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Sufficient Conditions for Stabilization of Interval Uncertain LTI Switched Systems with Unstable Subsystems

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ABSTRACT

This paper investigates the stabilization problem of an autonomous Linear Time Invariant (LTI) switched system with interval uncertainty and unstable subsystems. It is proved that the system would be stable by using a common Lyapanov function whose derivative is negative and bounded by a quadratic function within activation regions of each subsystem. First, a sufficient condition for the stability of an LTI switched system with interval uncertainty, based on the convex analysis and interval set theoretical approach, is presented and proved. Moreover, conservatism in the stability robustness bound is obtained. Then, a switching control law is designed to shift the LTI switched system among subsystems to ensure the decrease of the Lyapanov function within the state space. Finally, in order to decrease the switching frequency and to avoid chattering, the switching law is modified. Two examples are included to demonstrate the effectiveness of the theoretical findings.

Article Info

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Chattering, Interval Uncertainty, Lyapanov function, Stability, Switched System, Unstable Subsystems

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I. Introduction

Once In recent years, hybrid systems have seized highly increased attention. A hybrid theorem can be used to completely describe models of some systems, including bouncing ball, ON/OFF switching systems, or systems with conditional rules that cannot be defined by differential or difference equations[1, 2].

Switched systems are a specific class of hybrid systems with some subsystems and a switching law. The switching law can determine the active subsystem at each moment [3]. The subsystems can be divided into two types: autonomous systems and controlled (by input) systems. Autonomous switched systems arising from nonlinear systems, large-scale uncertain systems, and parameter-varying systems, have been investigated by studying of internal switching features [4, 5]. There are several constructive propositions in the literature to stabilize classes of linear switched systems without input signals [5].

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Stability and stabilization is fundamental issue in control systems. Consequently, One of the significant issues in the stabilizing of switched systems is to acquire necessary and sufficient conditions for the stabilization of the system under an arbitrary switching law [5, 6] And design of a stabilizing switching law [7]. In this paper, the method used to study the stability of switched systems utilizes a Lyapanov function whose derivative is negative and restricted in the region in which each subsystem is active (while can be positive outside of the active region). In switched systems' studies, this stability is often known as "quadratic stability"[8].

Extensive results concerning differential inclusions and stability problems of these systems can be found in[9-11]. The method presented by Wicks et al pointed out the existence a stable convex combination of linear subsystems, which denote the possibility of quadratic stabilization under a proper switching law [12]. Some switching law strategies have been rendered which restrain fast switching and chattering, as demonstrated by Wicks et al [7, 12].

In practice, the precise mathematical modeling of these systems seems to be difficult or even impossible. This is because of some reasons like excessive complexity, lack of enough information, the experimental errors, gradual changes of system parameters, neglected dynamics and so on. As a consequence, it is necessary to consider the uncertainties inherent within the analysis and modeling of systems. These uncertainties would be classified in different forms.

Z.Qiu et al demonstrated the conservatism in the stability robustness bound for state space models with interval uncertainty [13]. Y. Chen et al obtained the robust stability check for fractional order LTI systems with interval uncertainty [14] There are some results concerning stability and stabilization with Polytopic uncertainty in [15, 16] and norm-bounded uncertainty in [17]. Moreover, a switching law based on invariant space theory and an average dwell time (ADT) approach from Q. Yu and B. Wu is given in[18, 19].

In this paper, a sufficient condition for stabilizing of the integer order linear switched system with interval uncertainty, based on a convex combination technique and Lyapanov function, has been presented. We have introduced a conservatism criteria guarantying the robust stability. Next, a switching law associated with states is proposed that stabilizes the switched system with unstable subsystems. Finally, to prevent chattering, the stabilizing switching law has been improved.

The remainder of the paper is organized into several sections. Section 2 contains the problem description, and section 3 explores the sufficient conditions for stabilizing LTI switched systems. Then, a modified switching law is proposed to prevent chattering in section 4. Finally, the results are illustrated with two examples in section 5.

II. Problem Description

Consider an LTI switched system described by a state space equation as follows

$$\dot{x}(t) = (A_\sigma + E_\sigma)x(t), \tag{1}$$

Where $x(t)\epsilon R^n$ is the continuous state variable; $\dot{x}(t)$is a derivative of $x(t)$ relative to time; σ denotes a switching signal taking values as $\sigma = 1, 2, \dots, N$, within afinite set of matrices $\cap A$=:{A_σ: σ=1, 2,. . . N } and with an $E_\sigma = (e_{\sigma ij})$ interval uncertainty matrix, such that the interval constraint condition $-\Delta k_{mij} \le e_{mij} \le \Delta k_{mij}$, $i, j = 1, 2, \dots, n$, $m = 1, 2, \dots, N$,where Δk_{mij} are the known positive constants and $K_m = (\Delta k_{mij})$ is the matrix of the maximum interval constraint of E_m.

In interval model uncertainty, eigenvalues arenot fixed points in a complex plane; instead, they are a cluster of infinite points [14]. This paper presents sufficient conditions for stabilization of LTI switched systems with unstable subsystems and interval structured uncertainties. Then, a stabilizer switching law is achieved based on the convex analysis and interval set theoretical approach.

III. Sufficient Conditions for Stabilization of an LTI Switched System

The primary target of this section is to construct a continuous Lyapanov function for the switched system whose derivative is negative along each state trajectory. These regions cover the entire state space, and boundaries of each region are determined by the switching law. In this section, a sufficient condition is proposed to stabilize the case with two subsystems, and next, the condition is generalized to the case with N subsystems.

Theorem 1 [8]. The LTI switched system (1) without uncertainty is stabilizable if there is a stable convex combination of $A_k, k\epsilon N$ as follows

$$A_0 = \alpha_1 A_1 + \alpha_2 A_2 + \dots + \alpha_N A_N = \sum \alpha_i A_i, \quad \alpha_i \epsilon (0,1), \sum \alpha_i = 1, i = 1, 2, \dots, N. \tag{2}$$

The switched system is stable provided that all eigenvalues of matrix A_0 are located on the left side of the jw-axis.

Theorem 2. The LTI switched system (1) with interval parameter uncertainty and two subsystems is stabilizable if it holds the following conditions:

a) There is a stable convex combination such as

$$A_0 = \alpha A_1 + (1-\alpha)A_2, \quad 0 \le \alpha \le 1. \tag{3}$$

b) For the robustness bound of subsystems with interval structured parameters, using the interval set theoretical approach, we can obtain the following equation:

$$J_m = \sum_{i,j=1}^{n} (\Delta k_{mij} S_{max}(P_{ij})) < 1 \quad , \qquad m = 1,2, \tag{4}$$

where $P_{ij} = 0.5({A_{ij}}^T P + PA_{ij})$; the positive definite matrix P exists as the solution of the Lyapanov equation (5); A_{ij} is an $n \times n$- dimensional matrix with 1 in the ith and jth positions and 0 elsewhere. I is the identity matrix and $S_{max}(B)$ represents the largest singular value of matrix B.

$$A_0^T P + PA_0 + 2I = 0. \tag{5}$$

Proof. Using (3) we can rewrite (5)

$$P[\alpha A_1 + (1-\alpha)A_2] + [\alpha A_1^T + (1-\alpha)A_2^T]P + 2I = 0,$$

$$\alpha[PA_1 + A_1^T P + 2I] + (1-\alpha)[PA_2 + A_2^T P + 2I] = 0,$$

$$x^T\{\alpha[PA_1 + A_1^T P + 2I] + (1-\alpha)[PA_2 + A_2^T P + 2I]\}x = 0. \tag{6}$$

Or equivalently

$$x^T\{\alpha[PA_1 + A_1^T P + 2I]\}x + x^T\{(1-\alpha)[PA_2 + A_2^T P + 2I]\}x = 0. \tag{7}$$

In (7), at least one of terms is negative or both of them are equal to zero.

$$x^T[PA_1 + A_1^T P + 2I]x \le 0, \quad (8)$$

Or

$$x^T[PA_2 + A_2^T P + 2I]x \le 0. \quad (9)$$

Q_1 and Q_2 are defined as

$$Q_1 := PA_1 + A_1^T P,$$
$$Q_2 := PA_2 + A_2^T P. \quad (10)$$

Let $v(t) = x^T P x > 0$ be the Lyapanov function for the linear switched system (1), where P is given by (5)

$$\dot{v} = \dot{x}^T P x + x^T P \dot{x} = x^T({A_\sigma}^T P + {E_\sigma}^T P + PA_\sigma + PE_\sigma)x$$
$$\le x^T(-2I + {E_\sigma}^T P + PE_\sigma)x \le 2x^T(\textstyle\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij} - I)x. \quad (11)$$

If (11) is negative, the system (1) will be stable, as

$$S_{max}(\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij}) < 1. \quad (12)$$

Considering the triangular inequality of the largest singular value norm, we have that

$$S_{max}(\textstyle\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij}) \le$$
$$\textstyle\sum_{i,j=1}^{n} |e_{\sigma ij}| S_{max}(P_{ij}) \le$$
$$\textstyle\sum_{i,j=1}^{n} (\Delta k_{\sigma ij} S_{max}(P_{ij})) = J_\sigma. \quad (13)$$

It is evident that, if the inequality (4) holds, then the inequality (12) also holdsbased on the inequality (13).

Now regarding the above sufficient conditions,the switching law can be designated

$$\sigma = \text{argmin}\,\{x^T Q_1 x, x^T Q_2 x\}, \quad (14)$$

Where argmin stands for the index which attains the minimum index.

IV. Modified Switching Law

Since the chattering through infinitely fast switching may harm equipment in real world, to constrain the chattering phenomenon, the switching signal must be well-defined. [8]. If $(t_{i+1} - t_i) \ge \tau$ for each two successive switching times t_{i+1} & t_i, then a determinant switching signal could be considered with dwell time τ. It is apparent that each switching signal with positive dwell time is well-defined. By using two matrices Q_1 and Q_2 determined in (10) the switching signal $\sigma(t)$ is defined as

$$\sigma(t_0) = \text{argmin}\{{x_0}^T Q_1 x_0, {x_0}^T Q_2 x_0\}. \quad (15)$$

The first switching time instant is given by

$$t_1 = \inf\{t > t_0 : x^T(t) Q_{\sigma(t_0)} x(t) > -r_{\sigma(t_0)}(t_0) x^T(t) x(t)\}. \quad (16)$$

where $r_i \epsilon (0,2), i = 1,2$. If the set is empty, then let $t_1 = \infty$; otherwise define the switching index as

$$\sigma(t_1) = \text{argmin}\{x^T(t_1) Q_1 x(t_1), x^T(t_1) Q_2 x(t_1)\}. \quad (17)$$

Finally, we define the switching time/index sequences recursively by

$$t_{k+1} = \inf\{t > t_k : x^T(t) Q_{\sigma(t_k)} x(t) > -r_{\sigma(t_k)} x^T x\},$$
$$\sigma(t_{k+1}) = \text{argmin}\{x^T(t_{k+1}) Q_1 x(t_{k+1}), x^T(t_{k+1}) Q_2 x(t_{k+1})\}, \quad (18)$$
$$k = 1,2, \ldots.$$

Theorem 3. Under the above switching law, available stable convex combination (condition “a” in theorem (2) and following conditions, system (1) is well-posed and quadratic stable.

$$J_m = (\frac{2}{r_m}) \sum_{i,j=1}^{n} (\Delta k_{mij} S_{max}(P_{ij})) < 1, \quad m = 1,2. \quad (19)$$

Proof. The theorem for switched systems without uncertainty was proposed from Wicks et al in [7, 12]. This theorem has been proven with certain variations, in accordance with the systems’ uncertainties within this paper. We first proved the quadratic stability of the switched system; now, let us consider the Lyapanov function candidate $v(t) = x^T P x > 0$. Its derivative along the system trajectory is

$$\dot{v} = \dot{x}^T P x + x^T P \dot{x} = x^T({A_\sigma}^T P + {E_\sigma}^T P + PA_\sigma + PE_\sigma)x$$
$$\le x^T(-r_\sigma I + {E_\sigma}^T P + PE_\sigma)x \le 2x^T\left(\textstyle\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij} - \frac{r_\sigma}{2}\right)x. \quad (20)$$

If (20) is negative, the system will be stable, as

$$S_{max}(\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij}) < (\frac{r_\sigma}{2}). \quad (21)$$

Or equivalently

$$(\frac{2}{r_\sigma}) S_{max}(\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij}) < 1. \quad (22)$$

Similar to (13), we have that

$$(\tfrac{2}{r_\sigma}) S_{max}(\textstyle\sum_{i,j=1}^{n} e_{\sigma ij} P_{ij}) \le$$
$$(\tfrac{2}{r_\sigma}) \textstyle\sum_{i,j=1}^{n} |e_{\sigma ij}| S_{max}(P_{ij}) \le$$
$$(\tfrac{2}{r_\sigma}) \textstyle\sum_{i,j=1}^{n} (\Delta K_{\sigma ij} S_{max}(P_{ij})) = J_\sigma. \quad (23)$$

Clearly, if $J_m < 1$, m=1,2. Then in terms of the inequality (23), the inequality (21) also holds.

$$J_m = (\frac{2}{r_m}) \sum_{i,j=1}^{n} (\Delta k_{mij} S_{max}(P_{ij})) < 1, \quad m = 1,2. \quad (24)$$

To prove the well-posedness switching signal, suppose that t_k and t_{k+1} are two consecutive switching time instants. Let

$i = \sigma(t_k+)$ and under switching law (18) we have

$$x^T(t_k)Q_i x(t_k) = \min\{x^T(t_k)Q_1 x(t_k), x^T(t_k)Q_2 x(t_k)\}, \quad (25)$$

And

$$x^T(t_{k+1})Q_i x(t_{k+1}) \geq -r_i x^T(t_{k+1})\, x(t_{k+1}). \quad (26)$$

As $\alpha Q_1 + (1-\alpha)Q_2 = -2I$, it follows from (25) that

$$x^T(t_k)Q_i x(t_k) \leq -2\, x^T(t_k)\, x(t_k). \quad (27)$$

Let ϑ be any real number greater than 1. First consider the case

$$\|x(t)\| \leq \vartheta\|x(t_{k+1})\|, \quad \forall t\epsilon[t_k, t_{k+1}]. \quad (28)$$

In this case, define a function

$$g(t) = x^T(t)(Q_i + 2I)x(t), \quad t\epsilon[t_k, t_{k+1}]. \quad (29)$$

It follows from (26) and (27) that

$$\begin{aligned} g(t_k) &= x^T(t_k)(Q_i + 2I)x(t_k) \leq 0, \\ g(t_{k+1}) &= x^T(t_{k+1})(Q_i + 2I)x(t_{k+1}) \\ &\geq (2 - r_i)x^T(t_{k+1})x(t_{k+1}) \geq 0. \end{aligned} \quad (30)$$

And

$$\frac{dg}{dt}(t) = x^T(t)\{(A_i + E_i)^T(Q_i + 2I) + (Q_i + 2I)(A_i + E_i)\}x(t). \quad (31)$$

Denote $\nu_i = \|(A_i + E_i)^T(Q_i + 2I) + (Q_i + 2I)(A_i + E_i)\|$. Through the inequality shown in (28), we have

$$\left|\frac{dg}{dt}(t)\right| \leq \vartheta^2 \nu_i x^T(t_{k+1})x(t_{k+1}) \quad \forall\, t\, \epsilon\, [t_k, t_{k+1}]. \quad (32)$$

By plotting a line between two points $g(t_k) \leq 0$ and $(t_{k+1}) \geq 0$, the line slope will be $(g(t_{k+1}) - g(t_k))/(t_{k+1} - t_k)$. It is apparent that the $g(t_k)$ could not reach the $g(t_{k+1})$ with the slope $g(t_{k+1})/(t_{k+1} - t_k)$. Therefore $dg(t)/dt$ must be more than the $g(t_{k+1})/(t_{k+1} - t_k)$ in some $t^*\epsilon[t_k, t_{k+1}]$. Since (32) is held for all $t\epsilon[t_k, t_{k+1}]$ we have

$$\begin{aligned} g(t_{k+1})/(t_{k+1} - t_k) &= (2 - r_i)x^T(t_{k+1})x(t_{k+1})/(t_{k+1} - t_k) \\ &\leq |dg(t^*)/dt| \\ &\leq \vartheta^2 \nu_i x^T(t_{k+1})x(t_{k+1}), \quad t^*\epsilon[t_k, t_{k+1}]. \\ &\vartheta^2 \nu_i (t_{k+1} - t_k) \geq (2 - r_i), \\ &(t_{k+1} - t_k) \geq \min_{\nu_i}(2 - r_i)/(\vartheta^2 \nu_i). \end{aligned} \quad (33)$$

As E_i is variable, ν_i will be variable too. Now suppose that (28) does not hold, meaning that there is a $t^*\epsilon[t_k, t_{k+1})$ satisfying

$$\|x(t^*)\| > \vartheta\|x(t_{k+1})\|. \quad (34)$$

From system equation (1), we have

$$x(t^*) = \exp\{(A_i + E_i)(t^* - t_{k+1})\}\, x(t_{k+1}). \quad (35)$$

Using (34) and the fact that

$$\|\exp\{(A_i + E_i)(t^* - t_{k+1})\}\| \leq \exp\{\|(A_i + E_i)\|(t_{k+1} - t^*)\}. \quad (36)$$

It is obtained that

$$(t_{k+1} - t_k) \geq (t_{k+1} - t^*) > \min_i\{\ln\vartheta/\|(A_i + E_i)\|\}. \quad (37)$$

Based on the above explanation, we have

$$(t_{k+1} - t_k) \geq \sup_{\vartheta>1}\min_i\min_{E_i}\left\{(2 - r_i)/(\vartheta^2 \nu_i), \frac{\ln\vartheta}{\|(A_i + E_i)\|}\right\}. \quad (38)$$

Since the switching signal has a positive dwell time, it could be concluded that, it will be well-defined and be equivalent to well-posed in linear switched systems[8]. So under the above switching law, the chattering phenomenon does not occur. If r_i is close to 2, the dwell time will decrease, but it can raise the resistance of the system against uncertainty.Conversely, if r_i is small (close to zero), the results will be reversed. When there are more than two subsystems, theorem 3 can be generalized to the case of N subsystems.

Theorem 4. System (1) under switching law (18) is stabilizable and well-posed, if it holds two following conditions:

a) There is a stable convex combination as

$$\begin{aligned} &A_0 = \alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_N A_N = \sum \alpha_i A_i, \\ &\alpha_i \epsilon(0,1), \ \sum \alpha_i = 1, i = 1, 2, \dots, N. \end{aligned} \quad (39)$$

b) All subsystems satisfy the equation below

$$J_m = \left(\frac{2}{r_m}\right)\sum_{i,j=1}^{n}(\Delta k_{mij} S_{max}(P_{ij})) < 1 \quad m\epsilon\,\{1, 2, \dots, N\}. \quad (40)$$

Proof. The proof of the theorem with minute modifications in the theorem 3 is straightforward.

V. Simulation Results

In this section, the results are illustrated using two examples.

Example 1. Consider the system given by (1), with following subsystems and uncertainty matrices

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, 2K_1 = \begin{bmatrix} 0.3 & 0.22 \\ 0.28 & 0.24 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, 2K_2 = \begin{bmatrix} 0.32 & 0.4 \\ 0.16 & 0.2 \end{bmatrix}. \quad (41)$$

Where eigenvalues of subsystems are

$$\lambda(A_1) = \lambda(A_2) = \{-2,1\}. \quad (42)$$

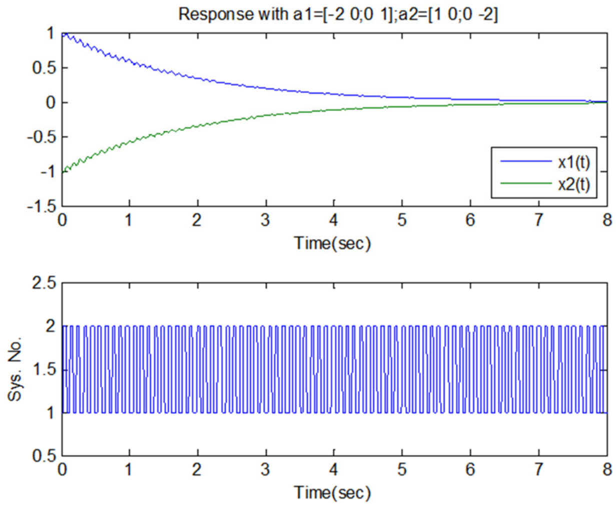

FIG. 1. The state trajectory and switching function between subsystems, response of example 1

It is clear that both of the subsystems are unstable. With positive coefficients of the convex combination $\alpha_1 = \alpha_1 = 0.5$, it is apparent that A_0 is stable.

$$A_0 = \alpha_1 A_1 + \alpha_2 A_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & -.5 \end{bmatrix}. \quad (43)$$

With $P = I_2, r_1 = r_2 = 0.8$ and computing of $J_1 = 0.9875$, and $J_2 = 0.9975$ the switched system under the proposed switching law is stabilizable. The state trajectory and switching function between subsystems have been shown in FIG. 1.

Example 2. Consider the system given by (1) with following subsystems and uncertainty matrix

$$A_1 = \begin{bmatrix} 0 & 2 & 0 \\ 4 & -12 & 2 \\ 6 & -16 & -6 \end{bmatrix}, 2K_1 = \begin{bmatrix} 0 & 0 & 0.16 \\ 0.04 & 0 & 0.2 \\ 0.2 & 0.2 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 2 & 0 \\ -12 & 0 & 2 \\ -10 & -4 & -6 \end{bmatrix}, 2K_2 = \begin{bmatrix} 0.04 & 0 & 0.18 \\ 0 & 0 & 0.16 \\ 0.2 & 0.2 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & -10 \\ 2 & -4 & -18 \end{bmatrix}, 2K_3 = \begin{bmatrix} 0 & 0.15 & 0 \\ 0.1 & 0 & 0.15 \\ 0.04 & 0 & 0.15 \end{bmatrix}. \quad (44)$$

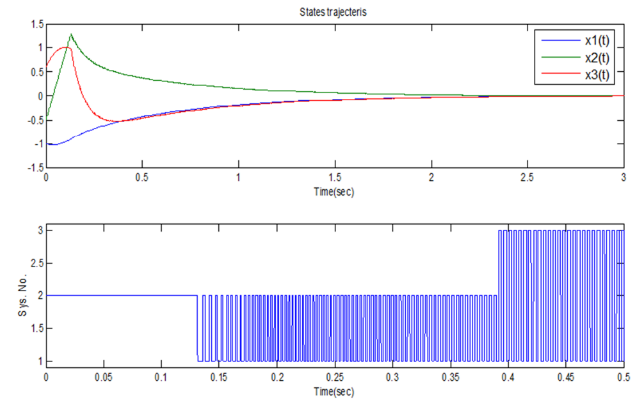

FIG. 2. The state trajectory and switching function between subsystems, response of example 2

Where eigenvalues of subsystems are

$$\lambda(A_1) = \{0.6642, -9.3321 + 4.6162i, -9.3321 - 4.6162i\},$$
$$\lambda(A_2) = \{-5.8798, -0.0601 + 5.5937i, -0.0601 - 5.5937i\}$$
$$\lambda(A_3) = \{-20.0901, 1.0451 + 0.9481i, 1.0451 - 0.9481i\} \quad (45)$$

According to the eigenvalues of the subsystems, all of the subsystems are unstable. With positive coefficients of the convex combination $\alpha_1 = 0.4434, \alpha_2 = 0.1182, \alpha_3 = 0.4384$. Obviously, A_0 is stable.

$$A_0 = \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 = \begin{bmatrix} 0 & 2 & 0 \\ 0.3552 & -5.3208 & -3.2608 \\ 2.3552 & -9.3208 & -11.2608 \end{bmatrix}. \quad (46)$$

With $P = \begin{bmatrix} 5.1789 & 3.3848 & -0.9351 \\ 3.3848 & 3.0596 & -0.9130 \\ -0.9351 & -0.9130 & 0.3532 \end{bmatrix}, r_1 = r_2 = r_2 = 1.8$ andcomputing of $J_1 = 0.9713$, $J_2 = 0.9871$ and $S_3 = 0.9535$ the switched system under the proposed switching law is stabilizable. The state trajectory and switching function between subsystems are shown in FIG. 2.

VI. Conclusion

In the present paper, we investigated the robust stability and stabilization of integer order linear switched systems with interval uncertainty. Firstly, we presented sufficient conditions for stability of such uncertain switched systems based on convex analysis and an interval set theoretical approach. Secondly, we designed a state-feedback switching law to ensure the decrease of the common Lyapanov function within the state space. Next, to decrease the switching frequency and chattering, the switching law wasmodified. Finally, simulation results demonstrated the main points of the proposed theoretical results.

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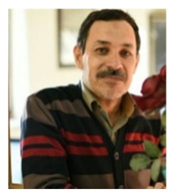

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