the convolution integral and impulse response properties, the system output $y(t)$ due to the input $x(t)$ can be written as follows:

$$y(t) = L^{-1}\{G(s)\} * x(t) = (L^{-1}\{G_1(s)\} + L^{-1}\{G_2(s)\}) * \left(ax_1(t) + bx_2(t)\right)$$

Then it can be simplified as the following:

$$y(t) = aL^{-1}\{G_1(s)\} * x_1(t) + bL^{-1}\{G_2(s)\} * x_2(t) + aL^{-1}\{G_2(s)\} * x_1(t) + bL^{-1}\{G_1(s)\} * x_2(t)$$

The signal $x_1(t)$ is designed based on the parameters of the subsystem $G_1(s)$. Then the third term has not dead-beat property. A similar justification can be existed for the signal $x_2(t)$ in the subsystem $G_1(s)$. Therefore the linear combination of the signals $x_1(t)$ and $x_2(t)$ would not provide a dead-beat response in the LTI system $G(s)$. Hence it is necessary to develop a general framework for designing of dead-beat control signal in the LTI system (4).

In the first order and second order systems, the mentioned results about the dead-beat response may be seem simple. Having known the step response of a general LTI system $s(t)$, the dead-beat response could be also investigated in the high order continuous-time system. Hence the main result of this paper will be concentrated on selection of a dead-beat control law in the general LTI systems. The proposed results will be presented in the next theorem.

Theorem 1: Consider the assumption 1 holds and the unit step response is described by the equation (10). In the LTI system (4), for some known positive constants β_i and switching-times t_i, $i = 1, 2, \ldots, n$, $0 < t_1 < t_2 \ldots < t_n$, if there exist some positive constants δ_i, $i = 1, 2, \ldots. n$ such that the equation $\mathcal{A}\mathcal{X} = \mathcal{B}$ admits a feasible solution $\mathcal{X}$. where the matrix $\mathcal{A}$ and two vectors $\mathcal{X}$ and $\mathcal{B}$ are given as:

$$\mathcal{A} = \begin{bmatrix} e^{\beta_1 t_1} - 1 & e^{\beta_1 t_2} - e^{\beta_1 t_1} & \cdots & e^{\beta_1 t_n} - e^{\beta_1 t_{n-1}} \\ e^{\beta_2 t_1} - 1 & e^{\beta_2 t_2} - e^{\beta_2 t_1} & & e^{\beta_2 t_n} - e^{\beta_2 t_{n-1}} \\ \vdots & & \ddots & \vdots \\ e^{\beta_n t_1} - 1 & e^{\beta_n t_2} - e^{\beta_n t_1} & \cdots & e^{\beta_n t_n} - e^{\beta_n t_{n-1}} \end{bmatrix}$$

$$X = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix}, \quad B = \begin{bmatrix} e^{\beta_1 t_n} \\ e^{\beta_2 t_n} \\ \vdots \\ e^{\beta_n t_n} \end{bmatrix}$$

Then with selection of the following input signal $x(t)$:

$$x(t) = \delta_1 u(t) + (\delta_2 - \delta_1)u(t - t_1) +$$
$$(\delta_3 - \delta_2)u(t - t_2) + (\delta_4 - \delta_3)u(t - t_3) + \cdots + \qquad (11)$$
$$(\delta_{n+1} - \delta_n)u(t - t_n)$$

the continuous system (4) would have a dead-beat response.

Proof: Considering the well-known superposition principle, the output of the LTI system (4) can be mathematically computed as follows:

$$y(t) = \delta_1 s(t) + (\delta_2 - \delta_1)s(t - t_1) + (\delta_3 -$$
$$\delta_2)s(t - t_2) + (\delta_4 - \delta_3)s(t - t_3) + \cdots + \qquad (12)$$
$$(\delta_{n+1} - \delta_n)s(t - t_n)$$

The term δ_{n+1} denotes the steady state value of the input signal $x(t)$. Then without loss of generality, the term δ_{n+1} can be assumed to be a constant value. Hence in term of the step response parameters, the system response $y(t)$ could be computed as:

$$y(t) = \delta_1\left(\alpha_0 + \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} + \cdots + \alpha_n e^{-\beta_n t}\right)u(t) +$$
$$(\delta_2 - \delta_1)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_1)} + \alpha_2 e^{-\beta_2(t-t_1)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_1)}\right)u(t - t_1) + (\delta_3 - \delta_2)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_2)} +\right.$$
$$\left.\alpha_2 e^{-\beta_2(t-t_2)} + \cdots + \alpha_n e^{-\beta_n(t-t_2)}\right)u(t - t_2) + (\delta_4 -$$
$$\delta_3)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_3)} + \alpha_2 e^{-\beta_2(t-t_3)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_3)}\right)u(t - t_3) + \cdots + (\delta_{n+1} - \delta_n)\left(\alpha_0 +\right.$$
$$\left.\alpha_1 e^{-\beta_1(t-t_n)} + \alpha_2 e^{-\beta_2(t-t_n)} + \cdots + \alpha_n e^{-\beta_n(t-t_n)}\right)u(t - t_n)$$

In order to have a dead-beat response, the system response $y(t)$ would be constant one after a finite time (i.e. for $t > t_n$). Then the system response $y(t)$, $t > t_n$ can be computed as:

$$y(t) = \delta_1\left(\alpha_0 + \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} + \cdots + \alpha_n e^{-\beta_n t}\right) +$$
$$(\delta_2 - \delta_1)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_1)} + \alpha_2 e^{-\beta_2(t-t_1)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_1)}\right) +$$
$$(\delta_3 - \delta_2)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_2)} + \alpha_2 e^{-\beta_2(t-t_2)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_2)}\right) +$$
$$(\delta_4 - \delta_3)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_3)} + \alpha_2 e^{-\beta_2(t-t_3)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_3)}\right)$$
$$\vdots$$
$$+(\delta_{n+1} - \delta_n)\left(\alpha_0 + \alpha_1 e^{-\beta_1(t-t_n)} + \alpha_2 e^{-\beta_2(t-t_n)} + \cdots +\right.$$
$$\left.\alpha_n e^{-\beta_n(t-t_n)}\right) \quad, t > t_n$$

By rearranging and taking some simplifications, it can be written as follows:

$$y(t) = \alpha_0 \delta_{n+1} + \alpha_1 e^{-\beta_1 t}\left(\delta_1 + (\delta_2 - \delta_1)e^{\beta_1 t_1} +\right.$$
$$(\delta_3 - \delta_2)e^{\beta_1 t_2} + \cdots + (\delta_{n+1} - \delta_n)e^{\beta_1 t_n}\left.\right) + \alpha_2 e^{-\beta_2 t}\left(\delta_1 +\right.$$
$$(\delta_2 - \delta_1)e^{\beta_2 t_1} + (\delta_3 - \delta_2)e^{\beta_2 t_2} + \cdots + (\delta_{n+1} - \delta_n)e^{\beta_2 t_n}\left.\right)$$
$$+\alpha_3 e^{-\beta_3 t}\left(\delta_1 + (\delta_2 - \delta_1)e^{\beta_3 t_1} + (\delta_3 - \delta_2)e^{\beta_3 t_2} + \cdots\right.$$
$$\left. + (\delta_{n+1} - \delta_n)e^{\beta_3 t_n}\right)$$
$$\vdots$$
$$+\alpha_n e^{-\beta_n t}\left(\delta_1 + (\delta_2 - \delta_1)e^{\beta_n t_1} + (\delta_3 - \delta_2)e^{\beta_n t_2} + \cdots +\right.$$

$$(\delta_{n+1} - \delta_n)e^{\beta_n t_n}\right) \quad, \quad t > t_n$$

It is clear that the system response $y(t)$, $t > t_n$ would be a constant value $\alpha_0 \delta_{n+1}$ when the following equations simultaneously hold:

$$\delta_1 + (\delta_2 - \delta_1)e^{\beta_1 t_1} + (\delta_3 - \delta_2)e^{\beta_1 t_2} + \cdots$$
$$+ (\delta_{n+1} - \delta_n)e^{\beta_1 t_n} = 0$$
$$\delta_1 + (\delta_2 - \delta_1)e^{\beta_2 t_1} + (\delta_3 - \delta_2)e^{\beta_2 t_2} + \cdots$$
$$+ (\delta_{n+1} - \delta_n)e^{\beta_2 t_n} = 0$$
$$\vdots$$
$$\delta_1 + (\delta_2 - \delta_1)e^{\beta_n t_1} + (\delta_3 - \delta_2)e^{\beta_n t_2} + \cdots$$
$$+ (\delta_{n+1} - \delta_n)e^{\beta_n t_n} = 0$$

The pervious equations can be rewritten as follows:

$$\delta_1\left(1 - e^{\beta_1 t_1}\right) + \delta_2\left(e^{\beta_1 t_1} - e^{\beta_1 t_2}\right) + \delta_3\left(e^{\beta_1 t_2} - e^{\beta_1 t_3}\right) + \cdots$$
$$+ \delta_n\left(e^{\beta_1 t_{n-1}} - e^{\beta_1 t_n}\right) + \delta_{n+1}e^{\beta_1 t_n} = 0$$
$$\delta_1\left(1 - e^{\beta_2 t_1}\right) + \delta_2\left(e^{\beta_2 t_1} - e^{\beta_2 t_2}\right) + \delta_3\left(e^{\beta_2 t_2} - e^{\beta_2 t_3}\right) + \cdots$$
$$+ \delta_n\left(e^{\beta_2 t_{n-1}} - e^{\beta_2 t_n}\right) + \delta_{n+1}e^{\beta_2 t_n} = 0$$
$$\vdots$$
$$\delta_1\left(1 - e^{\beta_n t_1}\right) + \delta_2\left(e^{\beta_n t_1} - e^{\beta_n t_2}\right) + \delta_3\left(e^{\beta_n t_2} - e^{\beta_n t_3}\right) + \cdots$$
$$+ \delta_n\left(e^{\beta_n t_{n-1}} - e^{\beta_n t_n}\right) + \delta_{n+1}e^{\beta_n t_n} = 0$$

Therefore, these equations would lead to the matrix equations $AX = B$. It completes the proof.

Remark 4: In this paper, it is assumed that the conditions $\beta_i > 0$ and $0 < t_1 < t_2 \ldots < t_n$ simultaneously hold. Then the rows of the matrix A would be independent. Hence the matrix A would be a non-singular. Therefore the equations $AX = B$ would always have a unique solution.

Remark 5: In the theorem 1, the switching-times t_i, $i = 1, 2, \ldots . n$ would be some design parameters which selected by the designer. There are many choices for selection of the switching-times t_i but the only limitation would be expressed as $0 < t_1 < t_2 \ldots < t_n$. Hence many control signals with dead-beat property can be obtained in the high order LTI system (4). In the theorem 1, the term t_n denotes the settling-time. Furthermore dense selection of switching-times t_i's usually leads to larger overshoot and also control effort. Hence a reasonable trade-off between the parameters of the system response as well as the settling-time, maximum overshoot and the maximum value of the control signal should be taken into account in a typical control problem. This issue is usually accomplished throughout solving of an optimization problem with some nonlinear constraints.

Remark 6: The Lemma 1 and the Posicast control law can be considered as a special case of the theorem 1. Then the dead-beat control signal can be computed in the theorem 1.

Remark 7: In the high order LTI system (4), the poles are assumed to be simple and located in the left half plan. Hence there is not any constraint on the system zeros. Then they may be some real or complex numbers. Therefore the proposed method could be used even in a non-minimum phase LTI system. The effect of the system zeros are only

considered in the step response $s(t)$.

Remark 8: In the theorem 1, a model based and open-loop control law is addressed to provide dead-beat property. The open-loop stabilization methods can be helpful to design an efficient control law. Hence the results of the theorem 1 can be used in a cascaded system like $G(s)G_c(s)$. Then the subsystem $G_c(s)$ is selected such that the some undesired poles of the subsystem $G(s)$ like complex or repetitive can be vanished. Therefore the dead-beat control design would be used in a simplified LTI system $G(s)G_c(s)$.

Remark 9: In the theorem 1, some reasonable restrictions as well as the asymptotic stability, non-complex and non-repetitive poles are considered in order to formulate the control problem. A similar method can be also suggested to design of dead-beat control policy in the high order LTI system with the complex and repetitive poles.

IV. SIMULATION RESULTS

In this section, the proposed procedure is applied in some dynamical systems in order to obtain a dead-beat control law. **Example 1**: Consider a first order continuous-time system with $T = 1$ and $K = 1$ as follows:

$$G(s) = \frac{1}{s+1} \tag{13}$$

Let select the constant parameter $\alpha = 0.3$. By applying of the result of the lemma 1, the switching-time t_s can be exactly calculated as $t_s = Ln\left(\frac{13}{3}\right) = 1.4663$. Due to the system uncertainty, the switching-time t_s may not be precisely determined. Hence, in the numerical simulation, amount of 20% uncertainty is considered about the calculated switching-time t_s. In the simulation of continuous-time system, the time increment sets as 0.01 seconds.

The simulation results are shown in the figures 1 and 2. It is seen that in a finite times (i.e. 1.4663 seconds), the system response $y(t)$ would be constant one. The switching-time is explicitly dependent on the system parameters.
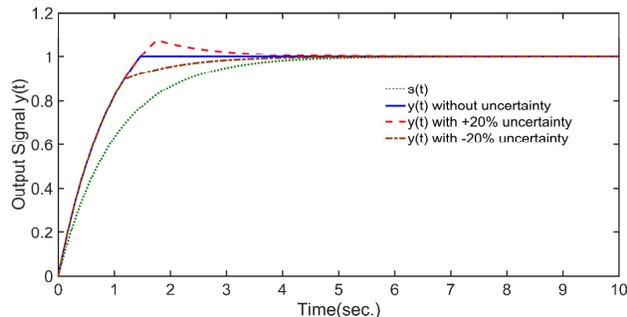
Fig. 1. The system response $y(t)$ in the example 1
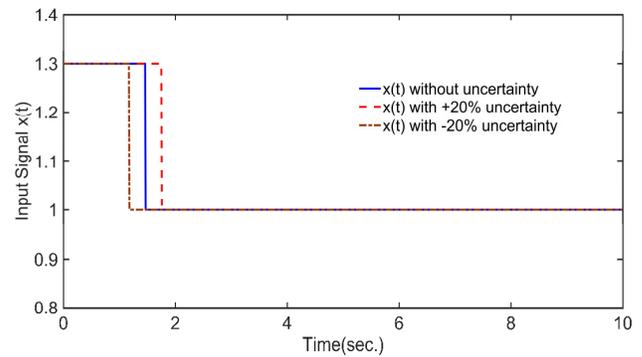
Fig. 2. The control signal $x(t)$ in the example 1

In the figure 1, it is seen that although the exact dead-beat response would be disappeared in the case of uncertainty. But the system response $y(t)$ is closely maintained around the dead-beat response. This property would be interested in some control applications as well as a guidance problems and power convertors. Therefore the proposed control law can be imagined a robust method in the presence of the uncertainty.

Example 2: Consider the following 4th order system:

$$G(s) = \frac{14s^3+80s^2+132s+48}{s^4+10s^3+35s^2+50s+24} \tag{14}$$

In the LTI system (14), the unit step response $s(t)$ can be computed as follows:

$$s(t) = (2 - 2e^{-2t} + 3e^{-t} - 4e^{-4t} + e^{-3t})u(t) \tag{15}$$

It is seen that the step response parameters can be found as the following:

$\beta_1 = 2$, $\beta_2 = 1$, $\beta_3 = 4$, $\beta_4 = 3$, $\alpha_0 = 2$, $\alpha_1 = -2$, $\alpha_2 = 3$, $\alpha_3 = -4$ and $\alpha_4 = +1$

In the proposed control method, the switching-times are design parameters. Let select the steady state value $\delta_5 = 1$ and the switching-times as $t_1 = 2$, $t_2 = 3$, $t_3 = 4$ and $t_4 = 5$. Then, by applying of the theorem 1, the control signal parameters are exactly calculated as follows:

$\delta_1 = 1.2482$, $\delta_2 = 0.9469$, $\delta_3 = 1.0027$ and $\delta_4 = 1.0$

In the simulation of continuous-time system, the time-increment is taken as 0.01 seconds. The simulation results are shown in the figures 3 and 4. Furthermore the control signal $x(t)$ will be analytically constructed as follows:

$$x(t) = 1.2482u(t) - 0.3013u(t-2) + 0.0558u(t-3) - 0.0027u(t-4) \tag{15}$$

The control signal $x(t)$ is also depicted in the figure 4. In the example 2, the settling-time is deliberately set as 5 seconds. In the figure 3, it is seen that the system response $y(t)$ would be exactly constant after 5 seconds. This method would be useful when no uncertainty exists and the system parameters are some known and fixed values.
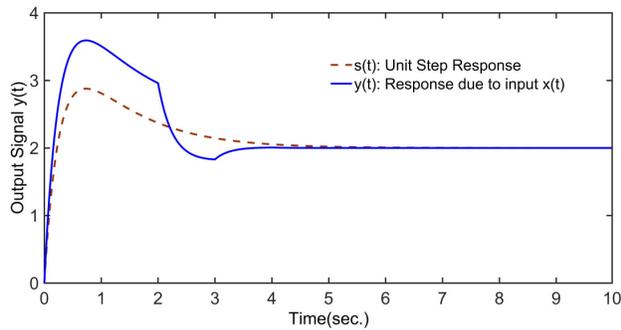
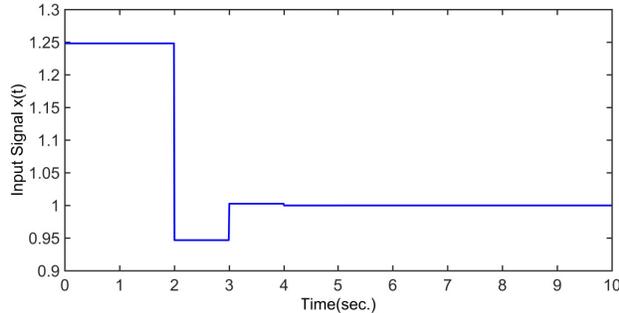**Fig. 3**. The dead-beat response $y(t)$ in the example 2

Fig. 4. The control signal $x(t)$ in the example 2

Next, the application of the proposed method is investigated in a typical mechanical system.

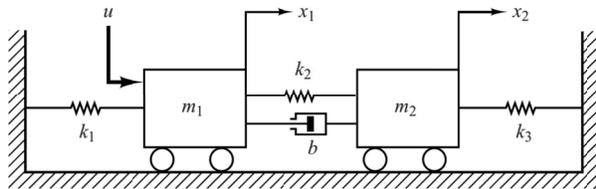**Example 3**: Consider the following mechanical system [1]:

Fig. 5. A typical mechanical system [1]

It can be shown that the transfer function between the control input $u(t)$ and the output $x_2(t)$ can be determined as follows [1]:

$$G(s) = \frac{X_2(s)}{U(s)} =$$

$$\frac{m_2 s^2 + bs + k_2 + k_3}{(m_1 s^2 + bs + k_1 + k_2)(m_2 s^2 + bs + k_2 + k_3) - (bs + k_2)^2}$$

The mechanical system parameters are selected as follows:
$m_1 = 1\ kg,\quad m_2 = 1\ kg,\quad k_1 = 5\ N/m$,
$k_2 = 1\ N/m,\quad k_3 = 1\ N/m,\quad b = 4\ N/m/sec$
Then the transfer function $G(s)$ would be found as:

$$G(s) = \frac{(s+3.4142)(s+0.5858)}{(s+7.327)(s+0.5029)(s^2 + 0.1698s + 2.985)}$$

An application of the remark 8 may be useful in this example. For this purpose, the compensator block $G_c(s)$ is simply chosen as $G_c(s) = 2.985\,\dfrac{(s+7.327)(s+0.5029)}{(s+3.4142)(s+0.5858)}$. Then by using of the Posicast control law as addressed in the equation (9), the control signal can be constructed for the cascaded system $G(s)G_c(s)$. The system response $y(t)$ is shown in

the figure 6. It is seen that the proposed method provides a dead-beat response while the step response of the mechanical system has consistently oscillatory behavior.
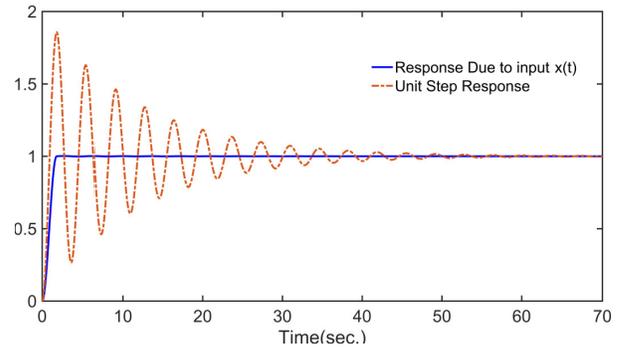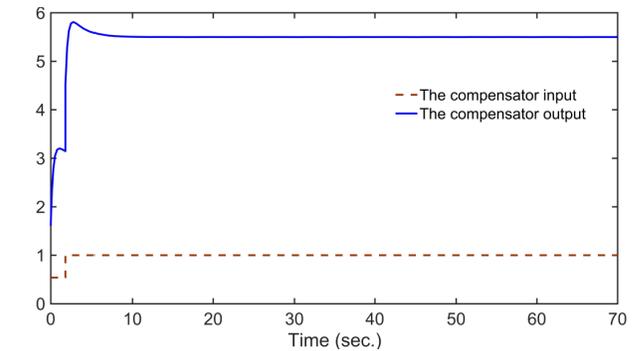

Fig. 6. Responses of the mechanical system

Fig. 7. The input and output of the compensator

The input and output of the compensator is shown in figure 7. The compensator output is applied to the mechanical system.

In these examples, the simulation results verify the effectiveness of the suggested finite-time control technique in the continuous-time systems.

V.   CONCLUSION

In the continuous-time LTI systems, a control signal can be selected such that the control goal would be met in a finite-time. The control synthesis problem may be converted to an algebraic matrix equation. Then the dead-beat control signal would be determined via solving the algebraic equation. The proposed control law is a model based method for designing of the dead-beat control policy. Hence it would be very efficient when a mathematical model exists for a dynamical system. The suggested approach is used in some control examples to show the effectiveness of the method.

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